HYDROGRAPHIC SURVEYING.

HYDROGRAPHIC SURVEYING:

ELEMENTARY.

FOR BEGINNERS: SEAMEN AND OTHERS.

BY

COMMANDER STUART V. S. C. MESSUM,
RETIRED, ROYAL NAVY,
INSTRUCTOR IN ELEMENTARY HYDROGRAPHIC SURVEYING AT THE
ROYAL NAVAL COLLEGE, GREENWICH.

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PREFACE.

It is admitted at once, that Nautical Surveying, like Navigation, and in fact any other profession or trade, cannot, nor ever will, be learnt by book only: nothing that has ever been written will teach a beginner to use instruments: and, quoting from J. K. Laughton's excellent small work, "the taking of observations on shore, even with the sextant, does not come altogether by the light of nature; whilst choosing suitable points for 'fixing' or for 'shooting up,' be they lumps of wood or carrion thrown up by the tide, or a twig; the numerous little artifices necessary to catch the light, even the correctly and quickly reading a sextant, are all things to be learned, and can only be learnt, by actual practice."

The purport of this work is a general explanation of handling instruments, their theory, the theory and practice of Nautical Surveying, and the application of one to the other towards a practical end.

It is only a fully equipped surveying ship, commissioned for that purpose, which will ever undertake the survey of a coast; that part of Nautical Surveying is therefore not included.

As a rule, however, the specially commissioned ship surveys all those harbours which have any pretensions to a present or a near future commerce.

In the course of time many changes take place in harbours; in the depths, and in consequence in the buoys and lights; and the details of the surroundings are constantly increasing or altering.

There are still nooks and corners, even in the British Islands, now perhaps too unimportant for the cost entailed in the employment of a surveying ship; their outline has been but roughly and perhaps incorrectly sketched by an enthusiast.

This book, then, is intended for :—

1. The guidance and reference to all those whose duty is to intelligently and correctly report to the proper authorities the changes that are occurring in the harbours, rivers, etc., over which they have control.

2. Those who, interested, enthusiastic, or progressive, wish to carry out an original work.

There are still others, in out-of-the-way parts, not quite included in the above, who may have occasion to survey river sources opened up perhaps for concessions, etc.; or others again who may be called upon to report the changes in the depths or shoals in channels which are out of the direct jurisdiction of local harbour authorities: for these too this is written.

In the case of original work it must be borne in mind that, since the outline of no two surveys is exactly alike, each must be approached practically, in a manner modified by the conditions, and this is where practical experience is so necessary; but the principle underlying the whole process of any survey is exactly the same, viz. building up, on the most accurate data under the conditions.

Theory is: what are the most accurate data to employ?

Practice is: how to obtain the data and how to apply theory: one is of no use without a knowledge of the other.

With ever so much practice, it is still a quite common fault to take, and to construct on, data theoretically not sufficiently correct; therefore the result is unnecessarily erroneous.

This book is intended to supply a very long-felt want of, theory, the explanation of elementary practice with instruments, and their combined application to Nautical Surveying.

For more advanced study of the subject, the late Admiral Wharton contributed a very complete work, for the use of surveying ships; while the late Admiral P. Shortland's *Surveying* is advanced theory, approached purely from a mathematician's point of view. The reader is referred to both for additional assistance.

Acknowledgment is due to Lieutenant P. O. Griffiths, R.N.R., for his valuable assistance and suggestions.

No hydrographic survey is possible without a sextant. Observing that this is the case, considerable space is devoted to its description and to its errors, so as to serve as a guide to the

purchase of the most suitable instrument required, and give full information in its use.

The use of a station pointer for fixing is entered into as fully as possible, bearing in mind its all-importance.

A knowledge of plane trigonometry and of spherical trigonometry is presupposed.

The practical part is taken in sequence as new elements are introduced, and as the magnitude or nature of the survey develops; each example and each explanation should be studied in the consecutive order they are placed.

The chapter on Tides is sandwiched in after "Plotting"; and since it bears an intimate relation to "sounding," which in the ordinary course of events follows "plotting," "Tides" is the connecting link between the plotted triangulation and the "sounding" which follows.

As this work developed, it was found that the finale of any given survey introduced matter which could be no longer called elementary; it is therefore given in order to bring the survey to its proper conclusion.

STUART V. S. C. MESSUM.

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PART I.

THE SEXTANT.

CHAPTER I.

GENERAL.

1. **Introductory, and Definition.**—The sextant is so called because the extent of the arc is about $\frac{1}{6}$ of a circle.

A quadrant was about $\frac{1}{4}$ of a circle.

A quintant about $\frac{1}{5}$ of a circle.

The latter two are now obsolete.

An octant is about $\frac{1}{8}$ of a circle; its use is uncertain.

A sextant may be required for shore 'observations,' for sea 'sights' or general purposes, and for boat work.

There have been but few improvements made in the sextant since its original introduction.

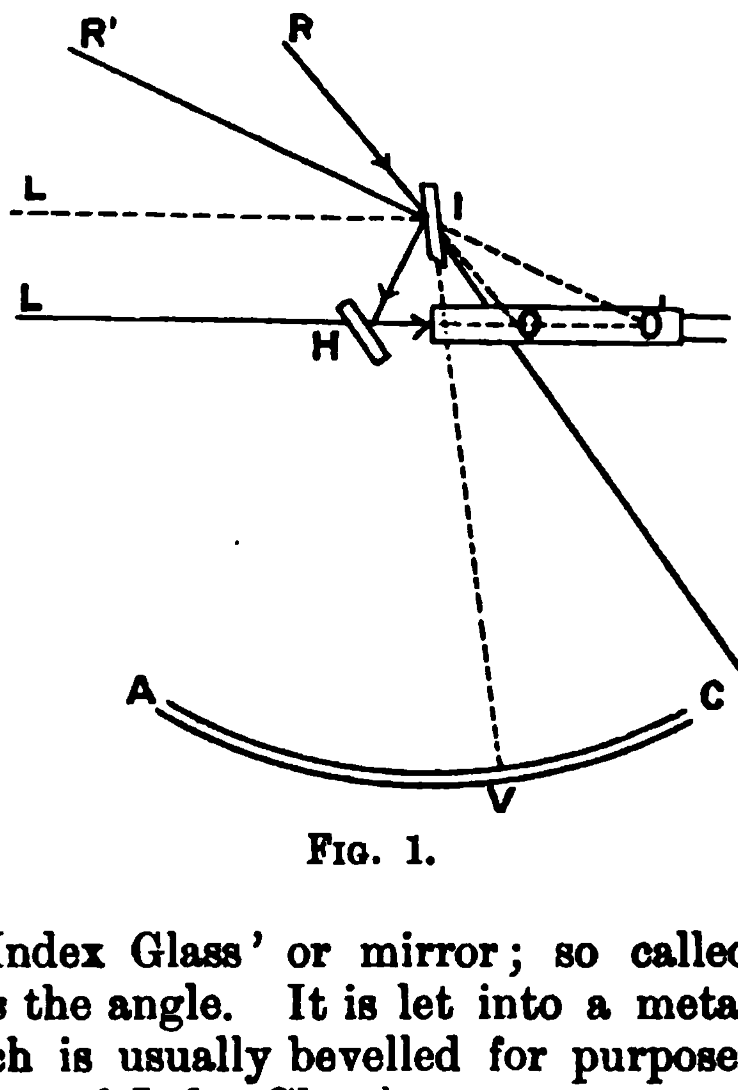


Fig. 1.

2. **Principle of Sextant.**—It is designed for measuring by reflection, through a system of mirrors, the angle between two objects in any plane.

I (fig. 1) represents the 'Index Glass' or mirror; so called because its movement *indicates* the angle. It is let into a metal frame, the right edge of which is usually bevelled for purposes of adjustment. (See *Adjustment of Index Glass.*)

'H is the 'Horizon Glass.'

In the sextants used for 'sights,' the lower half only is silvered; by being silvered in part only, the line of demarcation so produced does not obtrude in the line of vision when taking sights; but in those special sextants used for measuring angles only, *i.e.* for boat work, the whole glass is silvered; the interference of the top of the frame is not appreciable.

An image, R (R denoting right, because in measuring an angle, with the sextant the usual way up, it is the object R on the right that is reflected) is received into the mirror I.

By one of the laws of optics "the angle of reflection is equal to the angle of incidence": R is then reflected off at the mirror I at the same angle with the mirror that it impinged thereon (fig. 2).

Attached to I there is a radius bar I V, with an index at the end V; this index moves along the graduated arc A C.

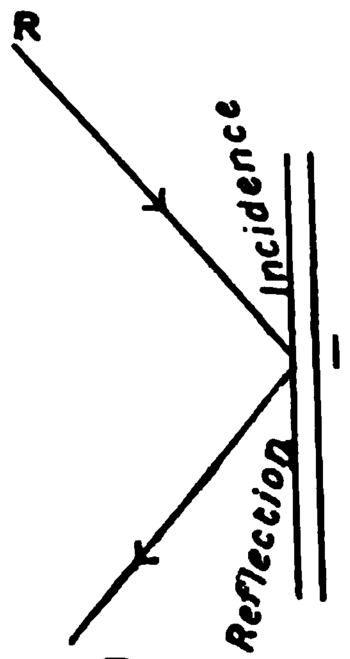


FIG. 2.

By moving this bar round, the angle of the mirror I is changed; and the change is so manipulated by the observer, that the reflected image of R is caught in the mirrored half of H, whence it is again reflected off H; the ray enters the line of the telescope, and is transmitted to the eye, and is there seen superposed on L, the left object, which is visible either through the clear part of the horizon glass, or just above it: the 'direct' object and the 'reflected' object are then in contact.

In practice, then, an observer with his eye along the line O L looks at the left object L, sees it either through the unsilvered portion of the glass H or over the top of H; by moving the radius bar I V he brings into the field of vision the reflection of R, seen through the silvered part of H; clamps the bar with the 'clamping screw,' and by a slow movement worked with a 'tangent' screw, brings R coincident with L: providing the 0° on the arc to be the position of V when H and I are parallel, then the reading at V is the angle at O, between R and L.

3. Place of Sextant when Observing.—By the diagram (fig. 1) it will be seen that if R I is produced to meet H O at O, then R O L is the angle at O between R and L.

Position of 'Receiving Angle.'—If R′ O′ L is another angle, between R′ and L, then the position of the point which 'receives' the angle is now O′. Therefore, for every angle observed the point of the 'receiving' angle is different; and theoretically the sextant would have to be adjusted in its position over a given spot, to accurately measure the angle between any two objects. This is not done in practice: either the index glass or the handle of the sextant is placed over the spot where the angle is taken from;

the error might be still further lessened if the collar of the telescope were placed over the spot; but whichever is used the error exists notwithstanding. (See Appendix I., 'Parallax' in a Sextant.)

4. The larger the angle, the nearer O will be to H; and conversely, the smaller the angle the further is O from H along the line H O.

5. Parallax in Sextant and Index Glass over Observation Spot.—Parallax in a sextant is the difference between the position of an object as seen from one point (H) and its position as seen from another point (I).

The measure of parallax is the angle at the object R, subtended by the distance H I.

When the distance to R is very great as compared with the length H I, the angle subtended at H being less than the least value obtainable on a sextant, viz. 10″, then parallax can be neglected, and the ray R H is considered parallel to and coincident with R I, the angle I R H being disregarded. When this is the case, the index glass (I), or in fact any part within the triangle H I O, may be assumed to be the point of the 'receiving' angle. The condition does not arise when H is at a distance of less than two miles.

6. Parallax when Neglected. Impracticability of Measuring Absolutely three Angles of a Triangle with a Sextant.—Under any other conditions, the error introduced, by not placing the eye at the position of the receiving angle O, varies with the distance of R and the size of the angle measured.

The error is a maximum when the angle is 105°; and with such an angle at 100 yards the error is nearly 6½′. Beyond two miles the error is an inappreciable quantity in an angular measurement, as, for instance, when finding index error by the method indicated in par. 66, and an angle is approximately correct if the index glass is placed over the point whence the angle is taken; within two miles the amount is appreciable, depending upon the size of the angle; and a sextant cannot practically be adopted for the purpose of measuring a correct angle on account of this 'parallax'; and three angles of a triangle the length of whose sides are less than 1000 yards (roughly half a mile), each angle being to the nearest minute, cannot be observed with a sextant so that their sum shall equal 180°; for further investigation see Appendix I.

7. Value of Graduations.—Though the arc of a sextant is only equal to that of a sector of a circle of about 60° the angle measured on it is twice that amount; for, as the radius bar is moved through 10°, both the reflected and incident rays have been each moved through 10° in opposite directions, travelling altogether 20°; then, to show that the movement of the angle is 20°, the gradua-

tions on the arc are doubled in value: or here is the usually accepted proof:—

Referring to fig. 3—

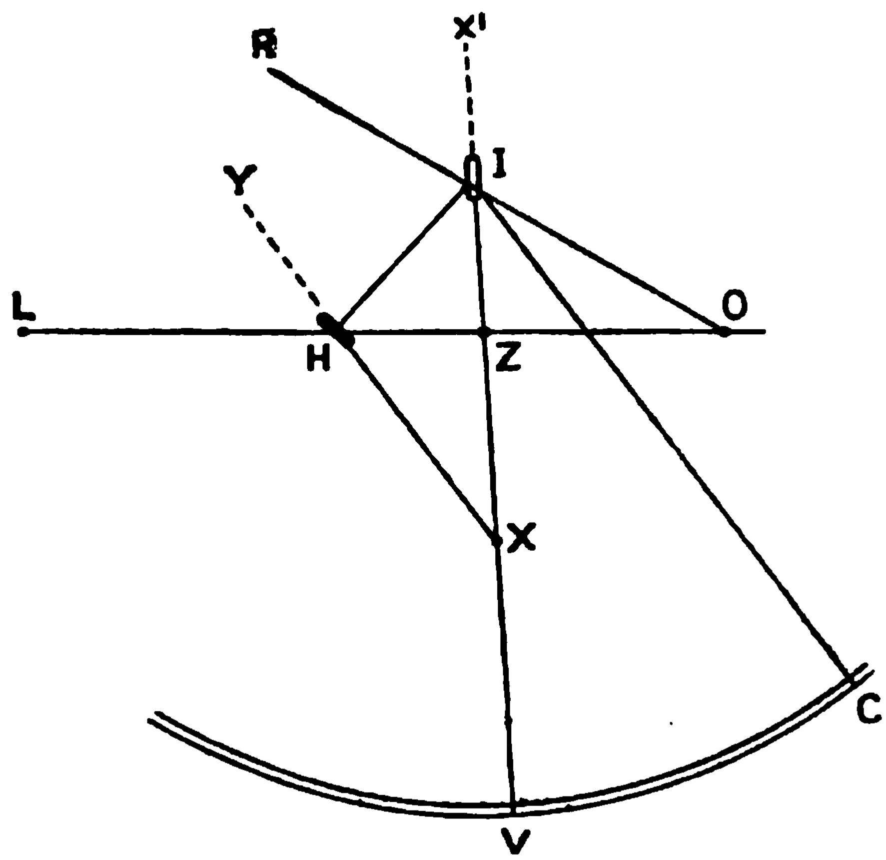


FIG. 3.

R O L is the angle to be measured; V I C is the angle moved through by the index glass and given on the arc.

Draw H X parallel to I C and produce it to Y.

$$\begin{aligned}
HZI &= HOI + ZIO \\
ZIO &= RIX' = ZIH \\
\therefore\ HZI &= HOI + ZIH \\
\text{also } HZI &= ZXH + ZHX \\
&= ZXH + IHY \\
&= ZXH + (HXI + HIX) \text{ or } ZXH + (ZXH + ZIH) \\
\therefore\ HOI + ZIH &= ZXH + ZXH + ZIH \\
\text{and } HOI &= 2\ ZXH\text{; but } ZXH = VIC \\
\therefore\ HOI \text{ or } ROL &= 2\ VIC
\end{aligned}$$

Showing that the angle observed H O I = twice that measured on the arc (2 V I C); therefore the graduations on the arc are doubled to show the correct angle.

Other Parts of the Sextant.

8. **Collar. Up and Down Piece.**—The ring with a screw holds the telescope, and is called the 'collar.' It is capable of being raised or lowered by the 'up and down' piece.

9. **Telescopes.**—Two telescopes are included with the sextant; the longer of the two is supplied with two 'eye-pieces,' one of higher magnifying power than the other: with either 'piece' it has a higher magnifying power than the shorter

telescope, and the objects seen through it are inverted; it has two pairs of cross wires. This long telescope is always used with the high-power 'eye-piece' for shore observations in connection with the artificial horizon, and generally afloat when taking 'sights,' with the lower-power 'eye-piece.'

10. Star Telescope.—It is customary to supply a star telescope, distinguishable from any other by the largeness of its object glass; this arrangement is for giving a brighter field so as to obtain a better and more distinct view of the sea horizon.

11. Attached Shades.—When taking shore observations the two images of the sun should be of equal brilliancy.

The shades fitted to a sextant are screens; the one set, placed between the index and the horizon glasses, is to shade the image reflected by the index glass; those in front of the horizon glass are to shade the sun seen directly through or over the glass.

12. 'Up and Down' Piece Regulating Brilliancy.—It often happens that owing to the bad silvering of the glasses, or to the insufficiency of the shades, or even to the quality and cleanli-

FIG. 4. FIG. 5.

ness of the mercury in the artificial horizon, the shades do not produce the equal brilliancy required; the 'up and down' piece is then resorted to, and the telescope raised or lowered.

Fig. 4 represents the appearance when the telescope is pointing midway between the mirror part of the glass and the unsilvered portion.

Fig. 5, the appearance when the telescope is raised, so as to get a clearer view through the upper half of the glass; this will also lessen the tone of the reflected image. In the converse, fig. 7, the reflected image, if the sun, will be brighter; if a terrestrial object, will be better defined.

Since the actual size of the reflected sun as it passes from the index glass to the horizon glass is only about $\frac{1}{8}$ in., the telescope may be raised considerably without the sun entirely disappearing from the field of view.

13. Raising Telescope for Star Observations. — When taking stars at night there is a difficulty in distinguishing the sea horizon; one aid is the larger object glass, and the second aid is to raise the telescope. (See fig. 6.)

Evidently, to be most efficient, the horizon glass should be the

size of the star telescope object glass, or the frame of the glass will interfere. (See fig. 6.)

In a great many sextants it is not so, and in such the advantage of the star telescope is not wholly effective.

14. Short Telescope for Measuring Angles or for Sea Sights.—There is also sometimes a short telescope, not inverting and of a *small magnifying power*, supplied in a sextant box. *It has no cross wires.* It may be used when great accuracy is not required in sea sights; and should always be used for measuring angles between terrestrial objects, for the objects observed are better defined and the position of the eye in the line of collimation is more definite. Also see par. 68 (note).

15. In measuring angles between two objects with a sextant held the right way up, the right object is reflected so as to coincide with the one on the left.

Distinctness Lost by Reflection.—In practice the right

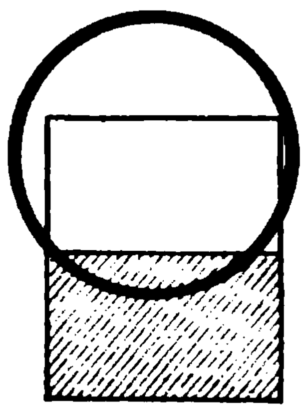


Fig. 6.

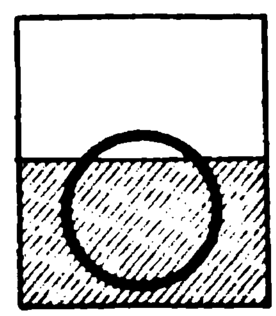


Fig. 7.

object is not always very distinct, and loses in detail when reflected; in this case the telescope will have to be lowered. (See fig. 7.)

Up and Down Piece in Taking Angles.—When the right object is too indistinct to be reflected, then the action of measuring an angle to it is reversed. The sextant is held handle up, and the left object is reflected to the right. It is a very unhandy way of obtaining the angle, and is not generally resorted to, though it is unavoidable in some conditions of lunar observations. A simple way is to reflect to each, another well-defined object which is to the right of both, and subtract the angles.

16. Tube Without Glasses.—The tube without glasses, sometimes supplied, is a substitute for the short telescope, and is for exactly the same purpose, though of course it is not quite so effective. It helps to concentrate the vision, and keeps the eye in its proper place at the sextant.

17. Coloured Eye-pieces.—The little coloured eye-pieces are substitutes for the shades in the sextant; these shades may be in error, and in the operation of taking shore observations both

sets of shades are used; any errors due to them are eliminated by using the coloured eye-pieces; the equal brilliancy being regulated by means of the up and down piece.

18. Arc and Vernier.—Finally there is the 'divided' arc, and the vernier for 'reading off' (fig. 8). The arc is of platinum graduated by machine into parts representing 10′ of arc. By the vernier, each of these 10′ on the arc is again subdivided into single minutes and tens of seconds.

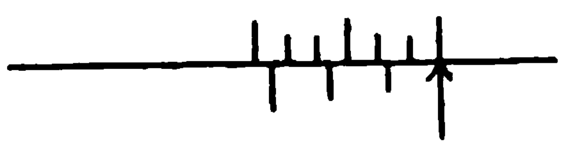


FIG. 8.

Vernier.—The result is obtained by making the space occupied by 119 divisions on the arc equal to the 60 divisions on the vernier, so that each vernier division then equals two arc divisions minus $\frac{1}{60}$ of an arc division (10′), and is consequently 10″ less.

For, suppose the 60 divisions of the vernier equal to 120 divisions on the arc, then each vernier division would coincide with two arc divisions; but since they are made equal to 119 arc divisions, then the vernier divisions will be $\frac{1}{120}$ of two arc divisions = 10″ behind; and, if the index of the vernier be set at a division of the arc, then the next vernier division will be 10″ behind two divisions further on on the arc; the next, 20″ behind the fourth division further on the arc, and so on; the 60th division, sixty times 10″ behind = 10′, or exactly one division of the arc. So that it has lost exactly one division in its whole measurement, and therefore equals 119 divisions of the arc.

If, now, the vernier is advanced so that its first division coincides with a division of the arc, the reading must have been advanced exactly 10″, and wherever the lines of the arc and vernier form a continuous line they are said to 'cut,' and that will be the reading in odd minutes and tens of seconds, beyond the tens of minutes which the index indicates on the arc. In coarsely divided sextants, the continuous line of the arc and vernier is not very evident, and notice should be taken of the fact at the time the instrument is purchased.

19. In the method adopted in 'sights' on shore, it is usual to 'clamp' or 'set' the index of the vernier at a given reading of an even 10′ on the arc. It is very necessary in such a case that the setting should be as accurate as possible; the vernier graduations to the right of the index are an aid to those on its left, to this end; for each division of *each* side of the index

should be equally distant from the arc division nearest to it; in fig. 9 the reading is set at 50° 10′.

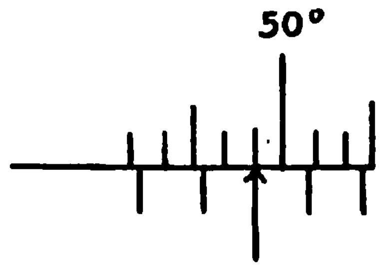


FIG. 9.

Arc of Excess.—These extra graduations to the right of the index are known as the 'arc of excess' on the vernier, and it is a very useful and necessary part of the instrument, and essential when properly understood.

20. Loss of Definition through Large Angles of Reflection.—The arc of the sextant is divided from 0 to usually about 150°, but too great dependence cannot be placed on angles beyond 130°, on account of the obliqueness of the ray from the images reflected. The loss of definition will depend upon the mirror, but it will not be noticeable if the angle between the front and back faces is less than 1″.

21. Arc of Excess on the Arc.—There is also an arc of excess on the arc; this is for measuring small angles up to about 5°, as will be explained later. If such an angle is measured both on the left of the 0 and on the right, the error of the position of the 0 is cancelled, and the mean of the two readings is the correct one; the difference between the mean reading and either one is the error of the index (see *Index Error*, par. 73).

If an angle measured to the left of the 0 reads 3° 30′, and measured to the right reads 3°, then the correct angle is 3° 15′, and the error in the position of the arc's index is minus 15′ to all readings made on the left, and plus 15′ to those made on the right.

22. Clamping and Tangent Screw of Vernier.—At the vernier end of the radius bar, and underneath, there is a screw for clamping the vernier to the arc. When the radius bar is moved, and a rough contact made between the reflected and direct image, the vernier is clamped. At the side of the vernier plate, tangential to the arc, is the 'tangent' screw; by its use a slow movement of the vernier is obtained, and the exact contact made.

23. Butting of Tangent Screw.—This tangent screw is only about 1 inch long; and at some period or another either end of it will butt, if constantly screwed in one direction; in which case it must be replaced in the centre or at either end of its run. When taking sights in an artificial horizon it is very

desirable to start the sights with the tangent screw at one end or another of its run, depending upon whether the altitude is increasing or decreasing.

In all 'screw motion' there is, unless counteracted, a 'back lash' known sometimes as 'lost time'; and as the threads wear, so will the 'back lash' be variable.

The effect of this is to modify the reading at the moment the tangent screw is let go, and its amount will vary in each thread; the error may be as much as 20″.

In the better class of sextants there is a counteracting spring, at the end of the screw, whereby a 'back lash' is practically eliminated.

Note.—There is in the market an improved movement for self clamping, with an endless tangent screw; obviating the double necessity of shifting the position of the hand to clamp, and of afterwards adjusting by the tangent screw.

CHAPTER II.

EFFICIENCY AND ERRORS.

THE efficiency of a sextant will depend upon its mechanical construction, both of the materials and putting them together; secondly, upon its being 'in adjustment'; and the errors in any observation will be made up of both of these combined with personal error.

24. Materials of Construction.—Most sextants are made of metal, and are cast; built frames are now rare and always dependable; but unless the metal is good, the instrument is liable to modify its shape, or not stand fair wear and tear in its parts.

25. Dividing and Centring Error, when Constructed.—The arc is divided by machinery, and each maker therefore has an individual error, owing to a faulty position of the pivot or centre from which the arc is cut. It is not, in fact, the centre of the circle of which the arc is a part of the circumference. Again, the divisions are not always 'finely' cut, and the reading therefore is ambiguous to within 20″ or even 30″.

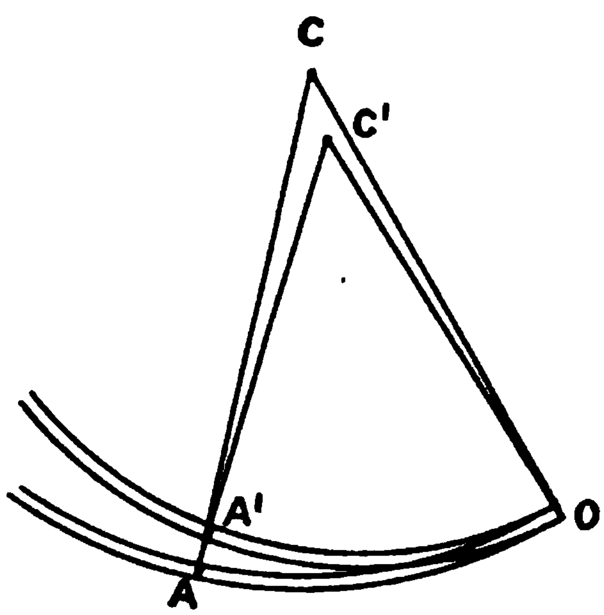


FIG. 10.

26. Centring Error. — When the radius bar that holds the vernier is placed in position its pivot will not coincide with the position of the pivot from which the arc is cut—it never will do so exactly. This is known as centring error.

If in fig. 10 C is the pivot from which the arc A O = 48° and C′ is the pivot of the vernier bar C′ A, known as the radius bar, then if an angle O C′ A′ is measured with the sextant it will be read off as O C A on arc O A, instead of on arc O A′. In this case the angle read off is too small. C′ may be anywhere. It is

probable that every sextant has, or will have in the course of time, centring error.

27. Effect of Centring Error in the Vernier.—One effect of centring error is that the vernier, if of correct size and correct fit at the 0° end of the arc, will cease to be correct as it is moved round; it will, in fact, be pushed or pulled out of place, according to the error of position of C′, and consequently will be too wide or too narrow, and the *value* of the divisions on the vernier will no longer be what they were at the 0° of the arc.

28. 'Play' in the Radius Bar.—All sextant makers make allowance for this displacement by a little 'play' on the length of the radius bar, where it falls over outside the arc.

29. Error of Planes, of Pivot, and of Arc.—There is also an error in connection with the pivot. When its vertical plane is not at right angles to the plane of the arc, this will give a twist to the radius bar; consequently, the vernier does not lie flat on the arc, and an error in the reading, due to parallax, is introduced. *This error can be produced by constantly lifting the sextant in and out of the box by means of the radius bar.*

30. Glasses.—In the putting together of the glass parts of a sextant, from a plate of ground glass the maker will select a small piece, as suitable for an index or a horizon glass; his only test in doing so will be to reflect on various parts of the glass plate, at a very large angle, any suitable object close at hand. If the reflection gives a well-defined image, the piece is cut from the plate, silvered, and placed in position. As a matter of fact such a piece of glass is anything but perfect.

31. Prismatic Error of Glasses.—It will be noticed in practice when observing the sun with an artificial horizon, when the angle measured is usually very large, that the image of one of the suns will probably show a distinctly blurred limb, or even a double limb; this defect in its minutiæ will be undiscernible in a shop where the instrument may be bought, or by almost any other test except by a star. The cause is due to the face and back not being parallel.

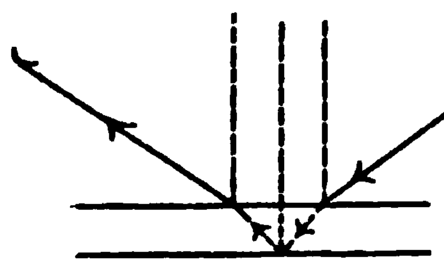


Fig. 11.

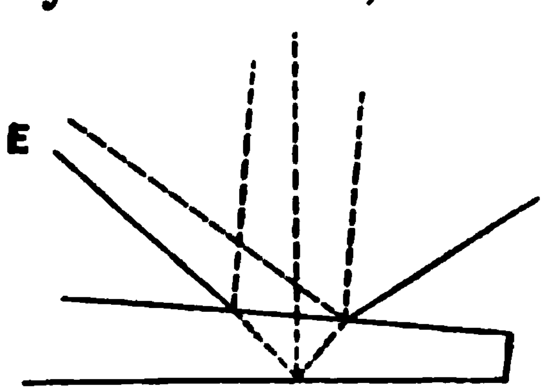


Fig. 12.

Fig. 11 is the correct path of a ray through a mirror, front and back being parallel.

Fig. 12 is the path through an imperfect mirror. The blurred image showing a double star is a combined image of both reflections, one part being from the upper surface of the glass, the other from the back.

If the figures be superposed, an observer in the position of E at fig. 12 would see the limb reflected from fig. 11 and that from fig. 12. This error will, of course, be very pronounced in a faulty index glass, when the angle observed is a large one, and the defect in its minutiæ can only be noticed in artificial horizon sights.

32. Prismatic Error at a Minimum in Horizon Glass.—A horizon glass similarly faulty shows no appreciable resulting error, because the angle of incidence and of reflection is very small: what there is will be absorbed in index error.

33. The ray coming from the index glass, and thence reflected to the eye, is a constant at the centre of the field; for any sextant the angle of incidence is generally about 15°, in fact half the angle at the centre of the horizon glass, between the centre of the index glass and the line of the telescope.

34. Convexity of Mirrors.—There is another error in the glasses, principally in the horizon glass, of frequent occurrence, due to the glass being 'squeezed.' The frame is made to hold the glass more or less rigidly, and the clips are supplementary grips; either through increase of temperature, the glass being irregularly heated, or owing to the adjusting screw being too tightly set up, the glasses may take up a convex or concave form, resulting in elliptical images of the body reflected. (See fig. 13.)

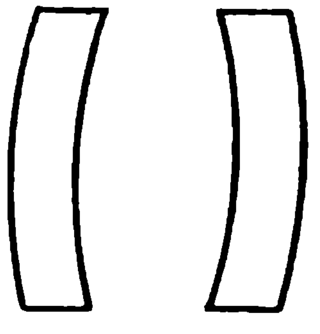


FIG. 13.

35. Cause of 'Squeezed' Glass.—The system by which a glass is adjusted by screws bearing directly on it, is conducive to such an error; this is, in fact, the case with the horizon glass. The only way of finding the existence of an elliptical image is by sun sights in an artificial horizon; for the moment of contact for either limb of the image reflected by the sextant with the image as seen directly from the mercury will be either too soon or too late. For further explanation, see remarks on Personal Error, par. 100.

Reason for Observations of both Limbs of the Sun.—The only way to eliminate the error is to take observations with both limbs of the sun reflected by the sextant.

36. As a rule, the error is not so gross as to be evident to the eye, but if it is, ease up all the adjusting screws.

If the frame is to blame, there is no present remedy.

37. Coloured Shades. Elimination of Errors by using Auxiliary Coloured Eye-pieces.—The shades also are liable to both of the above defects; they may be bent, *i.e.* convex or concave, and if their faces are not parallel the ray is bent out of its true course.

To find the error is too complicated for practical purposes. It exists; and in taking sights on shore, instead of the shades, the little coloured eye-piece is shipped on to the end of the telescope; this will eliminate the error.

The horizon-glass shades should, when in use, lie, not parallel with the horizon glass, but perpendicular to the line of the telescope; if they are not so, there is a liability of the introduction of 'ghosts.'

CHAPTER III.

ERRORS.

38. Kew Error.—It is essential that a sextant shall possess a Kew certificate. Usually the maker submits the new sextant, for testing, to the Kew Observatory. There, with the aid of a special instrument, Cook's testing apparatus, is tabulated for every 15° the amount of the error at the position where the index of the vernier registers any reading. It is, in fact, the difference between this reading and the absolute angle, and is therefore a total error. Kew does not investigate the contributory causes.

Kew Certificates.—Those sextants which show a maximum error of not more than 40″ are classed A; those of more than 2′, class B; and beyond that, the instrument is rejected. On an average one in eight is rejected.

39. Parts of Kew Error.—The Kew error is mostly made up of centring error; a combination of the makers' centring error, that is, of his marking machine, and of the error in the position of the pivot of the radius bar; theoretically the Kew error increases uniformly and varies directly with the size of the angle.

40. A purchaser should by no means be guided by the Kew certificate for a valuation of the instrument, beyond the fact that the glasses, vernier, arc and telescope have passed a fairly definite limit of test.

41. Effect of Centring Error on Vernier and Readings.—Kew error does not include the error of the misplaced vernier consequent on the centring error; and supposing the vernier at 120° 'cuts' at the 0 and 9, then the vernier divisions are $\frac{10}{9}$ths of what they are supposed to be.

Let the reading be 120° 4′ 30″; then $\frac{10}{9}$ths of 4′ 30″ = 5′; the correct angle is 120° 5′. If the Kew error is + 30″ then the true angle is 120° 5′ 30″.

42. The Kew error includes, within a definite limit, squeezed

and faulty glasses, and the residue of the error left after adjustments are made. All but the faulty glass are remediable and variable, and it is even possible to change the faulty glass.

43. Centring Error not Constant.—Centring error also will change in the course of time through wear and tear, depending probably on the material of construction, and on the workmanship—rigidity and durability, in fact. So that beyond the fact that a certain sextant has, at a particular moment, a total error as tabulated, the Kew certificate gives little assistance in valuing. It is no more than a qualifying certificate.

44. Selection of a Sextant.—In selecting a sextant, the first consideration is, What is it to be used for? If for boat purposes, or for sea sights, the requirements are not so rigid as for an 'observing' sextant: and a good observing sextant could not be purchased for less than £10.

45. Requirements in a Sextant. — For an observing sextant—

1. The glasses must be true, well silvered, and reflect very clearly.

2. The telescope should be good, with the maximum magnifying power for its length, and the object glass without focal error.

3. The arc and the vernier should be finely divided, so that the 'cut' appears unmistakably at one division only, and there should be an arc of excess on the vernier to assist in setting the index at any given reading.

4. The vernier must fit flat on the arc, at every portion of the arc.

5. The glasses must be firm in their frames, and the means for adjusting the horizon glass should not press on the glass; if a star telescope with a large object glass is a part of the fittings, then the horizon glass should be of corresponding size.

6. The clips holding the index glass should retain sufficient spring so that after being partly extended they will return to their original position. When the index glass is adjusted with the screw at the back of the glass, it often occurs in an inferior sextant that the clips are strained, and crippled as springs; the glass will then be quite loose.

7. The adjusting screws on the collar should be well clear of the flange of the telescope when shipped, so as to allow the collimation adjustment to be made without unscrewing the telescope.

8. The colour of shades is a matter of choice, those usually supplied satisfy most conditions; but see that when the index shades are put up they do not partly fall in front of the object glass of the telescope. The horizon shades must be at right angles to the telescope.

46. 'Observing' Value of a Sextant. — Its observing value can only be determined by observations.

47. Value of Error of Construction.—Supposing, then, a sextant class A with a Kew error at the date of its examination, which will exclude adjustment errors, to be 40″; this 40″ will be known as the total error due to construction.

NOTE.—A telescope is tested for focal error by standing the sextant on a table with the telescope shipped and pointing to any small, well-defined object. While twisting the telescope round, or revolving, by screwing and unscrewing the object glass, the object should remain in the focal centre; if it does not do so, the object glass is faulty.

CHAPTER IV.

ADJUSTMENT AND ERRORS.

THE next step is to adjust a sextant, and in the following order:

48. Order of Adjustment.—(1) The first and most important adjustment is to make the index glass perpendicular to the plane of the sextant.

Index Glass Error.—The radius bar is moved to a reading of about 40° on the arc; hold the sextant with the front of the index glass close to the eye, and the arc directly away from you in the position shown in fig. 14.

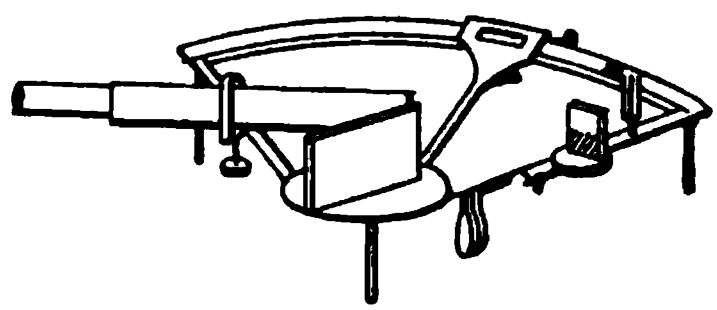


FIG. 14.

On looking into the index glass, in a line with the zero end of the arc will be seen the reflected image of the other end of the arc. With the sextant in this position the right edge of the index glass frame, it will be noticed, is bevelled off; and looking to the right of this bevelled edge, quite clear of it, the zero end of the arc will be seen.

Roughness of Method of Adjusting Index Glasses.—You must judge, then, whether that portion of the arc seen in the mirror is in the same continuous line as the part seen clear of it. It is essentially a matter of opinion, though it has been ascertained that an ordinary human eye can detect this error to within 2′ of the upright.

If these two parts are not in line, the index glass must be leaning one way or the other. There is a screw at the back of the frame on the upper half, and in, of course, the middle vertical line of the glass; with the aid of this, the glass is pushed or pulled upright, and the clips must follow this movement or they are useless.

49. Residue of Error after Adjustment of Index Glass.—If this adjustment is not made, or is made incorrectly, the

consequent error in the maximum angle (130°) is about $3\frac{1}{2}'' \times$ (minute of error in the inclination)^2. The error increases with the size of the angle, and always —. It is as well to note that on this adjustment all the others depend; but it does not require to be constantly made.

50. **Adjustment of Horizon Glass 'Side Error.'** — (2) The horizon glass has also to be perpendicular to the plane of the instrument. When it is out of adjustment, the sextant is said to have 'side error.'

51. To adjust it:—Place the radius bar with the index of the vernier reading about 30′ right or left of the 0°. Screw in the long telescope; and find a firm support for the sextant if possible.

Look at the sun through the telescope, with both the index and the horizon shades up, as found suitable. There will be two suns visible: the one seen directly through the clear part of the horizon glass; the other the reflected image of it, from the silvered part of the same glass; one sun will be seen abreast of or over the other, and nearly touching or overlapping it; if now the vernier tangent screw is worked, one sun will be made to move *over* the other; in this movement the upper and lower edges of the suns or right and left sides, technically known as their limbs, will coincide as they pass over each other. If, however, the limbs do not exactly superpose, the horizon glass is not upright; is, in fact, leaning forward or backward.

52. As in the case of the index glass, it must be pushed or pulled upright, by means of either a screw in the centre line which is in the upper half, so as not to press against the *silvered* portion of the glass, or else by canting the whole frame forward or back. In most modern sextants the screw device, working against the glass, is usually adopted; this, then, must be screwed up or back until the limbs of the two suns are seen to coincide.

Ineffectiveness of Adjustment.—As a matter of fact, in practice it is found that the operation is very hard to do with exact precision; a great deal of unsuccessful screwing is done, leaving a residual error; and, moreover, when made, it does not remain in that position; in fact, this adjustment seems constantly necessary. It is again a question of workmanship, of the clips, of the frame, and of the coarseness of the thread of the adjusting screw. The adjustment for side error is better made with a star, or even with a clear well-defined horizon line; in this last case the sextant must be held horizontally.

53. The adjusting screw at the back of the horizon glass is one of the two which, pressing on that glass, is a source of the 'squeezed' glass.

54. **Error in Index Glass contributes towards Side Error in Large Angles.**—When the vernier reads 0°, the resulting error in the angle due to an error in the adjustment

of the index glass is 0, consequently any error in that glass will not contribute towards the error in reading due to 'side error,' though it will do so as the angle increases.

NOTE.—Some makers retain the system by which, not the glass only, but the whole frame is adjusted for side error; the screw in this case is attached to the frame, and may be under the frame of the sextant.

55. These are the only two adjustments required to the glasses.

56. **Collimation Error.**—A sextant is not in an efficient condition until the axis of the telescope is parallel to the plane of the sextant.

57. **Telescope pointing to the Centre Line of Horizon Glass.**—In overhauling a sextant it is as well to see that the telescope points to the centre of the horizon glass, and that when the index shades are raised they do not screen the telescope.

58. When the instrument is put together, the collar which holds the telescope is placed at right angles to the arc by mechanical means; this is evidently not exact.

59. **Adjusting Collimation Error.**—After the long telescope is shipped, the following adjustment is necessary to make the longitudinal axis of the telescope parallel to the plane of the instrument; it is called "adjusting the line of collimation." It cannot be properly carried out without a 'sextant stand,' or some better means than the hand, to hold the instrument steady.

60. **Placing the Telescope Wires Horizontal.**—(1) Set the index at 0°, point the telescope to any star, with sextant held horizontally. If now the tangent screw is worked, there will appear both the direct and the reflected image of the *same* star in the field of view. With the tangent screw separate these two images by the diameter of the telescope, so that each star is just visible; turn the *eye-piece* of the telescope round, so that one of the images is at one end of the wire, and the other image at the other end; when this is done, the wires of the telescope will be parallel to the plane of the sextant.

(2) Select any two stars, and about 100° apart (the length of an average span of the hand with fingers extended from the tip of the thumb to the tip of the little finger, when the hand is held at arm's length, subtends at the eye an angle of about 20°). Make the contact between the stars, and place them both in the upper of the two wires in the telescope, as in fig. 15.

Suppose the reading of the angle to be 100° 48′. If the sextant now be ever so slightly canted, the images on being shifted to the lower wire will be found to have separated. Let the figure 15 represent their positions; again make contact on this wire; read off the angle, and suppose it to be 100° 44′. Now place the

mean reading on the sextant; this will be 100° 46′. By doing this, *half the error is removed by the tangent screw*, the appearance on the wires being as shown in fig. 16; and the telescope is in error, inclining upwards or downwards.

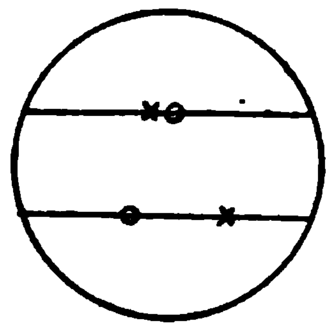


FIG. 15.

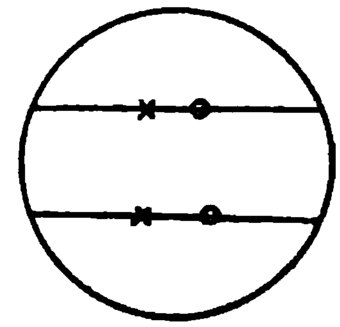


FIG. 16.

61. **Screws for Adjusting Collimation.**—On the *inner* surface of the collar is the screw holding the telescope; and this screw is also a sheath. The part of the collar holding the sheath is attached by two screws to a similar ring to the collar, and roughly parallel with it; and by the aid of the two screws holding these double parts of the collar together, the sheath can be perfectly adjusted to the plane of the sextant. Look at a sextant. Such an arrangement allows, of course, for only a very small adjustment; for two parallel plates held by screws can be moved from their parallel position only by a thread or two; and the greatest care is necessary that as one screw is tightened the other is slackened up exactly the same amount, or the sheath will not retain its shape, nor will the telescope screw into it.

62. It must be obvious, from what has already been explained, that, even supposing the sextant to be held in a vice or in a special stand, the operation is a very delicate one.

63. **Adjusting Screws Obstructed by Flange of the Telescope.**—To add to the trouble, sextants are invariably made so that the flange of the telescope, at its top, overlaps the two little adjusting screws on the collar; consequently, to make the adjustment the telescope has to be unshipped before any trial can be made, then screwed on again, then unscrewed, and so on until the observer is satisfied.

64. **Collimation Error always Existent.**—This constructional defect accentuates the fact that such a method of adjusting collimation error was not adopted by the maker; he was satisfied by a mechanical placement. That collimation error exists in all sextants there is hardly any doubt; and in the ordinary fair wear and tear in the life of a seaman's sextant the error is very liable either to be introduced or augmented.

65. **Additional Causes of Collimation Error.**—It has been remarked that occasions arise, especially at night when taking stars, loss of light necessitates the raising of the collar. When the sextant is replaced in the box and the lid does not close,

force is used, and the collar is bent; or, in the early years of a seaman's career, the sextant is sometimes carried by the telescope; this is a severe test for the material of construction, and collimation error results therefore to a greater or less degree. The operation of finding it and of adjusting it from time to time is in most cases almost impracticable.

66. Adjustment of Collimation Error Depending upon Correct Adjustment of Index and Horizon Glasses.—It should be noted here that by the above method of adjusting collimation error it is necessary that both the index and horizon glasses be themselves in perfect adjustment; and since there will be a residual error in those, under any conditions, there must be a residual error in collimation.

67. Adjusting Collimation Error in a Room.—Chauvenet gives a very simple method of roughly testing for collimation error, with rough appliances; with the advantage that it is independent of either the index or horizon glass. If two screws of the same length are stood on their heads, one at each end of the arc, the line joining their points will be parallel to the arc; and, looking along this line, a mark can be put on a piece of paper at the other end of the room to which the line directs. After screwing in the telescope, the collar is raised or lowered, so that the centre of the telescope shall be at the same height above the arc as the points of the screws are. Looking through the telescope, the optical centre should point exactly to the same spot marked in the paper as that indicated by the screw points. This can be done on a table in a room, with the distance of about 15 feet, and an adjustment made probably to within 10′. A small error in the height of the telescope is not very material.

68. For an Error of Collimation there is a Corresponding Error in the Observed-Angle.—The investigation for error in an angle due to an error in the line of collimation is beyond the scope of this manual.

The error at $130° = {\cdot}0006\theta^2$, where θ is the error of collimation in minutes; *e.g.*, for an error of $\theta = \frac{1}{2}° = 30' \theta^2 = 900'$ error in angle $= \frac{6 \times 900'}{10{,}000} = 32''$.

The error varies as the size of the angle, and is 0 at 0°, and increases uniformly; its sign is always minus.

Total Errors in Adjustment.—It should be noted here that, as previously shown, the index glass error varies directly as the size of the angle, and for an error in the glass of 3′ out of the perpendicular, the error in an angle of 130° is about 30″, *i.e.* $(3^2 \times 3\frac{1}{2}'')$; and the collimation error is of the same size and amounts to about 30″.

Residual Errors after Adjustments – 120″.—The residual error in an instrument which has been adjusted is a complicated

function of the two, and in the worst possible case may amount to $-120''$, and will be known as the error in adjustment.

The above are the only three adjustments necessary to a sextant.

NOTE.—The cross wires in a telescope are to guide the observer to the line of collimation. If an observation is made, with the sun or suns at either extreme of the visible wires, this practically introduces a separate error of collimation; and supposing the horizon glass to have a small residual error acting in the same direction as the collimation error introduced, the error of observation may be very large. For this reason it is not desirable to use the short, non-wired telescope for such observations.

CHAPTER V.

ADJUSTMENT AND ERRORS—*continued.*

69. A sextant, then, examined at Kew shows an error of, suppose, $+40''$ at $120°$; this is a total error; if the residual error of adjustment equals $-120''$, then the centring error of the instrument $=$ '$+2' 40$.'

70. The definition given by the glasses is judged within 'fairly definite limits,' but no test can be given for rigidity or durability; the author's experience in consequence of this is, that many B class sextants have been found more efficient for sight taking after a few years' wear and tear than those classed A.

71. Testing a Sextant.—The only way to value a sextant is by the results obtained with it; and although it may have a centring error of some magnitude, there are sometimes other compensating advantages; and in all observations calling for exact results, means are always adopted to *cancel* the total error; the more efficient, therefore, the parts giving the required definitions necessary to correctly observe and to clearly read off, the better will the results be.

72. Instrumental Errors Absorbed in 'Sea-sights.'—In ordinary sea observations it would be redundant to apply the total error due to errors of construction and of adjustment, if their maximum was only $2'$ at $130°$; for they are absorbed in other errors of far greater magnitude incidental to practical navigation.

A class B certificated sextant is here good enough.

For purposes of taking angles with the intention of projecting them, or for 'fixing' by, a reading correct to within $2'$ along any portion of the arc will suffice.

73. Index Error.—When the reflected image of any object is superposed on its direct image, and when it is then beyond the range of 'parallax,' the position of the index of the vernier on the arc is known as the 'Zero'; it is the point on the arc from which all angles are measured.

In other words, when the index glass and horizon glass are exactly parallel to each other the index of the vernier shows a specific reading on the arc.

But *change of temperature*, putting the sextant in its box, or taking it out, or putting it down not over gently, very slightly moves the glasses, however well they may be secured in their frames; and after such a change in their positions when the images are superposed, the index will not register the same reading as before, but some small amount on one side or the other of the 0° 0′.

In fact the zero position on the instrument will have shifted; and when the sextant is in constant use it is always shifting; the reading of the zero at the actual time of observation is known as the index error (I.E. for short).

74. Do not Adjust Index Error unless Necessary.—It is a very bad practice to reverse the order of things—that is, place the index at 0° 0′ and adjust the horizon glass parallel to the index glass.

75. This constant tinkering with the screws produces 'squeezed glass' (see par. 35), cripples the clips that hold the glass, and is altogether mischievous.

Some makers provide no screws for this practice.

76. Necessary to know the Zero of Readings.—It is obvious that the measurement of an angle cannot be of any value unless the reading of the zero is known; in other words, what is the I.E.? Just as, in fact, no distance can be measured without some zero, or starting-point, no matter what figure is given to it. If, then, the reflected image of some well-defined object beyond the range of parallax, say one mile, be superposed on the direct image; or in the case of the sea horizon if the reflected and direct horizon are made to form one line, the index and horizon glasses will be parallel, and the reading of the index will be the index error at that moment.

77. Sign + or − of the 'Index Error.'—If the reading is on the left of the 0° of the arc, the position of the 0° is too far to the right; and since most angles are measured to the left of the arc, the readings on that side will be too large by the amount of the I.E.; the I.E. is therefore −; conversely, if the reading is on the right of the 0°, the I.E. will be + to angles observed on the left.

78. Reading 'Off' and 'On' the Arc.—The left and right of the 0° on the arc are spoken of as 'on' and 'off' the arc.

79. Method of Finding I.E.—The above operation can be better carried out by using the sun, or a star, with the sextant in any position. In any case, 'side error' must first be removed. When, however, the suns are superposed their separate limbs are

not easily distinguishable, and a still better method is employed, viz., measuring the sun's diameter 'on' and 'off' the arc.

When superposed the angle is 0°, not necessarily the 0° on the arc.

If the right limb of the reflected sun is made to touch the left limb of the direct sun (see fig. 17), evidently it is the measure of one diameter. Suppose this to read 36′ 20″ '*off*' the arc (the sextant must be held horizontally). Now move the reflected sun across the direct sun, and by means of the tangent screw, make the left limb of the reflected touch the right of the direct sun; the reading now will be on the other side of the 0° mark on the arc, and will be a second measurement of the sun's diameter; suppose this to be 27′ 30″, '*on*' the arc.

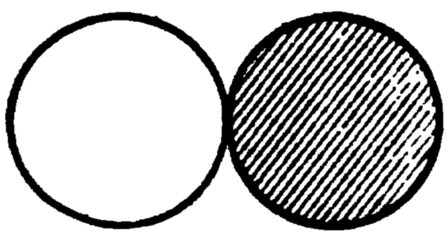


FIG. 17.

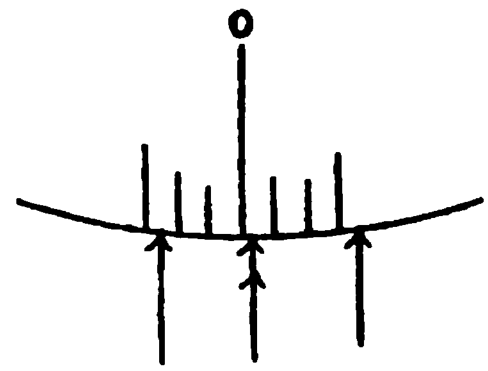


FIG. 18.

Since one measurement is 36′ 20″ and the other 27′ 30″ the mean reading must be the correct result of the observation; this will be half the sum, or 31′ 55″; the difference between this and the greater reading, or subtracting the smaller reading from it, will give the amount of the index error = + 4′ 25″.

The double arrow in fig. 18 is half way between the single arrows, each being the positions of the index for the measurements as above, and it must obviously be to the right of the 0° on the arc; showing that the 0° of the arc is 4′ 25″ to the left of where it should be, and that all angles measured 'on' the arc are 4′ 25″ too little. I.E. is then +.

In a simpler way, half the excess of one reading over the other is index error; $\frac{36' 20'' - 27' 30''}{2} = 4' 25''$; and if the greater reading is 'off' the arc, the sign is +. Conversely if 'on' the arc the I.E. is −.

80. Check to I.E. Measurement.—There is an additional advantage in this method; for since two measurements have been made of the sun's diameter, the mean of two observations, that is, half the sum, is one diameter; and quarter of the sum would be the sun's semi-diameter. In the *Nautical Almanac* the sun's semi-diameter is given for every day of the year; and if the observation is well made, the semi-diameter found should agree nearly with that given in the *Almanac*, providing the sextant is

fitted with a 'spring' tangent screw (see par. 23) and the horizon glass is not 'squeezed.'

81. **Residual Errors of Adjustment of I.E. ±10″.**—The residual error will be made up of squeezed glasses, side error, shade error, personal error, and the tangent screw, if without 'spring,' and should not be more than 10″.

82. **Readings 'Off' the Arc.**—In reading 'off' the arc, consider the 10 on the vernier as 0, 9 as 1, and so on, and read the vernier from left to right, instead of right to left.

83. Index error is constant for whatever angle is measured with a sextant, and forms part of the observation; without it, in an 'absolute' observation, *i.e.* the results of which depend absolutely on them, the sights are useless.

I.E. Always Found.—On every occasion of using the sextant the I.E. must be found, either by measuring the sun's diameter, or by a distant object, or the sea horizon; for sea observations a distant object or sea horizon is good enough.

84. It is, in theory, of no consequence what the size of the I.E. may be; but there are occasions in practice when it may be desirable to entirely neglect it, in which case it should be reduced to an inappreciable quantity, say 1′ or under.

85. **Occasions of Reducing I.E.**—Such an error would be disregarded when angles are taken for 'fixing' purposes, and usually a specially adopted type of instrument is used for such purpose; this, therefore, is the only occasion when it is advantageous to adjust the horizon glass and to remove or reduce the amount of the I.E.; and since there are no shades in the special sextant alluded to, the adjustment must be made either by the sea horizon or some distant well-defined object.

86. **Sounding Sextant.**—The instrument is known as the 'sounding' sextant; for the purpose intended it is more serviceable and handier in a boat than the ordinary sextant; usually does not rise to the dignity of a Kew error; has large mirrors; the telescope has a large object glass, and if it reads off to the nearest minute is accurate enough; it is smaller and lighter than the observing sextant, and costs much less.

CHAPTER VI.

RÉSUMÉ OF TOTAL ERRORS.

87. Total Errors of Construction.—In the error of construction there will be *assumed* an error of probably $\pm 30''$, made up of centring error, consequent vernier error, error in the glasses, shade error, and coarseness of the divisions.

88. Residual Errors in Adjustments and Total Error $\pm 1'\ 20''$.—In the residual error in the adjustments, probably $\pm 50''$; inextricably compounded error in index glass; collimation error, and that of side error included in the index error. This gives a total of $1'\ 20''$.

89. Incidental Errors $\pm 30''$.—There are, besides, errors incidental to any observations apart from personal errors. With artificial horizon sights there is the error through lack of magnifying power, and then there is the error due to the contact not being observed in the centre of the telescope (this is practically collimation error) or near the centre of the mercury in the artificial horizon; this last distorts the image; and there is, lastly, the error due to the glass in the roof of the artificial horizon; they may accumulate to approximately $\pm 30''$.

NOTE.—There is yet another error due to the force of attraction between large and small masses, owing to which the surface of the mercury may be attracted and incline towards such large masses.

90. Total Instrumental Errors in any Observation $\pm 1'\ 50''$.—The total errors in any angle may, then, if they are all of the same sign, amount to $\pm 1'\ 50''$; allowing some to be of opposite signs reduces the error to about $\pm 1'\ 30''$; whether $+$ or $-$ is not known, and hereafter will be alluded to as Instrumental Error.

CHAPTER VII.

ARTIFICIAL HORIZON AND PERSONAL ERRORS.

91. **Artificial Horizon.**—A quantity of mercury confined in any vessel will present a reflecting surface, which is perfectly horizontal if sufficiently remote from the action of attraction of large masses.

The receptacles for the mercury are made in all kinds of shapes and sizes, but taking the simplest form it is a rectangular iron vessel about $\frac{3}{4}$ in. deep. Into this sufficient mercury is poured from an iron bottle to well cover the bottom.

92. **Dross on Mercury.**—The first thing noticed will be that a dross appears on the surface of the mercury. Even the best mercury will tarnish, and motion increases its oxidisation; by wetting with saliva a small piece of paper and dragging it over the mercury the dross will stick to the paper.

93. **Cleaning Mercury.**—Mercury is easily cleaned by squeezing it through a piece of chamois leather.

94. **Capillary Attraction to Sides; Limit of Angle Observed owing to Height of Sides of Trough.**—It will also be observed that, through capillary attraction, the mercury on the edges of the trough will appear arched or rounded, and that unless a great deal of mercury is used the sides of the trough interfere with any image being reflected at an angle below about 30°.

95. **Shallow and Amalgamated Trough.**—To get over the last two difficulties—that is, the rounded edge and the high limit to the angles capable of being observed—a simple contrivance can be adopted, which can be made by any tinsmith; it consists of an oval, saucer-shaped receptacle of copper, of the same width and length as the iron trough, with the side $\frac{1}{4}$ in. high, and bottom flat. It is not advisable to make it rectangular or with corners, as the mercury eats holes in the corners; this tray will rest on the top of the iron trough, the weight of which is an advantage.

96. To Amalgamate a Trough for Mercury.—Before the tray can be used it must be 'amalgamated.' Take a solution of sulphuric acid, one part of acid to ten of water, and, wetting the tray with this solution, rub it well in; this will create a surface on the copper over which mercury will run freely. With a few drops of mercury rubbed on the surface the tray will be 'silvered'; then wipe in order to remove any mercury that may be left; leave a few hours, and it is ready for use. At first it should be rubbed with the residue of mercury after each time of using. When resting on the iron trough, pour sufficient mercury in the tray to cover the surface; this will be about $\frac{1}{20}$th in. deep, and there will be no rounded edges at the sides; in fact, the sides of the surface will be very slightly turned up.

97. Advantages of Shallow Amalgamated Trough.—There are several advantages in this form of artificial horizon. There is less mercury used, therefore less spilt; the observing surface is greater than it was in the iron trough; an altitude as low as about 12° can be observed in it; and it does not present the same effect of tremors as the deeper body of mercury, and moreover, the edges have a peculiar brilliancy, which when seen from the observer's eye through the telescope of the sextant act as a sort of frame to the sun seen in the mercury, and therefore assist the observer in keeping the sun in the middle of the surface.

All the patent devices in the market are intended to supply the advantages acquired by the shallow copper tray, and also reduce weight and improve the portability.

98. Roof of Artificial Horizon.—A roof large enough to rest on the ground is placed over the iron trough and the overlying copper tray (fig. 19). The glass is very carefully ground so that the surfaces shall be parallel.

Mode of Using.—Place the iron tray on the ground and turn it so that its shadow is in line with its length.

99. Pouring out the Mercury.—Then put the tray on the trough; unscrew the top arrangement of the iron bottle containing mercury; unscrew the inner stopper and put it aside.

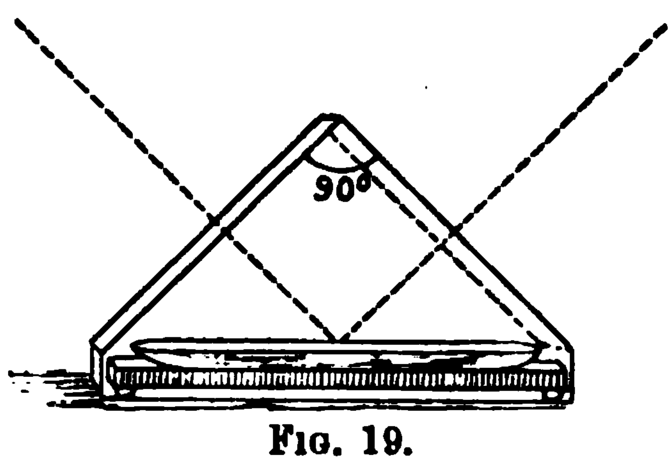


FIG. 19.

The top arrangement is a funnel. Place the finger over the orifice of the funnel; hold the bottle upside down, give it a shake; hold the bottle over the trough, and let the mercury flow into it.

To replace the mercury in the bottle, unscrew the funnel, reverse it, and screw the orifice on to the neck of the bottle; then

very carefully lift the trough, and pour. If held over the iron trough the spillings will be caught in it, and not be all lost. When there is sufficient mercury in the tray, about $\frac{1}{20}$th in., put the roof over; then finally, by the aid of the shadow it casts, turn trough and roof directly towards the sun.

100. Direction of Artificial Horizon.—It is essential to remember that the artificial horizon must be lengthways in the line of the sun, and to keep it so, must be constantly attended to. If this is not done, the ray from the sun strikes the glass of the roof at an oblique angle and exaggerates the distortion due to the faces of the glasses not being parallel.

101. Roof Level.—Also that it shall be as level as possible; if not, the effect is the same; for this end some artificial horizons are fitted with levelling screws, and the roof rests on the floor of the horizon.

102. Position of Artificial Horizon.—The spot selected should be free from tremors; *e.g.*, not too near a beach where there is a breaking sea. A passing train half a mile away will shake the mercury; the generating engine at Greenwich half a mile from the Observatory appreciably affects the surface of the mercury.

To be sheltered from the wind is equally advisable, both for the observer's sake and to avoid the resulting tremors; use a screen if necessary. Mosquitoes sometimes compel an observer to 'move on' from a spot otherwise well suited for observing. A stool and an assistant with a watch are the only accessories.

103. Observing in Artificial Horizon.—*How to Observe.*—Assume a sitting posture on the stool, and, with it, move until the sun reflected in the mercury is visible.

Screw in the long telescope, using the higher magnifying power tube, focus the eye-piece on the sun through the shades of the sextant, or through the auxiliary coloured shades.

Hold the sextant in such a way that each elbow rests on each knee; then looking along and over the top of the telescope through the horizon glass shade and through the clear part of the glass, will be seen the sun reflected by the mercury. At first this had better be done without the telescope.

Still holding it in this position, move the radius bar until the image of the sun reflected by the index glass comes into the field of view, and near the other sun; when near contact, clamp the arc; with only a slight simultaneous movement of the sextant and of the head, bring the eye to the telescope, and "voila" the two suns, either overlapping or separated.

104. Position of Observer and of Artificial Horizon in Relation to Sun.—Now the sun itself is moving in altitude and in azimuth; it is the latter movement that necessitates constant attention to the slue of the artificial horizon trough;

the movement in altitude necessitates a change in the distance from or towards the trough according as the sights are taken either P.M. or A.M.

In A.M. sights, for example, since the altitude is increasing, the observer will have to be constantly approaching the artificial horizon, so as to see the image reflected by the mercury; and in a northern latitude greater than the sun's maximum declination the sun will always change its azimuth from eastward through S.E. and S., then S.W. and westward, and the mercury trough will have to follow it round in that direction; the direction of this change in azimuth will vary as the latitude and declination.

105. Movement of Suns as seen in the Artificial Horizon.—Let H O, fig. 20, represent the sea horizon and M M′ the meridian of the observer, M the position of the sun in the meridian and at its maximum altitude.

At the instant of an A.M. sight the reflected image is 'brought down' to position A so that its lower limb touches the horizon. The time is taken and the altitude read off. Now if the sextant is untouched, the sun will be seen rising or moving away from the horizon to B and C and so on to M (see fig. 20).

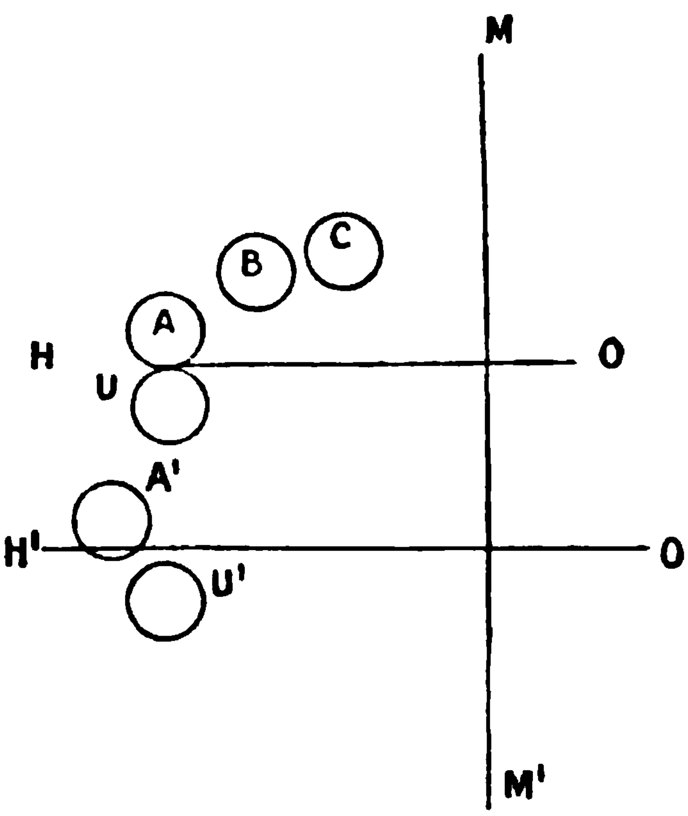


FIG. 20.

The lower limb is said then to be separating from the horizon; it is 'brought down' again, and at a certain instant of time, after the first observation, will have a larger altitude.

Now, in practice, the observation is made, not by 'making' or 'screwing up' to the contact (see Tangent Screw Errors, par. 23), but by overlapping the sun over the horizon and waiting, as it rises, till the contact appears; this is slightly more accurate than screwing up to it, because there is a certain amount of 'back lash' in the tangent screw, the effect of which is to displace the reading after the screw is let go. This is called 'advancing' the reading.

Movement A.M.—So then, when the sun is at position A′ on horizon, H′ O′ is the time to give the direction 'stand by,' and when it is like A on horizon H O, the observer says 'stop,' and the time is taken.

If, on the other hand, the upper limb is observed, since the

sun's motion is upwards in the forenoon, then U′ is the position of 'stand by,' and U the position of 'stop.'

Notice that the upper limb movement relative to the horizon is the opposite to that of the lower limb; whereas the lower limb is separating from, the upper limb is closing towards, the horizon.

Movement P.M.—In the afternoon the movement will of course be reversed, the sun taking the direction of the arrow (fig. 21); the lower limb will be closing towards, and the upper limb separating from, the horizon time.

Now substitute the artificial horizon for the sea horizon. The shaded sun, fig. 21, is the sun seen in the mercury, and is the artificial horizon.

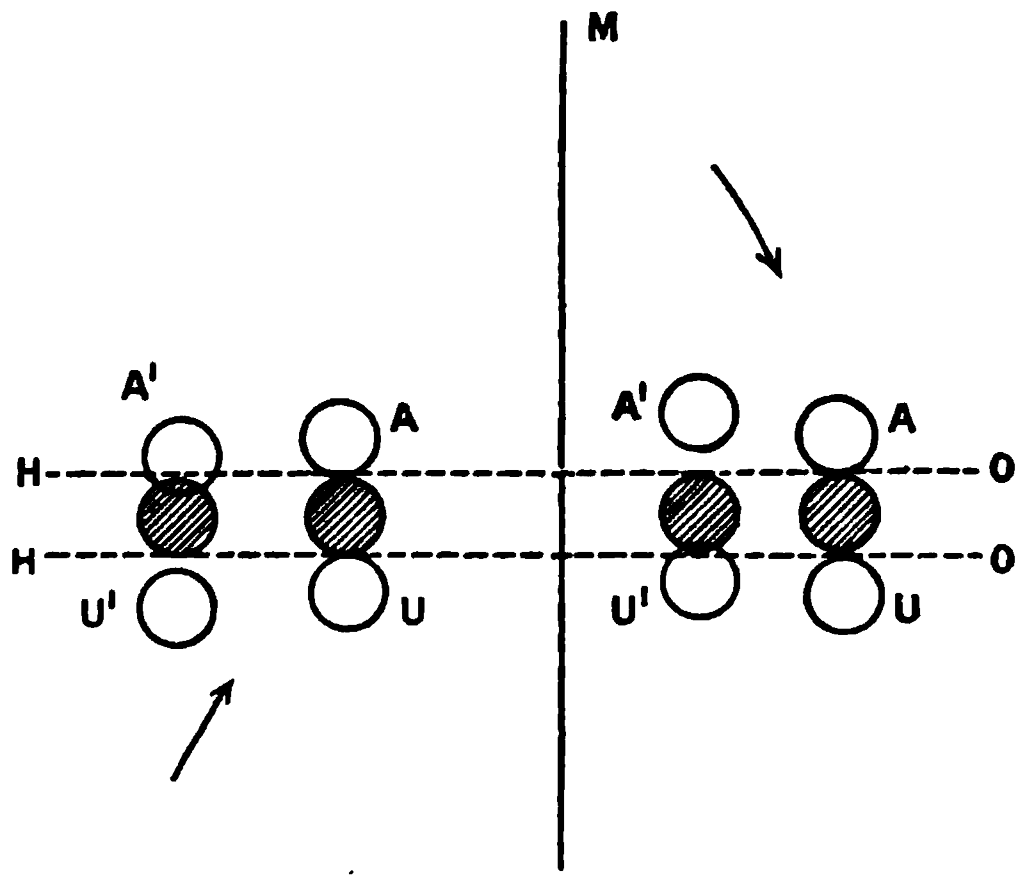


Fig. 21.

As before, the lower limb of the sun reflected down will overlap the artificial horizon, shown by A′ in fig. 21; the two will appear to separate, until A′ gets into position A; at this instant the observation is made: then, separating suns in the forenoon, must be the lower limb observed.

Upper limbs, on the other hand, will first appear as U′, then U, and evidently the suns will appear to be closing; the relative motions of A′ and U′ in the afternoon are, of course, reversed.

106. Personal Error.—An error of observation will arise through the indefiniteness of the sun's limb, and whatever the magnifying power of the telescope is, an error of this description will always remain; though of course the larger the magnifying power the less it will be.

But personal error is something separate from this.

One part of it may be termed defective judgment, either

permanently inherent, or defective through want of experience and practice; the other is a physical defect.

Imagine two balls of very fluffy wool placed in juxtaposition; standing away from them a few feet, it may not be easy to say whether they are touching or not. It is the same with the two suns seen through the telescope of a sextant; they appear to touch, and seem as if capillarily attracted; this phenomenon, called 'defraction,' is an unsolved difficulty in all observations (see fig. 22). Or, move the tips of the forefingers of each hand towards each other, and look at the space between them against a white background; it will be noticed that the sensation of touch occurs after the visible contact; with a shaky hand this is more involved.

If this error is made consistently, the resulting error is cancelled when observations are made on *both* sides of the meridian.

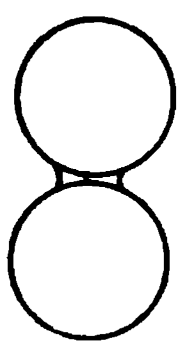


FIG. 22.

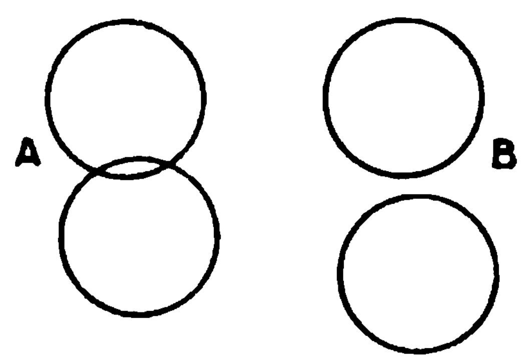


FIG. 23.

The second part of personal error has more to do with temperament; this may be permanent or merely transient; it is, in fact, either impatience, or "to-morrow-will-do": a personal equation.

In A.M. sights, supposing the lower limb to be observed, the suns are separating. If an observer is over-anxious or hurried he will say 'stop' when the suns are as in A (fig. 23). This will evidently be too soon, and the time from the meridian too great.

If sights are taken of the same limb, in the afternoon, with the same error, the suns will appear as at B (see fig. 23); here time and hour-angle are the same thing, and it is too early; therefore the time is too near the meridian, and the hour-angle is too small.

If the error is too far from the meridian in the first case, and too near the meridian in the other, the mean results shift the position of meridian; in other words, the results are in error.

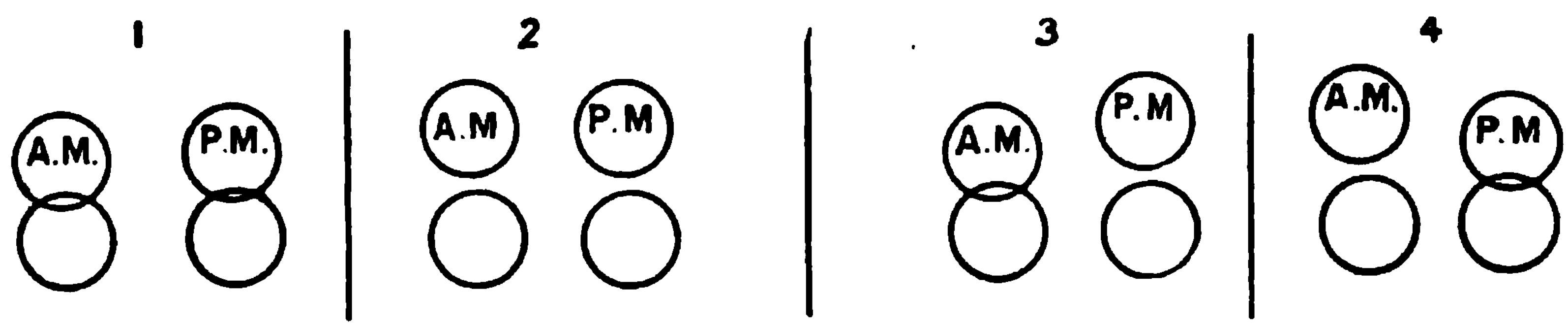
The results of taking the same limb both A.M. and P.M. too late produce the same resulting error; and whichever limb be used, will give the same erroneous results. Moreover, if one limb is taken, followed by a set of the other, this form of personal error will not be eliminated.

1 and 2 of fig. 24 is of the form first spoken of, more or less an accidental error; and, if consistent, the errors of A.M. and P.M. are cancelled; or it may be an inconsistent form of the

second kind of error, due to the error of separating and of closing motion being accepted differently.

3 and 4 of the same figure are personal equation error, and whichever limb or limbs be used, the errors accumulate if they are consistent. It is not possible to separate this error from the other errors, without the use of a special mechanical device for holding the sextant, such as a 'sextant stand'; the total error is probably $\pm 2'$, in the case of a fairly good observer, being reduced with practice and experience, and using a 'sextant stand,' to probably $\pm 20''$.

By bearing in mind that these two forms of personal and accidental error can be (inextricably) involved in each other, it may be possible that, by taking a number of observations of *each limb* in succession, comparing the result of one limb with that of the same in the afternoon, and, finally, taking the mean of the results obtained, the effect of personal error may be reduced to a



The Suns are Here Shown Relative to Contact Only

FIG. 24.

minimum. This is what is done in practice; a series of eleven sights are made of, first, the upper limb, where the movement is *closing* suns, followed by the same number of lower limb, the movement being *separating* suns; these closing and opening movements, both A.M. and P.M., will still further reduce the probable error.

It must be understood, without any manner of doubt, that any number of observations of the same limb reduces but infinitesimally the personal error of contact, with sun sights; the error is probably repeated sight after sight, and as the eye tires, is even more pronounced.

107. In Artificial Horizon the Observed Angle is Halved: Proof.—In artificial horizon sights the angle measured is twice the true angle; and the total error of observation, made up of $1'\ 30''$ instrumental error (see par. 90), and of $2'$ error of observation as stated above, totalling $3'\ 30''$, is applied to the observed angle; consequently the resulting error is halved.

At the end of Chapter VI. an explanation is given of instrumental error, accounting for $1'\ 30''$.

If to this be added $\pm 2'$, as 'personal' error, the total amount of error in any observation using artificial horizon with an ordi-

nary sextant, in supposed adjustment, in the hands of a fairly capable observer, will amount to $\pm 3' \, 30''$.

Parallax, *i.e.* the distance between the eye and the artificial horizon, is so small, as compared with the distance of the celestial object, that it is neglected; the angle, therefore, is the same whether taken from A (see fig. 25) or from E: S A H is the altitude of the body, but S′ E A is the angle measured, and S′ E is practically parallel to S A:

$$S'EA = SAX = SAH + HAX,$$
but $SAH = EAO$ (angle of incidence and reflection),
and $EAO = HAX$.
$$\therefore SAH = HAX,$$
and therefore $SAX = 2\ SAH$.

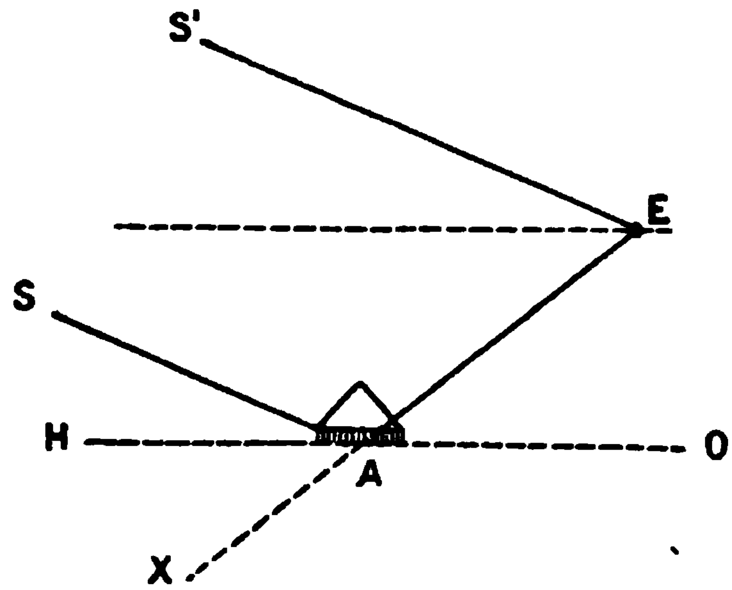


FIG. 25.

That is, the observed angle = twice the true altitude, and the observed angle, which includes all the errors, is divided by two to obtain the true altitude.

Advantage of Using the Artificial Horizon.—This, be it noted, is the advantage of using the artificial horizon; although the angles usually observed on it are those at which instrumental errors are a maximum, yet they are halved.

To distinguish Art. Hor. sights, they are shown as $\underline{\underline{\odot}}$ or $\overline{\overline{\odot}}$, being the lower and upper limbs respectively.

108. Purposes of Artificial Horizon Observations.—Now artificial horizon sights are taken for the purpose of determining the error of the chronometer, or for the ultimate purpose of finding the difference of longitude (see *Meridian Distances*, p. 415).

An observer with moderate practice should be able to obtain the error of the chronometer to within one second; and, if reasonably consistent in his observing, the difference of longitude between any two places to within ·2 of a second of time.

109. Observations for Longitude.—Usually the sun is observed, though equally good and perhaps better results are obtainable with stars, when an observer finds himself competent to take them. In practice stars are more troublesome; not so easy to 'catch' or to follow; it should be only attempted by ambitious observers.

Method of Observing.—Suppose, then, the sun is observed, and the time to be A.M. A set of eleven sights will be taken of the upper limb $\overline{\odot}$; then wait the interval that the sun takes to move two diameters, and take a set of eleven sights of the lower limb $\underline{\odot}$: the last 'sight' of the $\overline{\odot}$ will then appear , and without changing the angle, the first 'sight' of the $\underline{\odot}$ will be : the contacts are here shown as actual, not inverted.

These sets of eleven sights should be taken at intervals of 10′ of arc; the first sight being set at an even 10′, and the 'stand by' given just before the contact appears. The vernier is then advanced (for A.M.'s) exactly 10′ (see explanation of setting or reading a sextant and adjusting vernier screw, par. 105) as before, a 'stand by' given in good time; and so on for the whole eleven sights. Before commencing these sights, the tangent screw should be adjusted to one end of its run.

Suppose the first reading to have been 100° 30′, then the eleventh will be 102° 10′.

Leave this and wait for the passage of the sun from $\overline{\odot}$ to $\underline{\odot}$; this will occupy from 3 to 4½ minutes, and gives the eye a rest, and an opportunity to slue the artificial horizon as well as to readjust the tangent screw. With the $\underline{\odot}$ the first reading will be 102° 10′; the movement will then change from a closing to a separating one, and the tangent screw will be 'advancing' the angle as before. The next reading will be 102° 20′, and so on, the last being 103° 50′.

After giving the first reading, viz., 100° 30′, it is not necessary for the observer to read off 'again'; he merely advances the vernier an even 10′. When, however, the last 'sight' is made, it is advisable to read it off, to prevent mistakes.

Since the interval of arc, *if rightly set*, is exactly the same, the difference in time between each 'stop' should be very nearly the same (the changes of the intervals are really not quite equal, but they are smaller than can be measured by eye or ear) in taking the time for a set of eleven observations.

It is probable that owing to error of observing, the difference between the greatest and least interval will vary from 3 to 4

seconds; however, it is a help to the observer to see what is actually happening; for he may be either observing erratically or may be 'setting' the vernier too hurriedly and badly, or the artificial horizon may not be pointing right.

Taking into account instrumental and personal errors, the mean of these sights is roughly $\pm 3'$ in error, and the true altitude $1\frac{1}{2}'$ in error.

110. Let W E in fig. 26 represent the latitude of T. In conjunction with this, a line L N at right angles to it will give the most accurate position in longitude, because an error of observation will make the least corresponding error in longitude.

Suppose L N to be the correct longitude of T; to obtain the line L N, the position of the sun must either be at E or W.

Admitting, then, an error of $\pm 1\frac{1}{2}'$ of altitude, the 'observed' line of position will be at either of the dotted lines parallel to L N; if at $L^1 N^1$ the altitude is too large and the error is consequently $-$; if at $L^2 N^2$ the error will be $+$, and $T T^1$ is the error in one case and $T T^2$ in the other; but the sign of the error is unknown, therefore the observed position of L N is undeterminable.

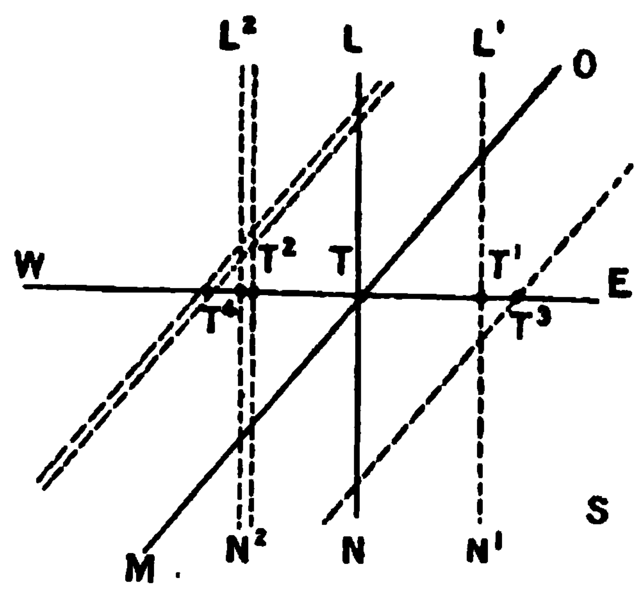


Fig. 26.

Now, if the position of the sun is not at E (for A.M. sights) but at S instead (the position at E may be at an inconvenient hour or it may not be visible on that bearing, either being obscured or below the horizon); and if the sights are absolutely correct, then M O is the correct line of position deduced; but there is an error of $\pm 1' 30''$ in the sights, and the line found may be either of the dotted lines parallel to M O; which of the two it may be is unknown, and T^3 and T^4 are the positions of intersection with W E: $T T^3$ being the error in one case and $T T^4$ the error in the other.

The sights, therefore, as they are, are incomplete, because they give no definite result.

The nearer S approaches to N—in other words, the nearer it is to the meridian or the smaller the hour-angle—the more will an error of observation increase the error of position, in any given latitude.

111. Elimination of Errors. Observations in Longitude.—Let us suppose that for one reason or another the A.M. sights are taken when the sun is at S^1, fig. 27.

Then M O is the true line of position, and either of the dotted lines parallel to it are the erroneous lines.

If the error happens to be − then $M^1 O^1$ is the line obtained; and $T T^1$ is the ultimate error.

Now if another set of sights is taken P.M., when the position of the sun is at S^2, and having the same hour-angle as at S^1, then $P Q$ is the correct line of position; and if the observer is consistent, and his total error is of the same sign and the same quantity as in the forenoon, then $P^1 Q^1$ is the erroneous line and $T T^2$ is the error of position.

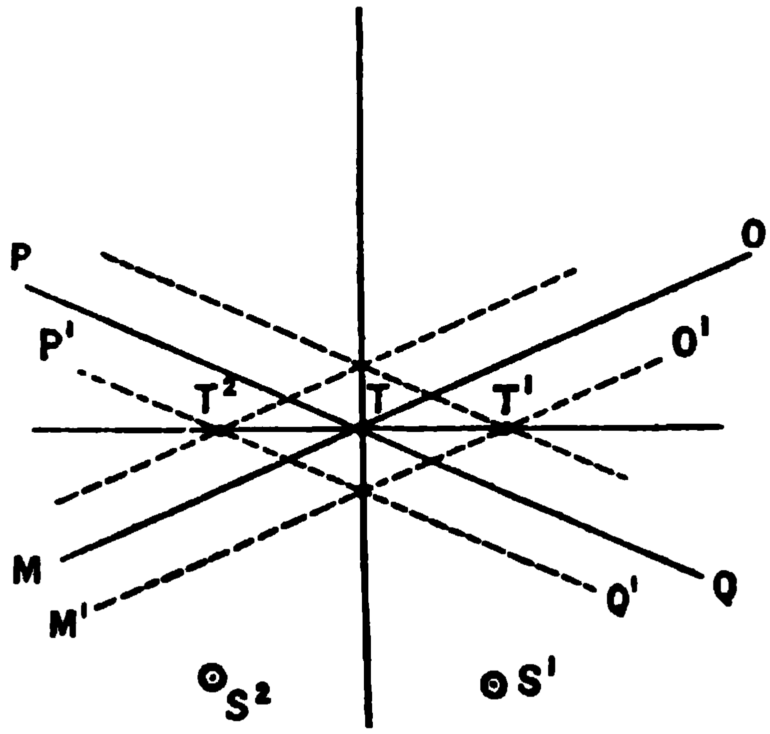


FIG. 27.

$T T^1$ will be equal to $T T^2$, if the hour-angles are the same, that is, if the altitudes are the same; and in this case provided the personal error is the same, then the whole error is the same.

112. In fact an equal altitude on each side of the meridian will probably produce an equal total error of the same sign—omitting personal equation—in which case then the mean of the two observations will be as near the exact position as is humanly possible.

Graphically, the intersection of $M^1 O^1$ and $P^1 Q^1$ on the meridian above or below T give the correct longitude.

113. Theoretically, this will be correct whatever may be the positions or hour-angles of S^1 or S^2, provided they are the same; but in practice, the nearer S^1 is to east, and consequently S^2 to west, the less will be the variation in personal error, *i.e.* the more consistent will the personal error appear to be; because, at E. and W. the sun's motion in altitude is more rapid than at S^1 or S^2, and the less is the time value of an error of contact.

114. *If the hour-angle of* S^1 *and* S^2 *are not exactly the same but nearly so, and consequently the altitudes nearly the same, practically the same results will be obtained.* (See *Meridian Distances*, p. 434.)

It is a question of a different method of calculating the results.

115. Eliminating Bad Pairs of Sights.—A set of sights is usually written down in the following manner, if equal altitudes thus:—

☉ Altitude.	Interval.	Time A.M.	Time P.M.	Interval.	Sum of Seconds.
° ′		h ′ ″	h ′ ″		
120 30		7 20 18·5	12 22 07·2		25·7
40	48	21 06·5	21 23·5	43·7	30·0
50	45·5	21 52·0	20 38·0	45·5	30·0
121 00	43·8	22 35·8	19 53·8	44·2	29·6
10	46·7	23 22·5	19 07	46·8	29·5
20	43	24 05·5	18 21·2	45·8	26·7
30	45	24 50·5	17 36·0	45·2	26·5
				Mean . . .	28·3

The last column is the sum of the seconds of A.M. and P.M. time of the same altitude; and since half the sum of A.M. and P.M. time of the same altitude should, if the observations are perfect, give the same hours, minutes and seconds, and taking it for granted that the hours and minutes will not be in error, then the sum of the seconds as observed should not differ materially from each other; if any one sum does differ widely from the general mean, it should be crossed out: a scrutiny of each separate interval of time, before and after the faulty pair, will expose the individual sight at fault.

If the altitudes are not equal they are written down in precisely the same manner, with an additional column for altitudes; owing, however, to the interval between each A.M. sight differing from that of the P.M.'s, the column for sum of seconds is useless; and this is a drawback to the value of the sights.

116. 'Meaning' Sights.—In the case of equal altitudes it is, however, unnecessary to mean all the times and altitudes: the altitudes are not required, and the mean of the sum of the seconds gives the data required.

When the altitudes are not equal, each set of altitudes and of times has to be meaned, and each set or each sight worked out separately, A.M. and P.M.; and the work is longer. (See p. 434.)

For methods of calculating equal altitudes or 'single' altitudes, see *Examples*, Appendix, pp. 459, 443, or refer to any standard work on navigation. It may be necessary to add that logarithms must be interpolated to the nearest second of arc.

117. Equal Altitudes taken on each Side of the Meridian.—Equal altitudes must be taken on either side of the superior (noon) or inferior (midnight) meridian, and *providing it is certain that the chronometer is not changing its rate*, theoretically, any number of hours or days may elapse between the sights.

For instance, a set of sights may be taken on Monday forenoon and the corresponding half on Tuesday afternoon; or on Monday afternoon and the following Wednesday or Thursday morning; in either case great care must be exercised in affixing the right signs to the parts in the formula. (See Appendix III. (*a*), p. 441, and *Example*, p. 385, par. 593.)

118. Determination of Latitude.—The artificial horizon is also used for the purpose of determining the latitude. But altitudes on each side of the meridian, *i.e.* ex-meridian of the body, would not get rid of the errors of observation.

In the case of ex-meridians of the sun, if S^1 is the position of the sun A.M., then M O is the correct line of position through T.

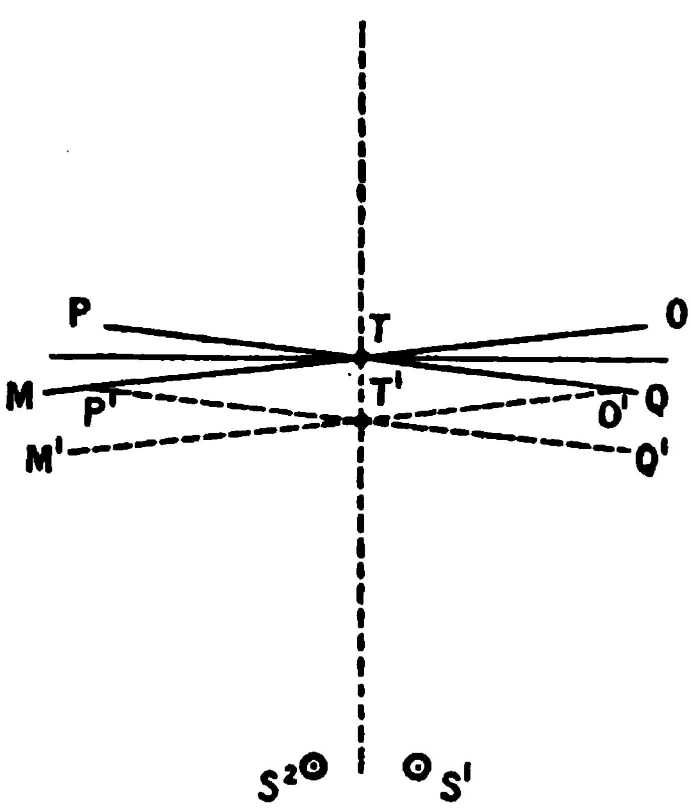


FIG. 28.

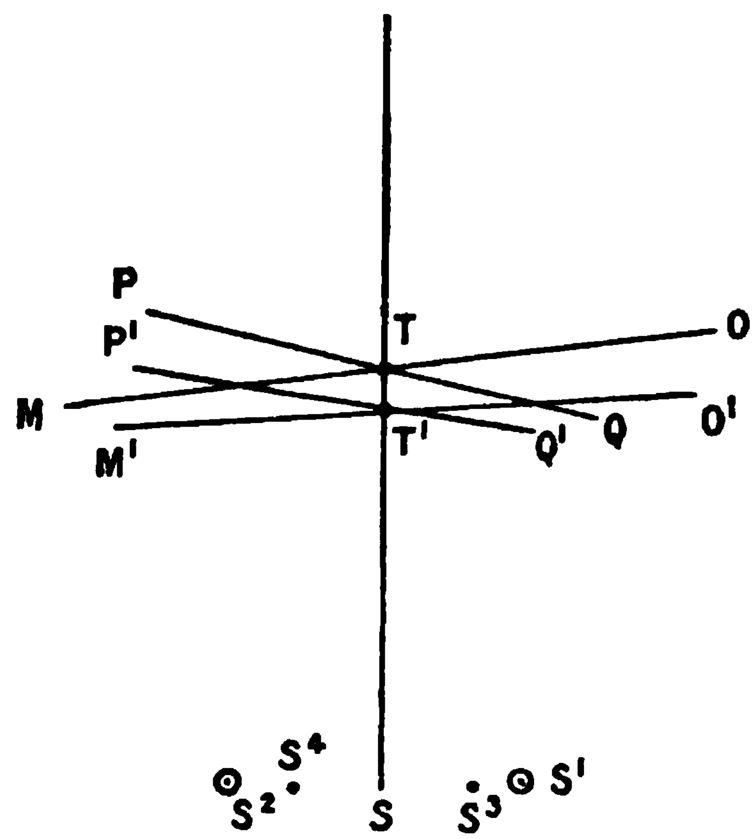


FIG. 29.

Suppose an error of observation to be —, then $M^1 O^1$ is the line of position obtained.

When the sun is at S^2, P Q is the position line, and $P^1 Q^1$ the erroneous line obtained if the error of observation is also —.

The intersection of these two lines is at T^1, and the error in latitude for an error of observation $T T^1$ remains the same for both observations. If there is an error of hour-angle when the observations are made on both sides of the meridian, the error of position is unaltered. (See fig. 29.)

For let S^1 (fig. 29) be the position of the sun A.M., and M O the true line of position through T.

The angle $S T S^1$ represents the hour-angle.

An error in the hour-angle supposes the sun to be at any place but S^1; let S^3 be the supposed position of the sun: then the line of position obtained, in addition to its error on account of error of observation, will be $M^1 O^1$, now not parallel to M O.

Now $S^1 T S^3$ is a small error in the hour-angle; and since $S T S^3$ is the erroneous, and $S T S^1$ the correct hour-angle, then the sign of the error is $+$.

If the same error is made in the hour-angle in the P.M. observation, then S^4 will represent the apparent position of the sun, and $P^1 Q^1$ will be the position line obtained.

The intersection of $M^1 O^1$ and $P^1 Q^1$ will be near T^1 as before, providing the error in hour-angle is small.

If the observation is taken on one side of the meridian only, an error in the hour-angle may increase the error of position found in the previous case more than it may if the altitudes are taken on both sides of the meridian.

In consequence of the above, ex-meridians of stars are resorted to: *one star being on each side of the zenith*, and their altitudes about the same.

The mean of each such pair of stars will eliminate 'errors of observation,' and if observed on both sides of the meridian the error in the hour-angle is reduced to a minimum. The requisite stars can be found in any star catalogue. (See Appendix IV.)

119. Seeing what is its use, then, for purposes of shore observations, the sextant should be the best obtainable; not solely judged by its certificate at Kew, but by actual results found in any set of sights without its complement; that would be, for instance, a set of A.M.'s, or a set of P.M.'s worked singly, in a place of known longitude with an exact error of the chronometer: or by a set of ex meridians of the sun, or even by a meridian altitude. The maker's valuation depends principally on the particular 'hands' who made the instrument, as well as on the material; and no doubt this is the best index to the instrument.

120. **'Ghosts.'**—It may be interesting to offer an explanation of what are known as 'ghosts,' or mock suns, often visible when using the artificial horizon.

The author has known more than once of complete sets of sights taken with these 'ghosts.'

When the image of the sun is reflected from the surface of the mercury, its path is through the glass to the eye; but under some conditions the glass on the roof of the artificial horizon acts partly as a mirror and reflects the image back to the mercury, from whence it is again reflected through the glass. In theory at certain altitudes there must be an endless number of these ghosts, but in practice only one, or perhaps two, may be very distinct, and a dusty glass will accentuate these images.

121. **Affixed Level in the Vernier Plate.**—Another point of interest is that in artificial horizon observations, whatever may be the altitude, the index glass is always vertical; and it is owing to this that in some 'observing' sextants a level is fitted in the vernier plate; so that if the approximate altitude of the star

is placed on the sextant and the instrument held so that the index glass is perpendicular, as shown by the level, then the observer has got hold of the right star; this contrivance, and a small electric light to throw a light on the arc, are the extra fittings on a sextant which are an aid in star observations.

122. **Sextant Stand.**—The 'sextant stand' has been alluded to: it consists of a pillar supporting an arm, at one end of which, through the handle, the sextant is screwed on; at the other end of the arm are adjustable counterpoise weights.

The arm is capable of turning in a horizontal plane, as well as a vertical plane *in the line* of the arms; and the sextant, pivoted at the centre of the handle, can be moved in a vertical plane at *right angles* to the arm; this gives three motions.

The pillar, at its base, is screwed into a socket, from which three legs radiate, having mill-headed screws at their ends.

These screws are for the purpose of giving the fine adjustment to the sextant in the two vertical planes, after the rough setting.

When first set up, the vertical plane of the sextant must pass through the artificial horizon when it is in alignment with the sun; to keep this alignment, since it is constantly necessary to slue the artificial horizon, the sextant, stand and all, has to be moved to one side or the other; it is also constantly necessary to change the position of the observer for the change in altitude; one movement is then advancing or retiring, towards or from the Art. Hor., and the second is to one side or the other; conveniently the sextant stand may be mounted on a double board, one a fixture and the other movable from it, to which is affixed a sideways motion screw, and a back-and-forward motion screw.

Since the sextant is held in a stand, the observer has then only to prepare in advance for the contact of the sun's limbs, or of the stars; and without touching the instrument, keeps his eye on the objects, and waits till they are in contact; any motion required is given by the mill-headed screws at the ends of the legs at the base of the pillar.

Necessarily, then, the observations will probably be more accurate than if the sextant is held in the hands.

123. **Error in Sea Sights.**—The 'errors of observation,' when using the sea horizon, can be very large.

There is, first, the instrumental errors, amounting to, as has already been explained, $\pm 1' 20''$.

The accidental errors would be due to limited magnifying power of the telescopes; and there is the wavy and not perfectly distinct line of the sea horizon; it is probable a combination of these two will amount to $\pm 1'$.

There is yet another error, which may be of very great magnitude, due to the change in the apparent position of the sea

horizon; it is caused by the difference between the temperature of the water and of the air at the surface.

In extreme cases this is known to amount to $+9'$ of arc (see table in *Inman's Tables*), and there are evidences of cases where the visible horizon is below the true, and the error therefore $-$. In other still more extreme conditions, such as arise in the tropics, the amount of the error is unknown. (See footnote, p. 99.)

On an average, the error due to the change in the apparent position of the horizon may be said to be $+1'$.

Personal error is probably another $\pm 1'$.

The combined errors *can* then amount to $\pm 4' 20''$, under ordinary conditions; and in the tropics may reach $+12'$.

N.B.—For twilight stars, however, the error may be reduced to $\pm 2' 30''$; but for night sights the error may be anything from $5'$ to $10'$.

Generally, all observations taken at sea, given the best conditions, can be said to be at least $\pm 4'$ in error: they range from $\pm 4'$ to $+12'$; this is equivalent to a minimum of four miles of latitude; the corresponding amount in longitude will depend upon the latitude, the declination, and the time the observation was taken, *i.e.* the H.A.

An error in time will change the direction of the line of position, and an error in altitude will move the line parallel to itself (see figs. 27 and 29).

Fixing Astronomically by Sea Horizon.—Two position lines, one being on each side of the meridian, will eliminate a great deal of the errors, though it may accentuate that of the change of position of sea horizon where the changes of temperature vary considerably, and will give the best longitude; whereas two position lines, one on each side of the prime vertical, the altitude being small, the errors of observation will be negligible, give the best latitude.

It must be evident that where there are such large errors incidental to fixing by sea horizon, the errors of construction and the residue of those of adjustment which reach the maximum at large angles, are absorbed by the grosser errors and can certainly be neglected, more evidently so, seeing that no angle can be over 90°; and a sextant capable of reading within $1'$ to $2'$ is good enough, *so long as it remains* in that state.

Questions.

1. On what occasion would it be preferable to adjust the sextant so as to reduce the index error to a minimum?

2. (*a*) Of what use is the blank tube usually supplied in sextant boxes?

(*b*) Of what use the coloured eye-pieces?

3. What purpose does the movement of the 'up-and-down' piece serve—

(*a*) When taking observations on shore?

(*b*) When taking stars at night, with the sea horizon?

4. At what exact spot on the sextant is the 'receiving angle' between two objects subtending an angle of 90°? the distance between the mirrors being 4 inches, and the angle at the horizon glass between the index glass and the axis of the telescope being 35°.

5. You require to buy a sextant costing not less than £10—

(*a*) Give any ideas you have in choosing one in the shop.

(*b*) State how you would overhaul it, adjust it, and try it 'at home.'

6. All side error being removed: if, in spite of the most careful observing, it is found that the check on index error measurement is 2′ in error, what is the probable cause? And how is it remedied?

If taking equal altitudes, would this error, if left in, affect the results?

7. In artificial horizon sights, with the sun's double altitude 130°, one of the suns shows distinct double limbs.

What is the probable cause? And how is it remedied?

8. What is the advantage of the large object glass to a star telescope? Is it necessary that the horizon glass should be a corresponding size? Explain why or why not.

9. The vernier coincides with the arc at 0° and 10°, when the index is at 0°; at 100° the vernier coincides at 0° and 7°.

(*a*) What is wrong with the sextant?

(*b*) Can the error be remedied?

(*c*) What would the actual angle be if the vernier read 102° 22′ 20″?

10. (*a*) What is the tabulated error in the lid of the sextant box?

(*b*) On what occasion would it be necessary to apply it?

(*c*) Is it a permanent error?

11. (*a*) What is collimation error?

(*b*) Suppose the difference of readings between the contact of two stars on the lower and on the upper line to be 4′. How is the correction made?

12. (*a*) What is the most accurate system of taking observations on shore with the sextant, and using the sun?

(*b*) Is personal error eliminated?

13. (*a*) What is the most accurate way of determining the error of chronometer for meridian distances?

(*b*) What is the most accurate way of determining the latitude of a place?

(*c*) In either case, is centring error eliminated? If so, in what way?

CHAPTER VIII.

ANGULAR MEASUREMENT AND FIXING.

124. Horizontal Angles.—Angular measurements with a sextant between the sun and any object are required for the purpose of finding the true bearing of the object. (See under *True Bearings*, p. 97.)

The angle between two terrestrial objects will be for the purpose of (1) projecting that angle from a line joining the observer and one of them as a line of reference; or (2) it is required for fixing the position of the observer.

In either case, and specially in the first, the angle projected must be the horizontal angle.

125. Difference between Angular Measurement and Horizontal Angle.—Let O be the position of the observer; X and Y two objects on the observer's horizon; P X, P Y, great circles through X and Y. Then X P Y or X O Y is the horizontal angle between X and Y.

If, however, X and Y have the same and a considerable altitude, then X^1Y^1 are their positions; and $X^1Y^1 = XY$: X^1PY^1 the horizontal angle between them $= X^1OY^1$ is $> XPY$ or XOY, because PX^1 is not parallel to PX, nor PY^1 to PY (see fig. 32 (*b*)).

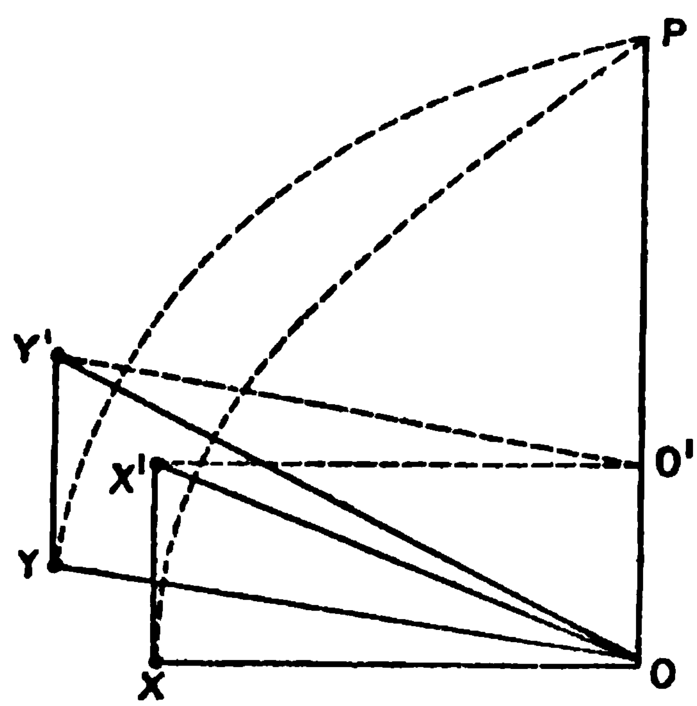


FIG. 30.

Hence X^1OY^1, the observed angle between X^1 and Y^1, is $> XOY$, and if projected as the horizontal angle between X and Y will be an error.

In fig. 32 (*b*) X and Y are on the observer's horizon and X O Y or X P Y is the horizontal angle between them $= X^2PY^2$.

Suppose X^1 and Y^1 (fig. 31) to be the summits of two hills, then $X O Y = X P Y = X^2 P Y^2$ is the projected horizontal angle between them, but $X^1 O Y^1 = X^1 P Y^1$ is the angular measurement, and $X P X^1 + Y P Y^1 =$ error.

Hence the angular measurement between two objects of the same altitude is not the horizontal angle between them, though the error is very small for small altitudes, when P X and P Y may be admitted parallel to $P X^1$ and $P Y^1$.

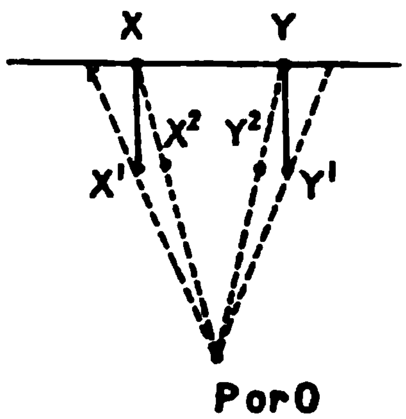


FIG. 31.

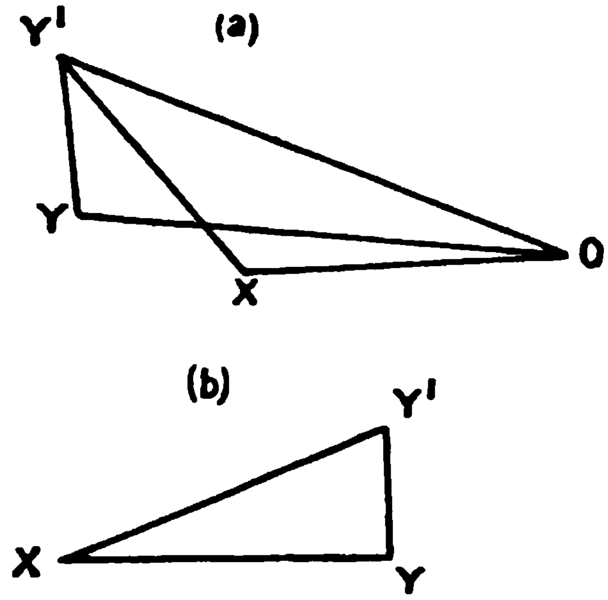


FIG. 32.

126. Objects of Different Altitudes.—When the objects are of different altitudes, taking X and Y^1 in fig. 32 (*a*) as an illustration, the height of Y^1 being 1000 feet: then the angle measured is between the distance $X Y^1$, and if X and Y are 1 mile apart = 6080 feet, then $XY^1 = \sqrt{(1 \text{ mile})^2 + (1000 \text{ feet})^2} = 6162$ feet.

Now whereas X O Y (the horizontal angle between X and Y^1) will subtend 60°, $X O Y^1$ (the measured angle) will be $> 60°$, because $X Y^1$ is $> X Y$.

The value of $X Y^1$ will depend upon the value of X Y (that is, the distance apart of X and Y) in combination with the altitude of Y ($Y Y^1$): and the length of $O Y^1$ will vary as the altitude of Y^1 (see (*a*), fig. 32); and likewise angle $Y X Y^1$ (see (*b*), fig. 32) will depend upon $Y Y^1$: and as $O Y^1$ increases with altitude, $Y X Y^1$ will increase in the same ratio when the angle is small; and

$$XY : XY^1 : : XOY : XOY^1 \text{ when } XY \textit{ nearly} = XY^1;$$

$$XOY = XOY^1, \frac{XY}{XY^1} = XOY^1 \cos YXY^1 \text{ when } YXY^1 \text{ is small,}$$

i.e. the horizontal angle between X and Y = observed angle cos $Y X Y^1$; and, the smaller the angle $Y X Y^1$, the nearer will the observed angular distance be to the true, the cos of $0° = 1$ cos $60° = \frac{1}{2}$.

127. Angle of Inclination between Objects of Different Altitudes.—Beyond 20° the cosine changes very rapidly, and

also the equation will be less exact, for X Y will not nearly = X Y^1; therefore the angle X Y X^1 should not exceed that inclination.

Hence, in taking angular measurements, the objects should, as far as possible, be of the same altitude.

128. Reducing Angular Measurements to an Horizontal Angle.—When the exact angle is required for plotting, and the objects are of different altitudes, it must be calculated. Fig. 33

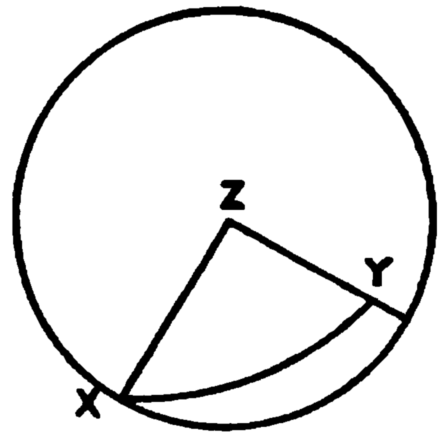


FIG. 33.

is on the plane of the horizon; X is a body on the horizon Z X = 90°; Y has an angle of altitude of 9° 20′. In the spherical triangle X Y Z.

ZY = 80° 40′, the arc X Y is the observed angle, X Z Y = horizontal angle.

Cos X Z Y = cos X Y. Cosec Z Y = cos observed angle. Sec altitude of Y.

Example 1.—Let the observed angle be 88° 47′; I.E. + 3′.

Then cos XZY = cos 88° 50′. Sec 9° 20′

$$\log \cos 88^\circ\ 50' = 9{\cdot}308794$$
$$\log \sec\ 9^\circ\ 20' = 10{\cdot}005788$$
$$\log \cos\ 9{\cdot}314582 = 78^\circ\ 05'\ 30'',$$

therefore the horizontal angle = 78° 05′.

Example 2.—Let the observed angle be 135° corrected for index error (see fig. 34), and suppose that, as before, the altitude of Y

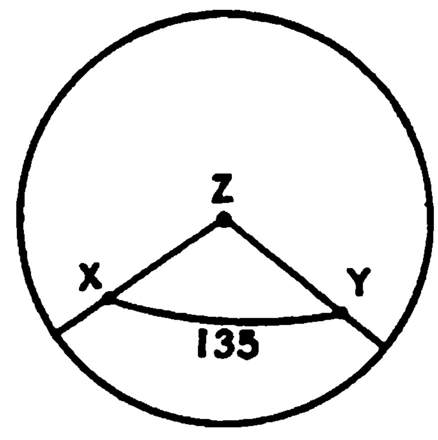


FIG. 34.

is 9° 20′, and let the altitude of X be 17° 50′; X Z Y is the horizontal angle between X and Y.

And log cosec	80° 40′	10·005788
„ „	72° 10′	10·021385
	8° 30′	
XY	135° 00′	
log ½ hav.	143° 30′	4·946937
„ „	124° 30′	4·977586
	log hav.	9·951696 = 142° 08′

Therefore the horizontal angle in this case = 142° 08′.

129. Horizontal Angles for 'Fixing' by Station Pointer.—Where the angle is taken for 'fixing' by station pointer, and the greatest possible accuracy is arrived at without exactness, it may be near enough to keep within the limit stated, *i.e.* of about 20° as explained above.

If, however, this condition does not exist, and an accurate angle is required, then, select a third object, to the left or right of the higher one, described as Y in fig. 32*a*, and measuring as large an angle as possible from X; observe the angle from X, and likewise take the angle from Y to the third object. The difference between the two measurements will be angle O X Y.

130. For, evidently, if the amount of the error which is the difference between the observed angle and the horizontal angle depends upon Y X Y^1 (see fig. 32*b*), the smaller that angle is the better when X is on the horizon and Y has an altitude, or where their altitudes differ materially; and its minimum will be where X Y = X Y^1 and X Y is the greatest possible—that is, X O Y is as large as the sextant will measure, viz. 130°, for then the inclination is nearly 0°, and the distances nearly equal.

131. Minimising the Difference of the Observed Angle and Horizontal Angle.—If, then, X is to the left or right of Y, select another on the side near the lower one, as far round from the higher object as the arc of the sextant will admit, and reflect both X and Y to it; the difference between the observed angles will give an approximately correct horizontal angle between X and Y.

132. Desirability of taking Objects of the Same Levels.—It is more desirable, however, so far as may be practicable, to use water-line marks with others on the water line, and summits with summits; and this, as has been stated, is more necessary when the angle is for the purpose of being projected.

133. Angles all Measured from an Initial Point called the Zero.—When more than one angle is required, it will be more accurate to take them all from the one object rather than from each to the next; that one is known as the zero—usually

marked ⊕. For every angle taken is in error; and if this error is carried from one to the other, each angle will have not only its own error but the errors of all the others from the ⊕.

134. A round of angles for plotting purposes might be shown thus, and taken with a sea serviceable sextant.

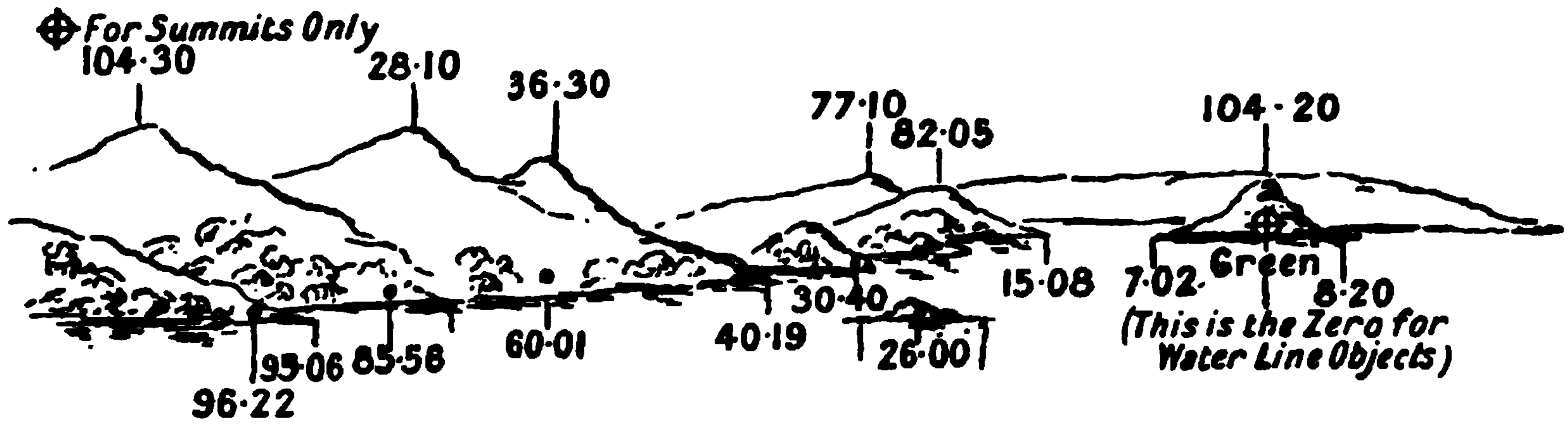


FIG. 35.

Each of the dots represents some definite object, to which a name can be affixed; and the zero is a 'fixed' object with a name, in the above sketch designated Green. To 'fix' the high summit on the left, ⊕ Green is the best available, and the angle is, Summit 104° 30′ Green; but all the other *summits* are taken with ⊕ left high summit. This gives the most accurate results; from Green to summit marked 82° 05′ or to 77° 10′ would be a bad way of measuring the angles.

It is a strange fact that, given equal-sized sheets of paper, two individuals will draw from nature the same view from the same distance a different size; and when the distances differ, the sketches sometimes seem ridiculously out of all relative perspective.

Now, two views, each with the necessary angles to the same objects shown on both sketches, are required to 'fix' any object; and when details such as hills have no distinguishing mark, they must be recognisable in each view so as to locate them by their general outline and relative positions, as well as by the size they are drawn in relation to distance.

In such a case, the sketches on the same scale as well as in separate notebooks save infinite trouble.

The scale adopted is an arbitrary one.

The graduations of $\frac{1}{4}$-inch units on the edge of a rectangle protractor may be used for a scale.

135. **To Make a Sketch.**—First, draw the outline of the distant hills at, for instance, half the distance apart as measured with the protractor when held horizontally and at arm's length. Starting from the left, suppose from one distant summit to the next on the right measured by the protractor held at arm's length is 20 units; then sketch it, measuring 10 units from the left, and so on; put in the water-line from the vertical height proportioned in the same way; then put in the middle distance, if necessary by

the same method, though it will be found often enough easy to fit it in between the background summits; finally, the nearer objects and those on the water-line, each fitting in in their respective places with regard to both angles and perspective distance.

By this means every object will be relatively and more or less correctly placed in angular measurement, and the relative perspective and scale of the two sketches will be very nearly the same.

Adopting half of the ¼-inch units as the scale makes the value of each unit about 1¼° and leaves room for writing in the angles.

For example, with the protractor at arm's length, the distance between two summits reads 20 ¼-inches.

The outline of these summits is drawn on the paper 10 ¼-inches apart; this will roughly represent a subtended angle of 12° at the observer's position.

Fig. 35 as here produced is on the very small scale of 1 unit = 10° roughly; the scale of fig. 36 is 1 unit = 3½°, and would be quite large enough for the particular purpose intended.

136. Sketch for Fixing one's Position.—For fixing purposes, on a published chart, where many hills and islets might bear no names, sufficient angles for fixing by station pointer or tracing-papers are all that is wanted.

The following sketch (fig. 36) will answer the purpose.

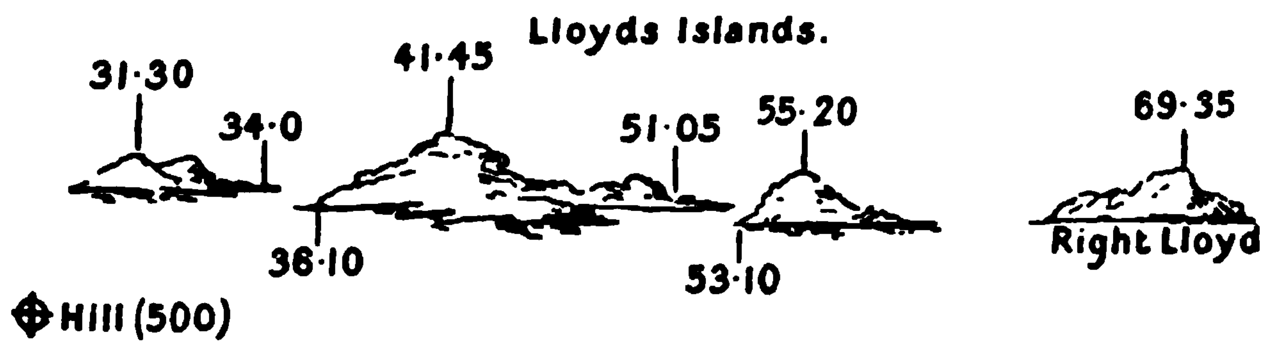


FIG. 36.

Here the object is to 'fix' a particular spot afloat, and not the land; and the extent of the sketch, the relative portions of islets, whether 'open' or 'shut in' or 'in transit,' is an additional and very important aid in 'picking up' that spot again.

The objects of the above two sketches are for different purposes. For this sketch a 'sounding sextant' suffices.

CHAPTER IX.

PROTRACTOR.

137. The primary use of a protractor is to 'set off' or 'lay off' any given angle.

The transparent horn protractor, either circular or semicircular, is graduated along its circumference, and, according to its size, either divided into degrees or half degrees.

The base line for these divisions is the line joining the graduations of 0° on each side of the centre; but the base of the semicircular protractor extends about ¼ inch below this line. It is not always safe to assume that this edge of the protractor is parallel with the base line of the graduations.

138. **'Projecting' Angles.**—To project an angle with any given line, first by the graduations along the edge. It must be admitted as a fact that it is not practicable to correctly 'produce' or 'extend' or 'continue' a line after it is once drawn, therefore it must be made the rule to draw the line joining the centre of any two marks or holes made in the paper, the full length the ruler will admit; and it should, as far as is practicable, approach Euclid's definition, "A line is length without breadth." This will depend upon the quality of the paper and upon the grade of the pencil used.

Let A B (fig. 37), drawn both ways beyond A and B, be a line joining A and B. From the point A it is required to project an angle 112° 18′, to the left of the line A B. This is written—at A, C 112° 18′ B; if the angle was to the right of A B it would be stated as—at A, B 112° 18′ C.

Place the base line of the rectangular or semicircular protractor along the zero line A B, and, with the centre mark at A, the base line will extend along A B produced. And here, it will be noticed, is the advantage of the length of A B; for the base line of the protractor is obviously more correctly placed than if the length A B only be used.

With the protractor in this position, mark on the paper or

'set off' with a needle-pointed pricker the angle as indicated along the edge of the protractor.[1]

In this case, since the angle is 112° 18′, and the graduations are only in 30′ or in whole degrees, the odd 18′ must be guessed.

Keep the 'pricker' in the mark with one hand, and remove the protractor with the other, setting its edge, or the edge of any ruler used, pushed up against the pricker and through the point A.

Before finally drawing this line, feel with the pencil point whether it will go through the centre of the hole made at A. Satisfied as to this, then draw the line *the whole length of the ruler.*

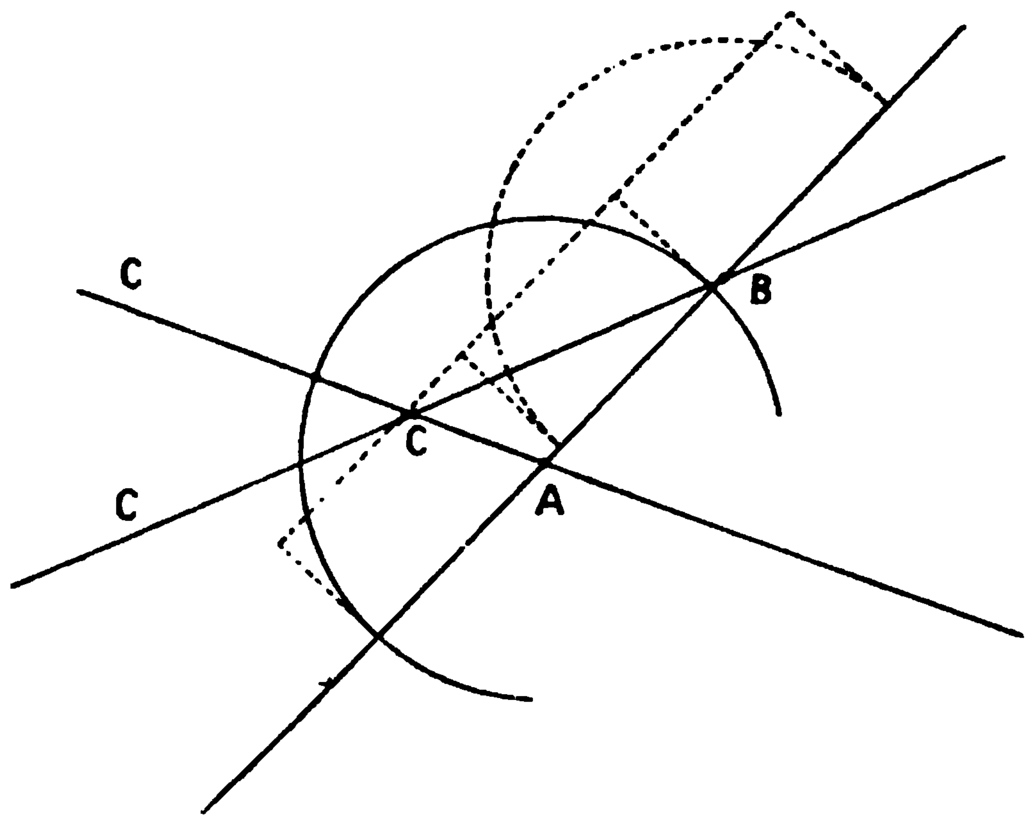


FIG. 37.

For any error made in 'setting off' the angle, the further the line is produced beyond the pricked-off point, the greater will be the error of position of any object from the correct line. (See par. 141.)

The object C will, if the angle is correctly set off, then be anywhere along the line A C.

139. Project a Number of Angles without Moving the Protractor.—If more than one angle is to be projected from the zero line A B, they should all be ticked off without moving the protractor. They are then all relatively correct to that zero line, which, if once moved, can never be replaced in exactly the same position as before.[2]

[1] The invaluable pricker is not often supplied in the ordinary instrument box, but it can be easily made. Take a disused wooden pen-holder; with the red-hot point of a needle make a hole in the butt end about ¼ inch deep; into this hole insert the eye end of the needle or the thick end after breaking off the eye; then, with sealing-wax, taper off the end of the holder to within ¼ inch of the end of the needle and it will be quite rigid.

[2] To ascertain if the base of the protractor is straight, lay off a line with the protractor towards you; then place the same edge of the protractor, but on the other side of the line: it should coincide with the first line drawn.

Again, if an angle is observed at B, between A and C, at B, A 22° 38′ C and projected from that point; the base line of the protractor is again placed along the line A B produced, with the centre at B. Here the use of the prolonged line again will be noticed, for without it there is only the short length A B to keep the base line straight.

As before, the required angle is pricked off, and, without moving the pricker, the ruler is placed in position.

Evidently the intersection of lines A C and B C will give the position of C, determined by two 'direct shots.'

140. Inability to Lay off Odd Minutes by Edge of Protractor-Receiving Angle.—But each angle laid off is in error by the amount of the guessing at the odd minutes; and the error of the position C, from the intersection of these two lines, will depend upon the angle of cut of the lines at C, known as the 'receiving angle' at C, and also upon the distances A C and B C, for obviously the longer they are the greater the error of their direction.

If A C = B C and the angle A C B is a right angle, then the error of the line of direction A C is equal to that of B C.

141. Errors affected by Error in 'Setting off' an Angle, and Distance.—Referring to fig. 37, it will be noted that the point of direction ticked off when the protractor was placed with its centre at A falls outside or beyond the position of C; hence any error made in that directing mark will produce a less error at the distance of C.

In the case of the angle 'laid off' from B, the point of reference along the edge of the protractor is nearer than the apparent position of C; therefore any error made in the directing mark will be increased the further the line is drawn beyond that mark; consequently the line B C, as far as the intersection at C, is not so correct as the line A C.

The more acute is the receiving angle A C B, the greater will be the error of the line B C, on account of the error in projecting the angle and the increased length of B C. For further explanation see par. 183.

This is independent of the additional ambiguity there will be owing to the thickness of the pencil lines cutting each other at very acute angles, called error in drawing (see par. 138).

142. Errors in Rectangular Protractor Greater than those in Semicircular.—In a rectangular protractor the error of 'ticking off' is still more accentuated when the angles are between 45° and 135°, because the radius of the protractor gets smaller and the directing line is shorter.

143. Size of Protractor.—From this it will be gathered that, with a protractor marked only in half degrees or degrees, when the point of reference for odd minutes can only be assumed it is

desirable that the edge of the protractor should fall outside or beyond the object to which the angle is projected.

The size or reach of the protractor required will therefore depend upon the nature and accuracy of the work in hand; those made of horn are usually larger and more accurate than the wooden or celluloid rectangular protractors.

144. Tables of Chords on Protractors.—But the best rectangular protractors have tables of chords and scales engraved on them, which provide means of laying off angles to a much greater degree of accuracy than the graduations on the edges; and this gives a wider scope to their utility.

Let us suppose that the protractor is not large enough to fulfil the requirement that its edge shall reach beyond the object to be plotted.

If means can be devised so that a protractor of any length of radius can be designed off-hand on the paper, such an arrangement will overcome any question of size of instrument at one's disposal as regards the reach of the graduated edge; the only limit being the size of the paper and the linear measurements necessary. If any protractor is, for instance, 8 inches long, a linear length of 7 inches can be measured from it, and it will then be possible to draw a part of the circumference of a circle with a radius of 7 inches, representing the rim of a circular protractor, a skeleton in fact, along the edge of which any angle can be set off.

145. Advantage of a 'Constructed' Protractor.—It will be borne in mind that if a rectangular protractor is 8 inches long, the centre of its base line is 4 inches from either side; and as the graduation of 90° is only about 3 inches or less from the centre, such an instrument can be said to have a radius of an average of 4 inches, allowing for the corners to be a little over that. If, then, a protractor with a radius of 7 inches is obtainable by design or construction, it will be an advantage as compared with the 4-inch radius ready-made instrument.

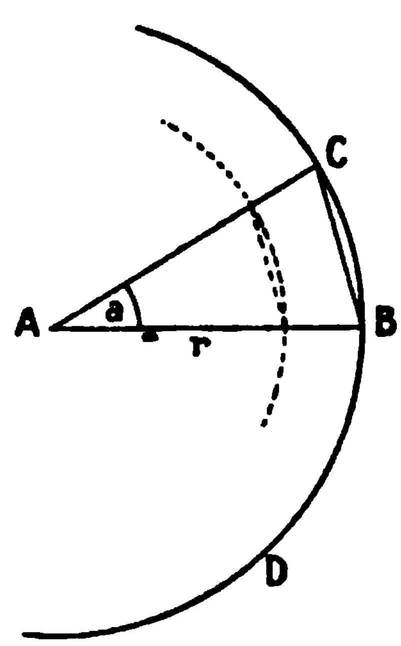


FIG. 38.

146. To Construct Chord Table on a Protractor.—Let C B D (in fig. 38) be the arc of a circle whose radius is A B.

Let C A B be any angle; then C B is the chord of that angle.

Evidently the length of the chord C B will depend upon the radius of the circle.

If a be any angle and r the radius, then (1) its chord $=$ 2 rad. $\sin \frac{a}{2}$, (2) and the chord of 60° will always $=$ radius $=$ $2r \cdot \sin 30° = 2r \cdot \frac{1}{2} = r$.

It is required to construct a table of chords for a radius of 7 inches. Evidently a line engraved on the protractor 7 inches long would equal the chord of 60° for a circle whose radius is 7 inches, because the radius = chord of 60°.

Along this same line is set off from the 0° the length of the chord of 10° and of 20° and so on up to 60°, according to the formula given above. By this means we have a chord scale engraved on the protractor, which may be graduated to 10′ of arc, and on it a measurement of 5′ of arc can be obtained by interpolation.

147. To 'Set off' an Angle by Means of the Chord Scale on the Protractor.—Referring back to where from B, A 22° 38′ C.; to lay this by the scale of chords. (See fig. 39.)

With B as centre, and the length of the chord of 60° as a radius (in this case assumed to be 7 inches), measured with a pair of dividers, sweep the arc of a circle *xy*.

It is not necessary to sweep the whole circle, but only such a part as is required for setting off the angle of 22° 38′ from line AB.

The arc will intersect BA at some point either within or beyond A.

Suppose it to intersect at X.

From the scale of chords, measure from the 0° the length of the chord of 22° 38′.

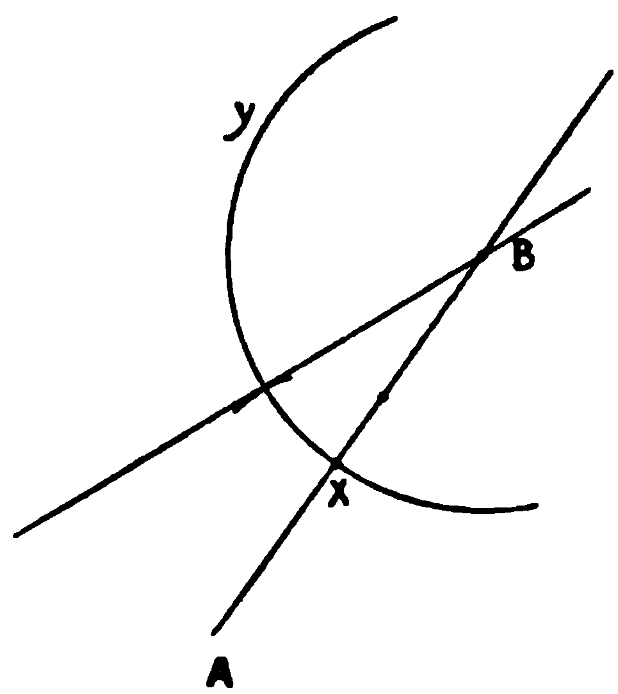


FIG. 39.

Now, with one leg of the dividers at X, sweep a small arc across the arc *xy*.

The point of intersection of these two arcs will be the mark on the part of a protractor *xy*, whose centre is B, representing the angle 22° 38′.

148. Advantage of Chord Scale in a Protractor.—The advantage of this scale is, in the first instance, a larger skeleton protractor produced from it; and, secondly, a closer graduation, with means of laying off an angle to within 5′ of arc.

149. Large Angle Measured by Two Chords.—In the case of the angle at A, C 112° 18′ B, it will be necessary to find the length of the chord in two bits.

With centre A and length as before (7 inches), or length of chord of 60°, sweep the circle *xy* (fig. 40).

Without any fresh measurement, place one point of the dividers at X, and sweep an arc across *xy*, intersecting *xy* at Z.

Now *x*Z is the chord of 60°, and Z is then the mark of 60° on that portion of the arc *xy* with centre A.

The angle required is 52° 18′ beyond the 60°. Take off the length of the chord of 52° 18′ from the scale, and measuring onwards from the 60° mark, that is, from Z to y, sweep across Zy.

Then $y\mathrm{A}x = 112° \ 18'$.

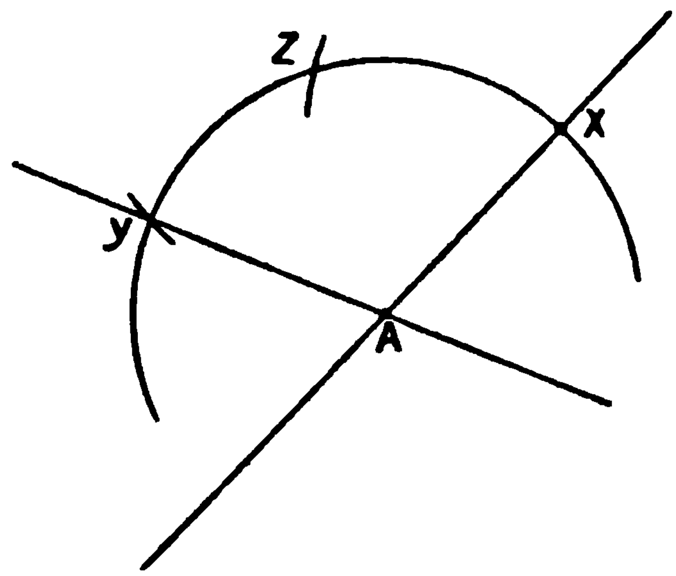


FIG. 40.

150. Chord Scale not beyond 60°.—This will also explain why the chord scale need not extend beyond 60°; still, just as is the case when using the edge of the ready-made protractor, the circumference of the constructed one may not fall beyond the object C when the angle is projected from B (see fig. 41); but the radius cannot be prolonged beyond the *range* (in inches) the protractor admits of; longer than 7 inches, however, introduces work on a scale beyond the scope of this manual, where more special instruments would be brought into requisition; and the 7-inch protractor is therefore long enough for elementary work.

151. Accurate Plotting by Chords not on the Protractor.—But suppose, with the aid of the same protractor, it is desirable to plot the angle to the *nearest minute*, for the sake of the utmost accuracy; then the measurement of the length of the chord must be taken from the 'diagonal' scale instead of the prepared scale.

Take the case before alluded to, where at B, A 22° 38′ C. With a radius of 7 inches, the arc of a circle is swept, with centre b cutting A B at x.

The length of the chord of $22° \ 38' = 2r \times \sin \dfrac{22° \ 38'}{2}$ (see par. 146) $= 14 \times \sin 11° \ 19'$.

In *Inman's Nautical Tables* there are natural sine tables. Then take out the natural sine $11° \ 19' = \cdot 1962$; Multiply by $14 = 2 \cdot 747$ nearly. This is in inches.

From the 'diagonal' scale on the protractor take off this measurement; and with centre x sweep across the arc xy.

At the intersection is the point of reference for 22° 38′ exactly.

152. Length of Radius for Chording Angles.—In practice, if the work undertaken is strictly within the scope of this work, it is seldom necessary for a radius of longer than 5 inches to be taken; and, moreover, 5 inches has the advantage that $2r = 2 \times 5 = 10$; and 10 multiplied by the natural sine of an angle merely means shifting the decimal point one figure further on; so that, *for a radius of 5 inches*, the natural sine table is the chord table, with the small additional trouble of multiplying the logarithm by 10.

If a 10-inch radius is used, the natural sine of half the angle is multiplied by 20. But the protractor generally in use does not exceed 8 inches: this, then, must be the limit for the radius used with it; nor will the line joining any two points in an elementary survey usually exceed 9 inches, which is just sufficiently long for sweeping a semicircle with a radius of 8 inches.

153. Occasions for Uses of Edge, or Table of Chords, or Natural Sine Table.—Each method of projecting an angle, whether using the edge of the protractor, or using the scale of chords engraved on the instrument, or using the linear scale and plotting to the nearest minute, will have its time and place.

154. Other Scales on a Protractor (Protractor not less than 8 Inches).—Beyond the graduations on the edge of a rectangular protractor, the scale of chords, the diagonal scales, and the linear measurement of inches, the other tables on most protractors serve no purpose to a seaman. For practical work with any pretensions to accuracy a protractor should not be *less* than 8 inches long. For navigational purposes, the horn protractor is the more serviceable.

It is seldom that the graduations on a protractor are quite correct, and less frequently the centre is out of position—they should always be tested (see *Testing Station Pointer*, p. 65).

Examples in Laying Off Angles by Chords (the figures are on $\frac{1}{2}$ natural scale.—(1) On a base A B, length 4 inches, construct a triangle having the angles C A B = 35°, C B A = 20°. With A as centre, describe an arc of a circle of radius $1\frac{1}{2}$ inch cutting A B in a.

It is required to lay off C A B of 35°. The nat. sine of $\frac{35°}{2}\left(\frac{a}{2}\right)$ $= \cdot 3007$. The chord of $35° = 2r \times \sin\frac{a}{2}(r = 1\frac{1}{2}'') = 3 \times \cdot 3007$ $= \cdot 902$ inches.

With one leg of the dividers at a, 'sweep' across the circle drawn with A as centre, cutting A C at a'. Join a', the point of intersection, with A. Then C A B = 35°. Repeat the operation at B. Describe a circle of radius $1\frac{1}{2}$ inch cutting A B in b. From b measure the chord of 20° to b'. Join Bb', and produce it to meet Aa' in C.

Then A B C is the triangle whose angles are C A B = 35°, C B A = 20°.

In fig. 41, if there is the smallest error made in measuring the chord aa' or bb', producing the lines Aa' or Bb' magnify that error

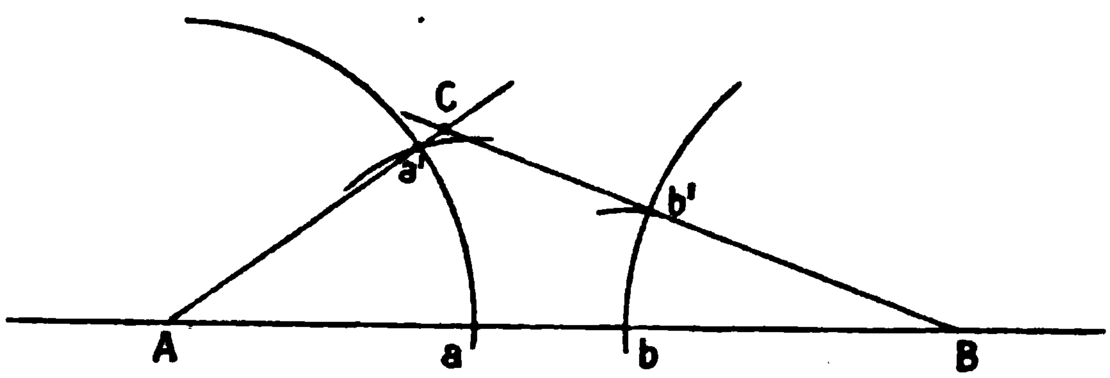


FIG. 41.

the further the lines A C, B C, extend. On the other hand (see fig. 42), when the measurements aa' or bb' are in error, the result of the error is diminished the further the point C falls within the

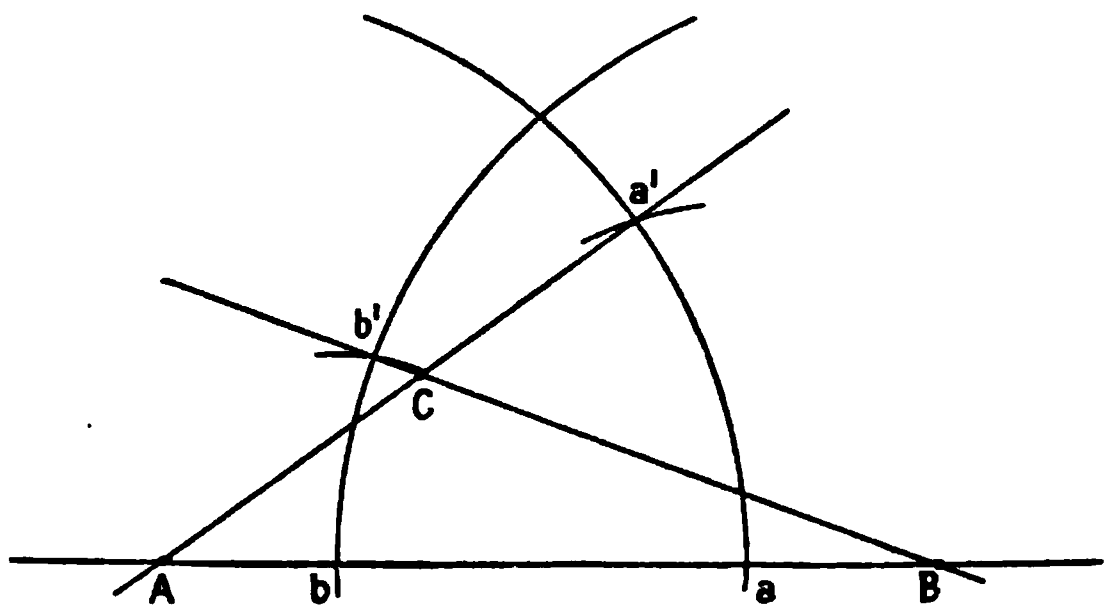


FIG. 42.

described arcs; hence the radius used should fall 'without' the station to which angles are to be projected.

(2) On a base A B (fig. 43), length 2 inches, construct a triangle having the angles C A B = 80°, C B A = 90°. The figure is on $\frac{1}{4}$ the natural scale.

With centre A, radius 6 inches, describe an arc of a circle cutting A B produced in a. With centre a and same radius (6 inches), sweep a small arc across the former circle. This intersection will be the measurement of an angle of 60°. Starting from this mark of 60°, measure the chord corresponding to 20° intersecting at a'. Then a'Aa is 80°. Carry out the same operation from B and b, making bBb' equal to 90°.

The cut of the lines A B and B C at C is not well pronounced, and the angle A C B is 'ill-conditioned'; this method of constructing the triangle is therefore not good.

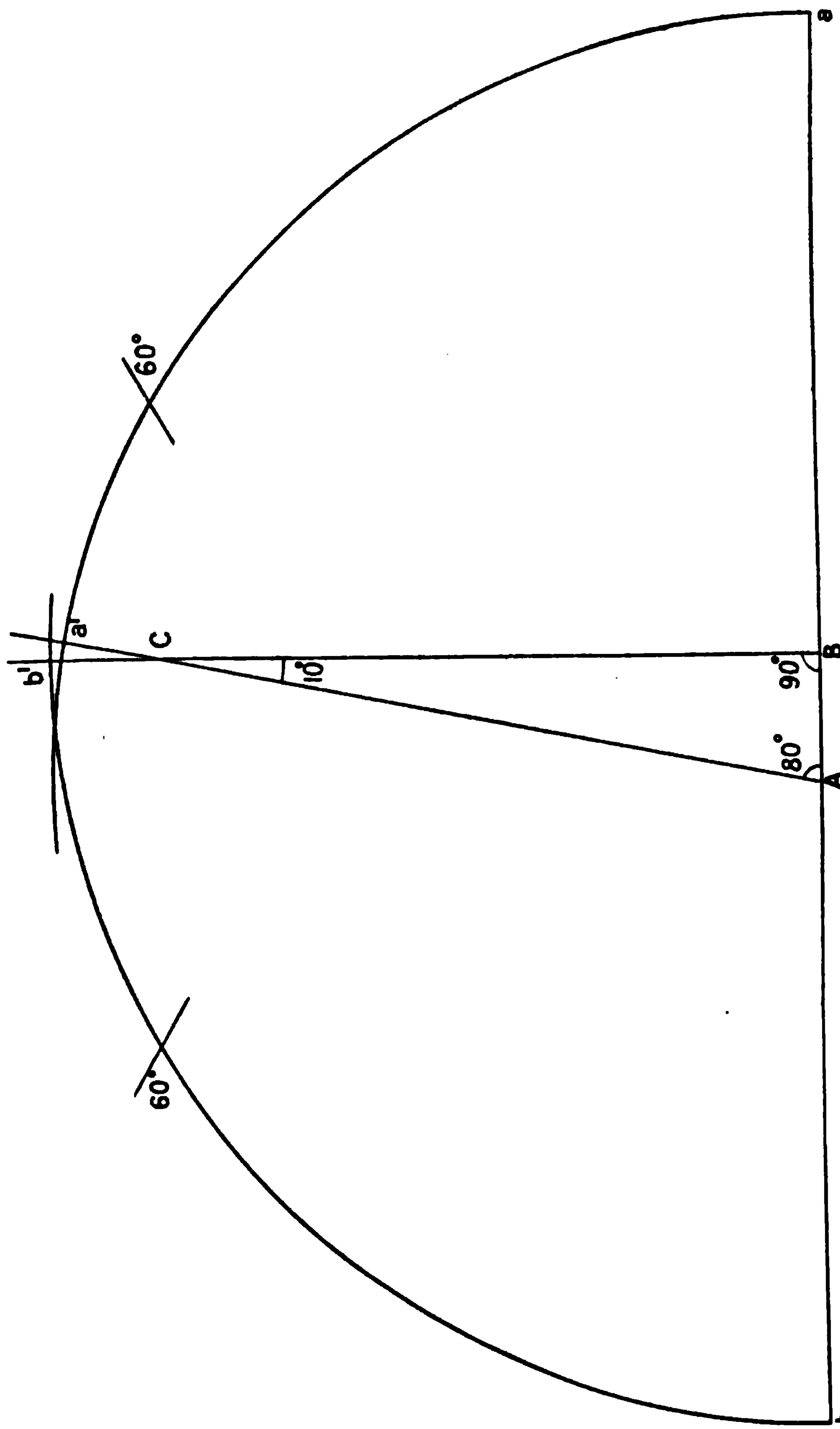


Fig. 43.

CHAPTER X.

FIXING.

155. Definitions of Expressions 'Fixing,' 'To Fix,' 'a Fix.'—'To fix,' or 'a fix,' is the intersection of two lines or of two arcs at a point, but includes also the series of angles taken at or to a position, afloat or on shore, which will determine that position; such a spot is known as 'fixed' (see *Plotting*, p. 198).

'A fix' by the intersection of arcs of circles, is either plotted with a station pointer (see *Station Pointer*, pars. 170, 187) or with tracing-paper; or 'a fix' by the intersection of straight lines, may be plotted by one direct line of reference, and a line obtained from a 'calculated angle' (see further on, under *Plotting*).

156. Method of Writing down a Station Pointer Fix.—'Fixing' by station pointer is written down thus:—

C 63 40 B 52 10 A;

or "Hill (500) 41 43 Summit Lloyd, 27 50 Summit Right Lloyd" (see fig. 36 B). These are station pointer 'fixes'; in the first one, A being the right object, B in the centre, and C to the left, their relative positions being as they are written down.

157. Tracing-paper Fix.—

B	52	10	A
c	60	15	"
d	68	40	"
e	72	12	"
f	110	27	"

is a fix for use with tracing-paper. The angles are projected with a protractor from an initial line A; taking care that each angle is pricked off without shifting the protractor from the position in which it is once put down.

When the lines are drawn, and labelled, they will serve as a station pointer with six legs; and each line must pass through the object indicated by the angle.

158. To 'Fix' an Object by 'Shooting it up.'—To 'shoot up' an object is to take an angle at the 'fix' of the observer, between a mark already determined and the unfixed object.

And, when an object is 'shot up' from three or more fixes, its position is said to be 'fixed' by direct angles or direct shots.

NOTE.—The intersection of two lines fixes an object; the third line is a 'check' (see par. 159).

'Shooting up' a number of objects from two fixes will appear thus:—

$$\left.\begin{array}{l}\text{Hill (264) 32 56, hill (96) 117 48,}\\ \text{hill (500) 34 26 peak}\end{array}\right\}\text{chart No. 2920.}$$

(This is the fix of the first position on the chart stated, from whence a number of objects which follow were 'shot up.')

$$\begin{array}{ll}\text{Hill (96)} & \text{10 20 a hillock.}\\ & \left.\begin{array}{l}\text{33 40}\vdash\text{of a house}\\ \text{34 10}\dashv\quad\text{,,}\end{array}\right\}\begin{array}{l}\vdash\text{is the left edge.}\\ \dashv\text{is the right edge.}\end{array}\end{array}$$

Hill (96) is a summit, the position of which is marked on the chart; the other objects being 'shot up' from and to the right of it, since they are written down on the right.

With the angles necessary to 'fix' placed on a station pointer, or plotted on a piece of tracing-paper, the observer's position is determined (see *Station Pointer*, p. 67).

A line is drawn joining this position with hill (96) (this is known as the zero for the angles that follow); and, placing the base line of the protractor along this line, the centre being over the fix, a mark is made with a needle-point pricker on the paper, at the edge of the protractor, where each angle is indicated; and each angle is projected as explained under *Protractor*, Chapter IX.

At the next fix it is found more convenient or more accurate to use a different zero (see par. 136)—abbreviated $\oplus$.

$$\begin{array}{rrl}\text{Castle 40 36 Beacon} & \text{50 52} & \text{Abbey.}\\ \dashv\text{House} & \text{63 40} & \text{,,}\\ \vdash\quad\text{,,} & \text{65 60} & \text{,,}\\ \text{Hillock} & \text{66 20} & \text{,,}\end{array}$$

The first line of angles is the 'fix,' and those that follow are all with Abbey as $\oplus$; these are projected from the line joining Abbey and the fix, and to the left of Abbey; where the lines from the first fix intersect each one of these through the same objects, is the apparent position of the objects 'shot up.'

159. Use of 'Check.'—So far, there is no certainty that either of the fixing angles is correct, or that there has not been some mistake in the identity of the objects used in fixing.

If the angles at the fix are taken wrong, the station pointer will still give a definite position; and if the objects used are not those you think they are, the station pointer will still give a definite position when the legs are over the correct objects.

And from any two definite positions, though they may be completely in error, two lines projected will intersect somewhere.

To overcome the case of using wrong objects, if there is any doubt as to their identity, it is better to take a check angle to a fourth object; and when the position of the fix is obtained by the two angles between three objects, adjust the station pointer angle for that observed to the fourth object without moving the centre of the instrument.

If, then, from the fix found, the third angle goes through the object used as a check, the position is probably correct.

Supposing the objects to be the correct ones, there remains the errors incidental to observing the 'fixing' angles, which will produce an error of position (see *Station Pointer*, Chapter XII.).

From these erroneous positions, the angles projected to the same object will intersect somewhere, although at either or both positions the angle may be in error (see further on); the error may be a gross one or merely an error incidental to observing.

Thus it will be necessary that there shall be a 'check,' and this check is derived from a third 'fix.'

The observer then 'fixes' a third position, and again 'shoots up' the same objects.

160. Errors in 'Fixing' and 'Shooting up' and 'Cocked Hats.'—Now if—

(1) the positions of the originally fixed objects used to 'fix' by are correct;[1]
(2) the observer does not shift his position, and the fixing angles are correct to the nearest minute;
(3) the station pointer is perfect;
(4) the objects 'shot up' are sharply defined and the same spot in the object is taken from all three fixes;
(5) the 'shooting up' angle is correct to the nearest minute;
(6) and the angle projected from the fix is set off by chords to the nearest minute:

then the intersection of the three lines from the three fixes will cut exactly.

In practice, not one of the above conditions exists; hence the intersecting lines will form a triangle of varying dimensions.

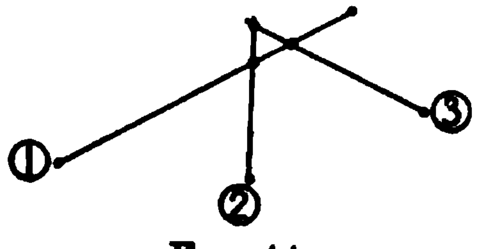


FIG. 44.

This triangle is technically known as a 'cocked hat,' and there are three positions defined (see fig. 44).

If (1) *the same three objects* have been used to fix with, and each 'fix' is equally good, and from each fix the best zero is used to 'shoot up' from (see App. II.),

[1] Every published chart, in the process of engraving, is more or less distorted, and it must not be expected that the objects thereon are in their exact positions.

and each projected angle is laid off by the length of its chord, with a suitable-sized radius, from each fix, then the area of the 'cocked hat' represents the errors of observing, and the centre of the inscribed circle is, under the conditions, the probable position of the object.

If the angles are not chorded, the value of any one position plotted from two fixes, will depend upon the receiving angle at that position, and the length of the lines projected through it from each fix (see par. 183). In fact, each line drawn will have a value—the shorter they are the better; and the combination of any two lines will have a value—the nearer 90° they cut the better, *i.e.*, the nearer to 90° is the receiving angle *at* the point desired to 'fix' (see p. 78, par. 183).

The usual conditions, however, which call for this method of fixing an object, do not call for straining at accuracy.

They are adopted either in the case of a ship at anchor, or even with weigh on, when 'shooting up' some large object on the shore, either practically inaccessible, such as, for instance, a church steeple, a monument, a building or summit, or a weather shore; or absence of landing; or inaccessible because it may be otherwise inadvisable (for further explanation of this see par. 579); or in boat work, where the objects or things 'shot up' have no defined point.

For the third method of 'shooting up' an object and fixing it by calculated angles, see *Plotting*, p. 203.

CHAPTER XI.

STATION POINTER.

161. Description of Station Pointer.—A station pointer consists of three flat metal legs, one side being bevelled or chamfered, pivoting from a common centre.

To enable either leg to be placed at an angle with the other, a circular plate graduated to degrees and tens of minutes, the centre of which should coincide with the pivoting centre of the legs, is affixed to the middle leg of the three; leaving the other two free to revolve, and enabling them to be set at any angle with the middle one.

Where the graduated circle meets the movable legs there is a clamp and a tightening screw attached to each leg so that it can be set to any angle desired, as indicated by the position of an index mark.

Those instruments that are used for navigational purposes have merely an index mark, which is in line with the bevelled side of the leg; and the arc is graduated to 10′.

An angle, then, can be 'set off' to the nearest 10′, or approximately 5′, of arc, which is sufficiently accurate for navigation (see *Fixing*, p. 69, par. 173).

162. Degrees of Accuracy in Setting Station Pointer.—But in a station pointer used for work calling for consideration of the nearest minute of arc, there is a vernier, which, as in the case of a sextant vernier, enables the angle to be read off to the nearest minute.

This vernier is secured to the leg by two screws set in elliptical holes.

163. Testing.—A testing sheet may be made by chording a number of angles at every 10°, with a radius sufficiently long to reach almost to the end of the legs: 10 or 12 inches, for example.

Placing the middle leg over the 0°, from whence the angles are chorded, each leg in turn can be placed over the lines on the testing sheet; and should the reading of the index not be the

same as that on which the leg is placed, the position of the index is in error; and since the holes which secure it are elliptical, the index can be moved. (See par. 167.)

164. Centring Error.—The station pointer is, however, subject to considerable centring error; the centre from which the legs revolve not always coinciding with the centre of the arc as placed.

The error of reading in such a case, when it is 0 at 0° and 180°, will uniformly reach a maximum ± at 90°.

165. To ascertain the Existence of Centring Error.—Place an angle of 90° by the arc reading on each leg; then draw a line along the edge of both, through the centre of the station pointer. Draw a line along the bevelled edge of the middle leg, so that it shall extend through the centre of the instrument, and be prolonged through the 180° graduation; the intersection of this line with that joining the legs set at 90°, gives the correct centre of the arc. Any error of this kind could not be adjusted by moving the vernier.

166. Error in Graduations.—But the instrument has a regular, or irregular, error in the graduations also; or also an error in the position of the index (0°) at the centre leg, in which case the error is constant for all readings; and the combination of the centring with these errors will sometimes produce variable total errors for each 10° of reading, such as +5′ at 10° and −2′ at 20°.

167. Adjusting Station Pointer.—It is only that part of the whole error which is common to all the readings that can be removed by shifting the index of the vernier.

For instance, if the error at 10° is 10′+
" " " 20° is 12′+

and so on, uniformly increasing or decreasing, while the sign remains the same, the least + reading can be removed from all, by shifting the index; but if the signs are different there is no remedy.

168. Error Supplied on the Lid of Box.—The instrument is easily strained if badly handled; hence the error supplied on the lid of the box is not necessarily the permanent error.

It should therefore be frequently tested on the test sheet.

169. Extension Legs.—There are extension legs sometimes supplied in station pointer boxes; they are joints, and can be 'shipped' when necessary.

Each joint has projecting sockets which fit into corresponding holes in the legs; and there is a mill-headed screw to increase the rigidity.

Each joint has one or two indentations punched on it, and the leg on which each fits has similar marks, so that a joint will only fit a particular leg.

Station pointers should never be held by the legs beyond the graduated arc; and care and practice is required in replacing them in their box.

170. Theory of Station Pointer.—If the horizontal angle be measured between any two objects, then the observer must be on a segment of a circle which can be described through the two objects and himself.

Let A, B, be any two objects. Suppose an observer O finds the angle between A and B = 40°.

Then, if A, B, and O are on the segment of the same circle, and at O, anywhere on the circumference, the angle between A and B is 40°, then the angle at the centre of that circle will be 80°.

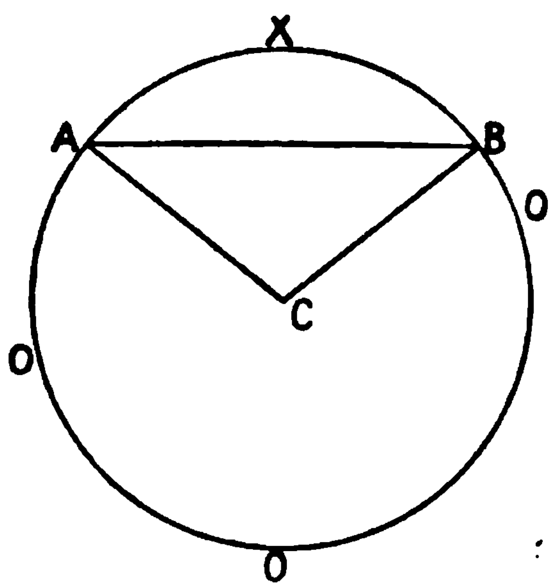


FIG. 45.

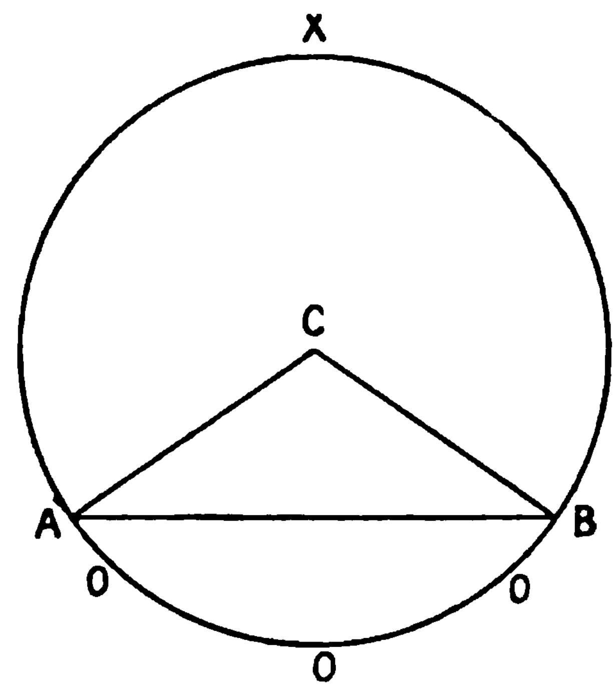


FIG. 46.

The angle at the centre of a circle = twice the angle at the circumference when both angles stand in the same arc.

Let C be the centre of the circle.

$$\text{Then } ACB = 80°, \text{ and } ACB = 180° - (CAB + CBA).$$
$$\text{But since } CA = CB, \text{ then } CAB = CBA,$$
$$\text{and } ACB = 180° - 2\,CAB \text{ or } 2\,CBA.$$
$$\therefore\ 2\,CAB = 180° - ACB = 180° - 80° = 100°.$$
$$\therefore\ CAB = 50° : 50° = 90° - 40° \text{ (the observed angle).}$$

At A lay off from AB an angle BAC = 90° - observed angle (AOB); at B lay off the same angle ABC.

The lines will intersect at C; and C is the centre of a circle the circumference of which goes through A, B, and O; the angle ACB = 80°, and the angle AOB = 40°.

If the angle observed at O is > 90°, then from each end of the line AB (fig. 46) lay off an angle = observed angle - 90°, on the *opposite* side to O.

Let AOB = 122°.

Now the sum of the angles standing on arc A O B and on arc A X B = 180°. If the angle A O B on arc A O B = 122°, then the angle standing on arc A X B = 180° - 122° = 58°;

and, as before, A C B = 116°,

and $BAC = ABC = \frac{180 - 116}{2} = 32°$

$= 122° - 90°$ (the observed angle $- 90°$).

Therefore at A (fig. 46), on the further side of A B from O, lay off B A C = 32°, and the same angle A B C at B.

They intersect at C; and C is the centre of a circle whose circumference goes through A B and O, where A O B = 122°.

171. The fact then is deduced that, if an observer measures the horizontal angle between any two fixed objects A and B, the circle can be described which will go through the three places A, B, and O.

A and B are fixed points on the arc, and O is anywhere along the circumference A O B.

If the observed angle A O B is set off on either leg of a station pointer, and the two legs measuring the angle are placed so that the bevelled edge of the one goes through A, and of the other through B, then the centre of the instrument which represents the position of the observer will be *anywhere* along the arc A O B; this demonstrates practically that 'all the angles in the same segment of a circle are equal': if through A B the portion A O B were cut off the circle, then A O B is a segment of a circle.

This segment A O B is defined as an 'arc of position,' along any part of which O may be (see note 'Danger Angle.'

NOTE.—**Horizontal Danger Angle.**—Conversely, at a definite position O, the angle subtended between two points A and B on a chart, can be measured; and so long as this angle remains the same, the observer will be on the arc A O B; if he approaches C (the centre of the circle drawn through A B), the subtended angle will increase, while if he recedes from C the angle will decrease.

With the knowledge of this fact, a circle of any radius can be drawn, whose circumference will fall a definite distance clear of a known danger, and along any portion of a segment of which the angle between two selected objects will remain constant.

The best position of the centre of the circle is anywhere along a line drawn at right angles to the observer's course, and through the danger; and the length of the radius merely depends upon what two 'fixed,' well-defined objects can together be involved on the circumference.

If another angle—say 50°—is taken at O between B and an object C to the right of it, this will produce a second 'arc of

position' B O C, also along any part of which O may be; and since these arcs intersect at O, this must be the position of the observer, for it fulfils the conditions that O is on both arcs.

The 'fix' is written A 40 B 50 C; B being the middle object, and, viewed from O, A is to the left and C to the right of it.

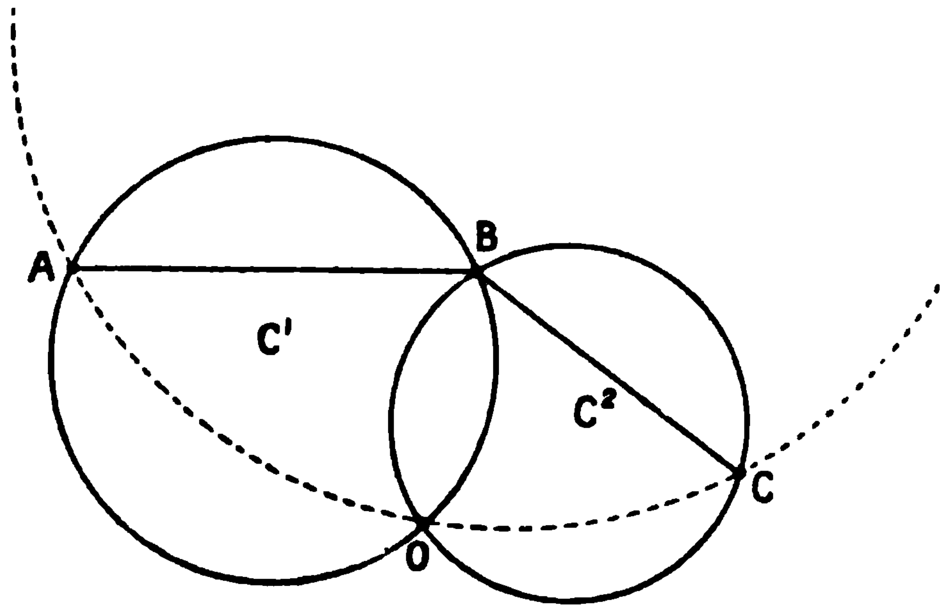


FIG. 47.

It would be possible to draw each circle in the manner described, but the station pointer saves us this trouble.

For if the left and middle leg pass over A and B respectively, they provide the arc A O B; similarly, the middle and right leg give the arc B O C; and if the left, middle, and right legs pass through A, B, and C respectively, *i.e.* the left, middle, and right objects as viewed from the observer's position, the centre of the instrument gives the position of the intersection of the arcs and also of the observer.

There is besides A O B and B O C a third circle, shown in dotted line, fig. 47, which in this case is 'derived' from the sum of two angles observed; each angle in this case being measured from the middle object, and A O C being the sum of the two.

In fig. 48 the *whole* angle A O C is measured, followed by one of the others, in this case A O B; the observation being $\left.\begin{matrix}\text{A } 100^\circ \text{ C}\\ 60^\circ \text{ B}\end{matrix}\right\}$, then from the ends of the line A C, *i.e.* from A and C, lay off 10° (100° – 90°) on the opposite side to O (see par. 170); at the intersection of these lines will be the centre C^3 of a circle going through A, C, and O.

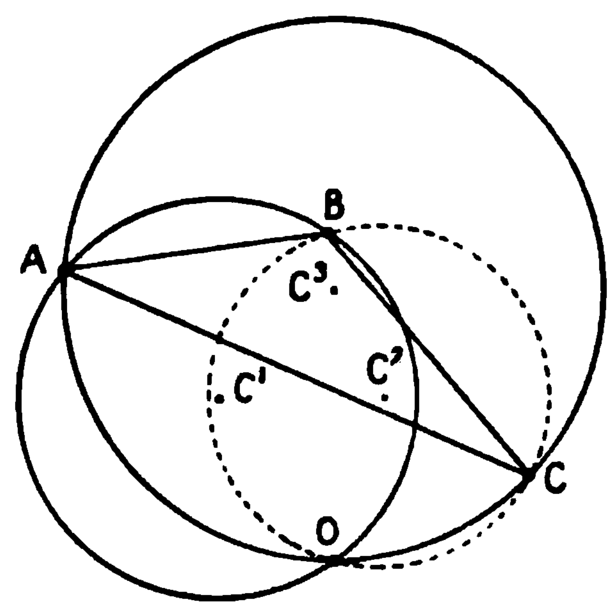


FIG. 48.

From the ends of the line A B lay off 30° (90 – 60°) on the side towards O; and C^1, the centre of a circle, will be found, the circumference of which goes through A, B, and O. At the intersection of the two circles drawn from each of these centres C^3 and C^1 will be the position of O, found from A 100° C and A 60° B.

The dotted circle is a third circle which is derived from the difference between 100° (the whole angle), and 60° (the left angle; in

this case = 40°) ; it is in no way a check on the accuracy of the observed angles, though in actually drawing the derived circle it would be a check on any error of construction in the other two.

The facility and rapidity in taking the angles nearly simultaneously, in setting them quickly, and in fixing a ship's or a boat's position is one of the main advantages of fixing by angles and station pointer.

172. In fixing by the station pointer there are certain relative positions of objects A, B, C with O which give better results than others. For a good fix, similar conditions to those used in fixing by the intersection of lines of position, are necessary ; they depend upon the angles at which the circles intersect, the dimensions of the observed angles, and the distances of the objects from the observer.

In the best cases, an 'error of observation' makes a small consequent error of position.

Errors of observation are here intended to include *all* the errors incidental to fixing by station pointer (see pars. 187 to 190).

For any given error in the observed angles only, from which the circles can be drawn, the corresponding error in the position of the observer is a minimum where these circles cut each other at right angles. But other conditions are involved besides this error in the observed angles; and, *without exception, that fix which is readily found, that is, easy and quick to plot with the station pointers, is a good fix.* The best is therefore the easiest; for it exhibits not only the fact that the cut or intersection of the circles is nearly at right angles, but also that the positions of the objects are so situated both in size of angle, and in distance, that a small displacement of the centre of the station pointer, will make a *large* or larger movement of either, or both legs, at the position of the objects, or, in other words, that an 'error of observation' makes the least corresponding error in the position of the observer. This fact that an easily plotted fix is a good fix is very important, and in the knowledge of it lies the whole crux of the matter, as well as the only tangible foundation on which one can learn to distinguish between inferior and good fixes. Constant familiarity of testing a fix by this means, is the only possible manner of eventually learning which objects to choose.

173. **Rapidity of Fixing with Station Pointer an Advantage, and Setting the Angles.**—When navigating, a vessel 'fixes' while under weigh, and the angles are not usually taken simultaneously; also, the errors from all sources in the angles observed are large, and, since it is not then material to within a diameter of 400 or 500 yards where the true position of the ship is, the station pointer need not be, and is not, set with too great deliberation.

A navigating station pointer is, for this reason, without a vernier, while measuring to the nearest 10′ of arc suffices.

174. But judgment is required in the selection of the best objects, with a view to reducing to a minimum the error of position due to an 'error of observation.'

175. Right and Left Station Pointers and Using the Pointers.—The index at the middle leg, the zero of the angles, is in line with the bevelled side; and on that side where the bevelled edge of the movable leg meets it, an angle can be set off to the smallest dimensions.

Owing to the width of the middle leg, the square side of it will be some distance from the zero. This varies in different instruments from 2° to 14° or 15°; therefore, on that side, the smallest angle that can be set off will be limited by this width.

The bevelled edge of the middle leg may be on either side of it: if to the right, then the left leg cannot be set to the minimum angle; if to the left, it is the right leg that cannot be so set; and station pointers are known as *left* or *right*, according as the least angle can be set off on the left or right leg.

There are occasions, then, when one of the angles observed, if inside the limit stated, cannot be placed on a particular pointer; and the possessor of that instrument should be aware of this before he takes the angles; for, in that case he had better adapt his angles to the station pointer.

176. Reversing the Legs when the Angle cannot be 'Set off.'—It may still be possible, however, by changing the name of the legs, to put the angle on.

For example, A 10 B 100 C.

It is found impossible to put 10° on the left leg; the right side of the middle leg must then be the bevelled one.

In this case, call the right leg the middle one; lay off on that side 10°, and use the instrument with the middle as left, right as middle, and turn the left leg round, setting it 110° from the index, that is, 100° from what is now the middle leg. In this operation, the position of the vernier is reversed, and will have to be read from left to right (see par. 82).

177. Limits to the Method of Reversing the Legs.—There are limits to this method, because the tangent screws of the movable legs abutting will not allow of a nearer approach of the indices than about 80°.

178 Fixing at the Back of the Paper when Other Means Fail.—There is yet another method of overcoming the difficulty, though it is not always practicable.

Given the same fix A 10 B 100 C, and that the left angle cannot be set on the station pointer, owing to the reason given above.

Prick through to the back of the chart, or paper, the positions A, B, C; turn the paper face downwards and mark A, B, C.

Place the angles on the pointer in reversed order, that is, 100° on the left, and 10° on the right leg, and fix the position with the objects as shown on the back of the paper; prick this position through the paper on to its front.

179. **Fixing with Tracing-paper.**—Failing this, recourse must be had to tracing-paper.

In a ship with a well-found, roomy, and sheltered chart-house this is easy enough, though errors in projecting lines and angles with a protractor are introduced; but in a place exposed to wind, and when space is limited, tracing-paper does not commend itself.

In most cases, then, it is better to adapt the angles to the station pointer.

Fixing with the use of tracing-paper is unavoidable in the case where more than three angles are required to fix by; this arises when three objects, by themselves, are in such bad positions that they would give a fix that cannot be admitted, or that would be doubtful.

'Set off' the angles thereon from a common zero line, and use the lines so drawn, just as is done with the legs of a station pointer (see par. 157).

180. **Angles taken Simultaneously.** — Where accurate fixing is required, and this is especially the case in large-scale plans, the angles must be taken simultaneously, and set on the station pointer with all the accuracy possible.

181. **To Use the Station Pointer.**—Set off on each leg the angles as observed.

In the fix A 40 B 50 C the angle between A and B is known as the 'left angle'; that between B and C, the 'right angle,' or 'angle on the right.'

Place the three legs with their bevelled edges over the objects, the objects being marked with a small dot, and the leg should go through the centre of this dot; the centre of the instrument is then the position of the observer.

182. There are two methods *practised*, in handling the station pointer, for sliding the legs over the objects.

One is to place the pricker through the middle object, and hold it there with the left hand.

The right hand, holding the pointer by the arc, manipulates the right and left leg, keeping the middle one pressed against the pricker, so that he may slide the instrument backwards or forwards, and turn it as well; the eye in this case travels from the right to the left leg.

The other method is to place the pricker through the left object, or, better still, hold it there loosely with the forefinger and thumb of the left hand, allowing the leg to pivot, as well as to slide, through the fingers; bearing against that or them, manipulate the centre and right leg. In this method the eye

has not to travel so far from one object to the other, and it may therefore be the more expeditious way of using the instrument: there is, however, the very necessary caution that the left leg must be firmly clamped to the instrument.

There are six possible ways of *rapidly* manipulating the station pointer, allowing a margin for probable errors:—

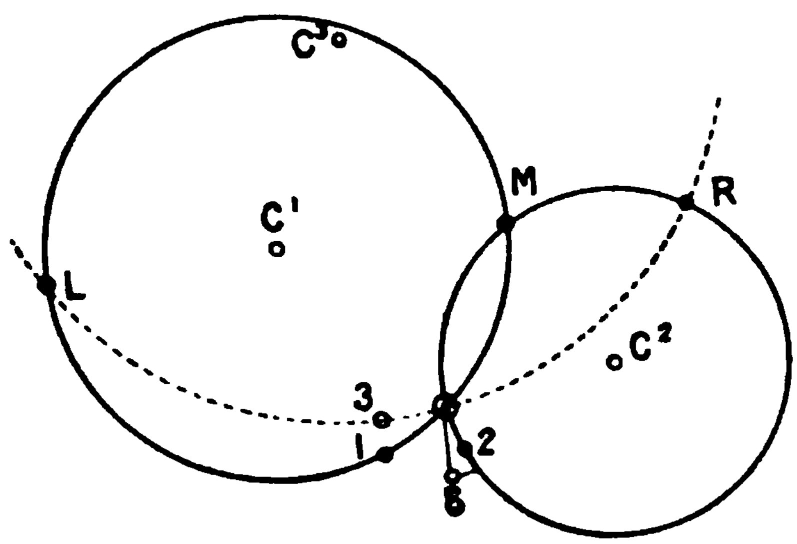


FIG. 49.

1. Pivot from the middle and left leg, *i.e.* from centre C^1 (fig. 49), and slide the right leg over R; the centre of the pointer approaches from 1 towards O.

2. Pivot from the middle and right, *i.e.* from centre C^2, and slide on to the left object; the centre moving from 2 towards O.

3. Pivot from left and right object, *i.e.* from centre C^3, and slide the middle leg over, as, for instance, from 3 towards O; L O R being the 'derived' circle in all cases. In this, the only, case the station pointer fixes from the whole angle or circle L O R and one of the others.

4. Pivot from the left leg only, slightly extending the length of pivot C^1 O on the circle L O M, then approach circle R O M; the movement of the centre is from 4 towards O (fig. 50), and is almost identical with 1 towards O.

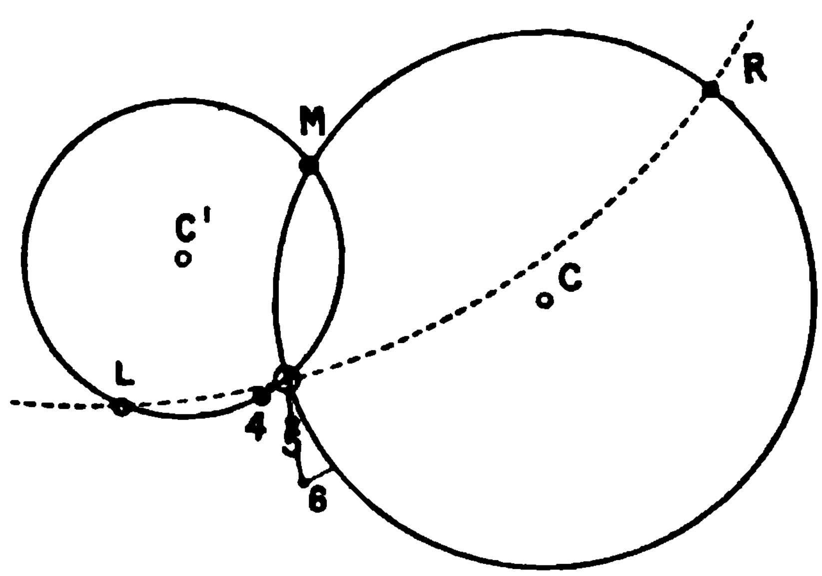


FIG. 50.

5. Pivot from the right leg only, on an extended radius of the circle R O M; the movement of the centre will be from 5 towards O.

6. Pivot from the middle leg only on an extended radius from the third circle, and approach either one of the others; the movement of the centre is from 6 (fig. 50) towards the centre of the third circle.

If there is no systematic way of handling the instrument, and the fix is plotted at leisure in a room on shore, the movement of the centre is a mixture of all the movements 1, 2, 3, 4, 5, and 6 towards O, and the value of the fix practically depends upon the accuracy of the angles.

In practice when 'fixing' is rapidly carried out, left and right combined (3) is never used, and will consequently be left out of the discussion.

1 (Middle and left) and 4 (left only). If the distance of approach from 1 or 4 towards O is small, these are practically the same thing; for when using 4 (the left only), the middle leg is usually placed over the object before any attempt is made to place the right leg; immediately this is done, 1 and 4 combine. The same remarks apply to 2 (middle and right) and 5 (right only) if there are any left-handed people who would use the pointer so. They will be left out of the discussion; their case being the same as 1 and 4. The angle of approach of 1 towards O, represented by 1 O 2 (fig. 49), will depend upon the cut of the right and left circles; and the angle of approach of 6 towards O will depend upon the cut of the whole circle L O R with either one of the others, whichever one the instrument is inclined towards (see small dotted arc from 6); for instance, from 6 towards O might intersect 1 to O or 2 to O, and will without doubt at some period affiliate with one or the other; but 1 to O will only use 2 to O in deciding the position of O.

Seeing, then, that a station pointer rapid 'fix' is contained by only two cuts of the arcs of circles, the question arises, Which two circles shall be taken?

With the infinite number, and sizes, of circles that can be made to cut each other at an infinite number of angles, it arises that perhaps the left and right circles cut each other at the best angle, *i.e.* that nearest to 90°; while again it is the third (here the derived circle) that cuts either one of the other two at the best angle (see figs. 49 and 50); L O R with M O R is the best; but this third circle is not used, except by method 3 (left and right legs), so the determination of O as regards the angle of cut, depends, in practice, entirely on the left and right circles.

As regards the movement of the legs for a small displacement of the centre.

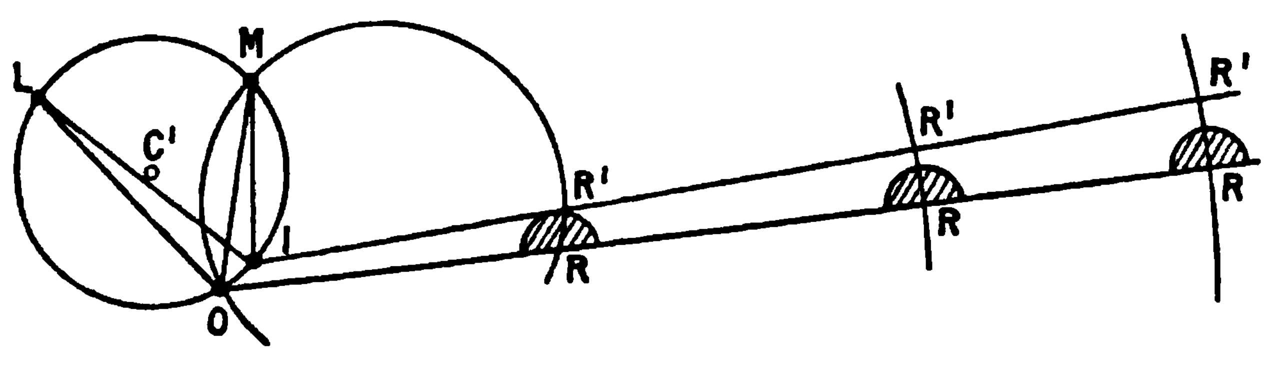


FIG. 51.

Let C^1 (fig. 51) be the centre of the left circle, C^1O the radius on which left and middle legs pivot. If the centre of the station pointer is moved from 1 towards O, the right leg moves from R^1

to R; and the greater the distance of R the more will the movement of 1 to O indicate.

Hence, if pivoting from a short radius C^1O, the movement of the legs is magnified at the distant object $R R^1$: and, conversely, pivoting from a long radius, the corresponding movement conveyed to the shorter is less.

Now when a small displacement of the centre shows a large movement of the legs at the object, the fix is less in error for any given error in the position of the leg over the object, and is therefore a good fix to adopt.

For example, in fig. 51A, moving the centre from O to 1, *i.e.* pivoting on R, the distant object, will make a movement of say x in the leg at L; x is shown by the shaded portion. In fig. 51 the same amount of movement of the centre, but pivoting round the smaller circles, will make a much larger movement than x at

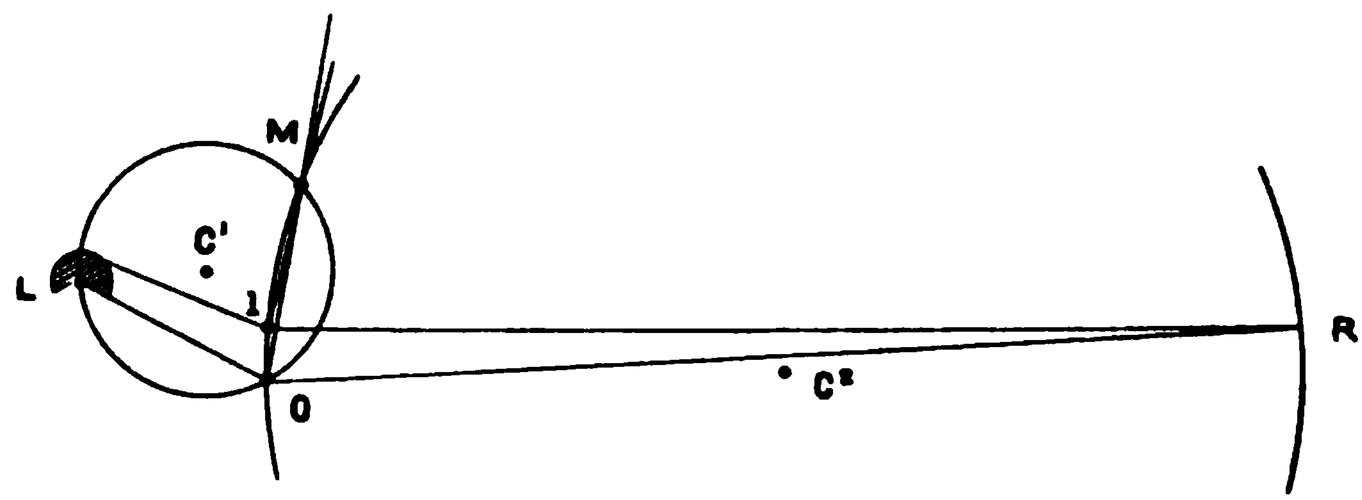


FIG. 51A.

the object R, because C^1O in fig. 51 is shorter than C^2O in fig. 51A, supposing the R's in both figures are equally distant beyond the circle L O M.

Consequently, if the legs are pivoted from the shortest radius, or, which practically means the same thing, through the nearest objects, the position of the centre is the more 'sensitive' to any displacement of the leg over the distant object. In fig. 51 the left and middle legs, or what is practically the left leg, are the best to pivot from, and the middle is the worst. If the middle leg were used, the centre would practically incline towards the right circle and eventually coincide with it, and, when kept one on the right and the other on the middle object, would be made to approach on that circle, *i.e.*, the larger, until it abutted against the circle L O M.

Using the middle leg only, is then but an apology for either the left and middle, or right and middle.

The use of one leg, *i.e.* left or right, in preference to another does, without doubt, make a difference in the placing of a fix; for if celerity, combined with a reasonably probable error, is indispens-

able in the handling of a station pointer, putting aside errors in observed angles, it is practically the same as admitting a small error in the position of each object used, but principally that one which is approached with the leg; in which case, a movement of the centre should show the maximum movement at the leg where it is supposed to touch the object: and then the 'fix' also is easy to place: the position of objects C, M, and A in fig. 51A, or L, M, R in fig. 51, represent exactly what an ideal 'fix' will be at O, and pivoting with the left leg.

The conclusion is, that it is easier, and more 'sensitive,' to pivot from *either* the left (practically the left and middle), *or* from the right and middle; but the right and middle combined is a left-handed way of handling the instrument. It may arise, however, that using the middle leg only, becomes a substitute for what is eventually the middle and right, as is shown in fig. 51; and in most cases, by using the middle leg only, the tendency will be for the centre to attach itself to the circumference of the greater circle; therefore it is usually the least accurate pivot for fixing.

CHAPTER XII.

STATION POINTER. ERRORS OF POSITION DERIVED FROM THE INTERSECTION OF LINES, DUE TO AN ERROR OF OBSERVATION.

183. Let O (fig. 52) be the position of an observer; O X a determined line of reference, or a line of position.

Let O C be a selected line of reference from C, to fix the position of O.

Then C O X is the 'receiving angle,' and the *true* angle at O between X and C.

Suppose the *observed* angle to be $C O^1 X$; then O^1 is the derived position, and $C O^1 X = C O X + O C O^1$, *i.e.* the observed angle = true angle + $O C O^1$.

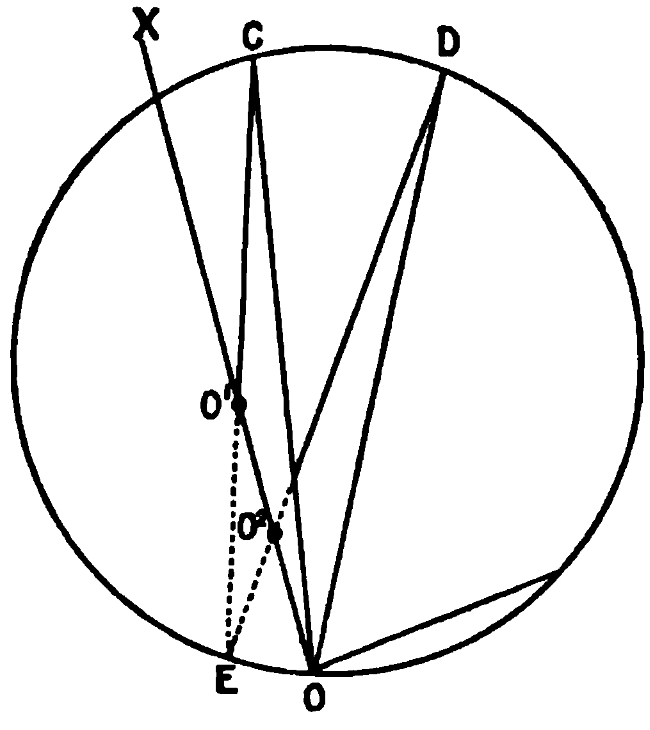


FIG. 52.

$O C O^1$ is therefore the 'error of observation,' and in consequence of this error, $O O^1$ is the resulting error of position in an angle whose dimension is $C O^1 X$ or C O X ('the error of observation' being very small as compared with C O X), when the distance = O C.

Now if, instead of using O C as a line of reference to fix O, the line O D is adopted, D O X being larger than C O X, but O D remaining = O C.

Produce $C O^1$ to meet the circumference of any circle drawn through O and C, at E.

Join D E cutting O X in O^2; since O C E = O D E, both angles standing on the arc O E, and both being the error of observation.

Then $O O^2$ is the error of position due to the same error of

observation as before, when the angle D O X is > C O X, though OD = OC.

Hence, for an error of observation, the greater the receiving angle at O the less is the corresponding error of position when the objects of reference are equidistant from O.

The error of position $O\,O^1 = O\,C \,.\, \sin O\,C\,O^1 \operatorname{cosec} C\,O\,X$. If the error of observation is constant, then $O\,O^1$ varies as distance $O\,C \,.\, \operatorname{cosec} C\,O\,X$; and the greater the distance the greater will $O\,O^1$ be. The cosec of 20° is 2·9; of 30°, 2·0; of 40°, 1·3; and of 60°, 1·1. So that the smaller the angle the greater will the value of the cosec be, and hence the greater the value of $O\,O^1$.

Hence the greater the distance, and the smaller the angle, the greater will be the error of position in the intersection of two lines.

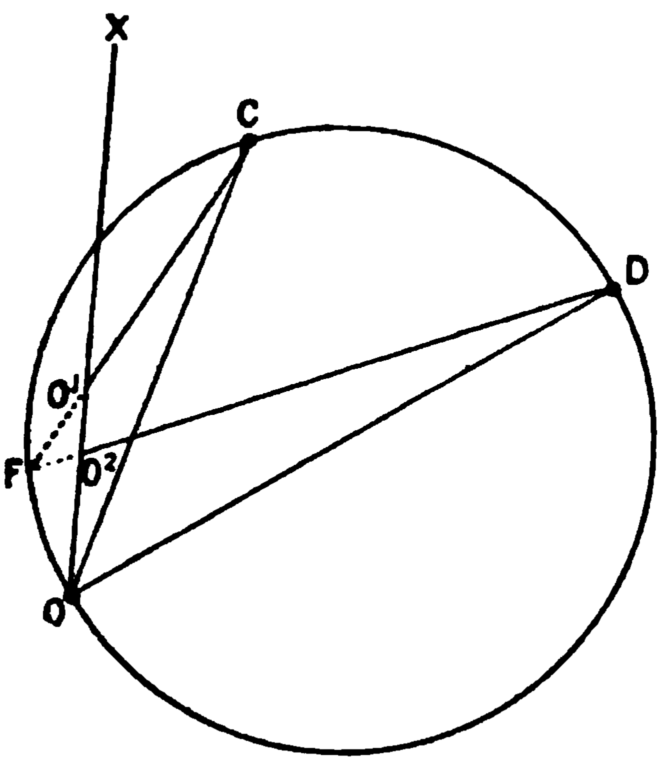


Fig. 53.

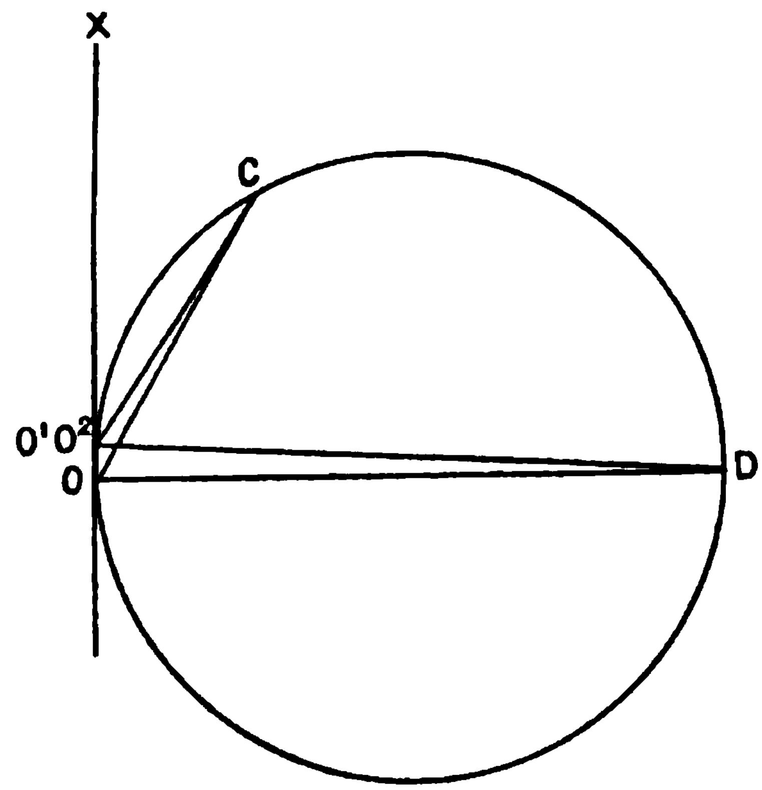


Fig. 54.

This is explained graphically by the three figures 52, 53, 54.

In fig. 52 the distances O C and O D are equal, but C O X is < D O X; and

$O\,O^1 = O\,C \,.\, \operatorname{cosec} C\,O\,X \,.\, \sin$ error of observation;

$O\,O^2 = O\,D \,.\, \operatorname{cosec} D\,O\,X \,.\, \sin$ error of observation;

$\therefore\ \dfrac{O\,O^1}{O\,O^2} = \dfrac{\operatorname{cosec} DOX}{\operatorname{cosec} COX}$; that is, $OO^1 : OO^2 :: \operatorname{cosec} COX : \operatorname{cosec} DOX$.

In fig. 53, case 2, the receiving angle C O X is smaller than D O X as it is also in fig. 52; but the distance O C is now < O D. The difference between the error of positions is now O^1O^2 which is smaller than O^1O^2 in fig. 52. For, using the formulæ

$O\,O^1 = O\,C \operatorname{cosec} C\,O\,X \sin$ error,

$O\,O^2 = O\,D \operatorname{cosec} D\,O\,X \sin$ error,

$\therefore\ O\,O^1 : O\,O^2 :: O\,C \operatorname{cosec} C\,O\,X : O\,D \operatorname{cosec} D\,O\,X.$

$$\frac{OO^1}{OO^2} = \frac{OC}{OD} \cdot \frac{\text{cosec } COX}{\text{cosec } DOX}$$

$\frac{OC}{OD}$ is < unity because OD > OC; in fig. 52 $\frac{OC}{OD}$ was made = to unity. $\therefore$ $\frac{OO^1}{OO^2}$ in fig. 52 is $> \frac{OO^1}{OO^2}$ in fig. 53.

Finally, in case 3, $O\,O^1 = O\,O^2$.

If $O\,O^1 = O\,O^2$ (fig. 54), then each = O C cosec C O X = O D cosec D O X.

If O C = $\frac{1}{2}$ O D, cosec C O X must = 2, *i.e.*, C O X = 30°,
and cosec D O X must = 1, *i.e.*, D O X must = 90°.

Hence, every line will have a plotting value depending upon the size of the receiving angle, and the length of the line.

Any point can be taken between C and D so that O C may be of any proportion to O D, and a value can be found for C O X relative to D O X, when D O X = 90° and $O\,O^1$ shall = $O\,O^2$; from which the deduction is, that, given a line OX, and another O D with reference to it making a right angle at the point O, a

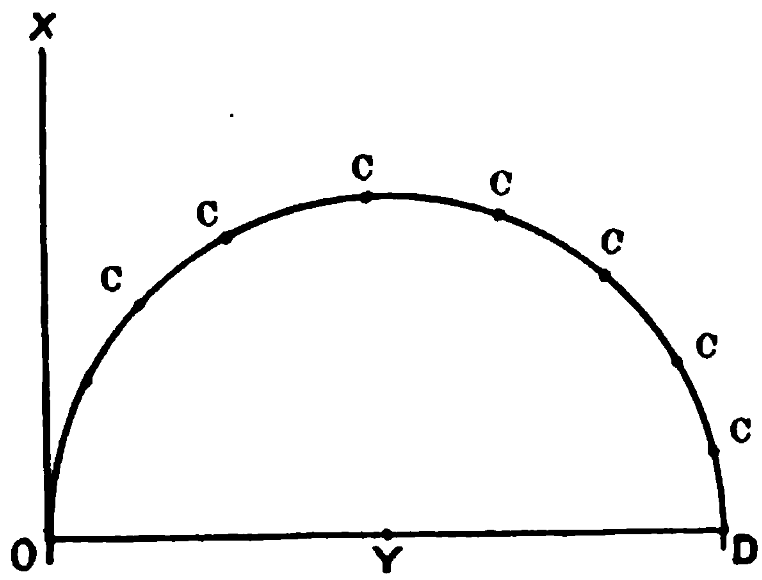


FIG. 55.

cosecant curve can be evolved which will represent a curve of equivalent errors (O^1O^2) of position O, with regard to the angles and proportional distances of any point C.

Thus in fig. 55 let O X be a determined line of reference. O D is one at right angles to it.

184. Arc of Equivalent Distances for the same Error of Position.—Bisect O D at Y. With Y as centre and O Y as distance describe a semicircle.

On any part C of the semicircle an object can be chosen as a point of reference from which to project the line C O intersecting X O, and the resulting error of position at O for any given error of observation will be identical. And see par. 190.

As an illustration of its practical utility, suppose a ship anchored at O. A *bearing* is taken of X determining the line of position X O drawn from X.

Let D be a distant summit, and X O D = 90°.

The first impulse is to take the bearing of D, because X O D is a right angle, the purpose being to obtain a 90° receiving angle.

Now through O D (fig. 56) draw a semicircle of equivalents, represented by the dotted line. Then on any part of that semicircle, the object would be quite as 'good' as D, from which a line of reference can be drawn; and a line projected from C would evidently be more accurate than from any other point along the circumference.

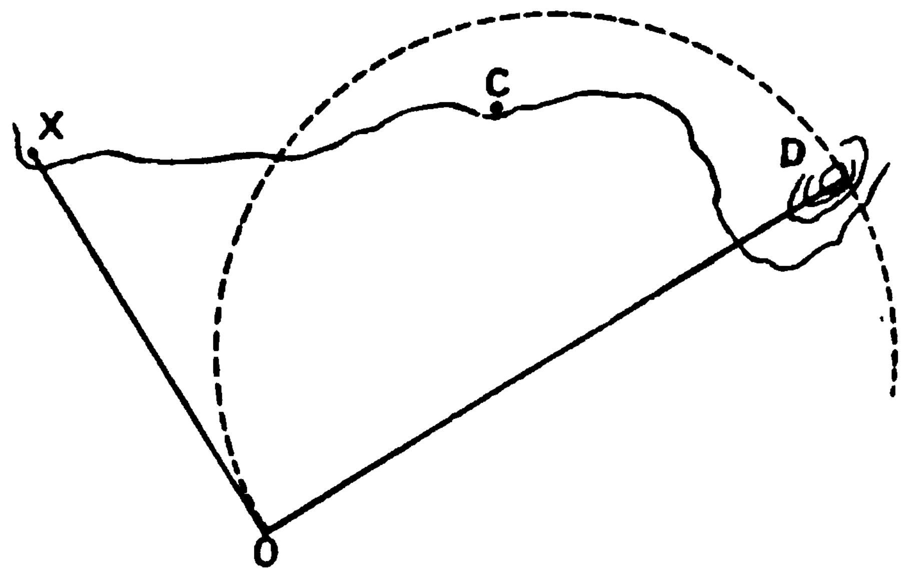


FIG. 56.

Moreover, D is an ill-defined summit, and C may be a well-defined flagstaff, or church steeple; so that under such circumstances, C would be doubly the more accurate to take.

185. Error in the Arc of Reference due to an Error in Observation.—Precisely the same result and law is deduced when a circle, or arc of reference, is drawn through two objects and the observer.

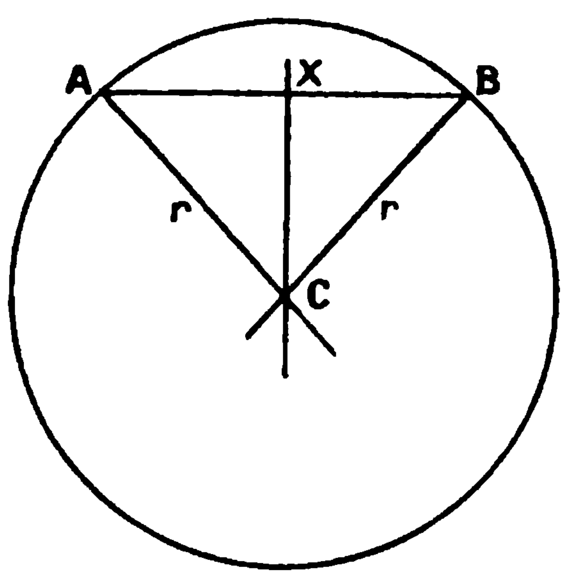


FIG. 57.

Given an error of observation in one angle, the nearer the angle is to 90° the less will be the corresponding error in the radius of the circle, and in consequence in the arc described with that radius.

Let A B (fig. 57) be two objects subtending any angle a.

From the ends of the line A B lay off A B C and B A C, each being 90° − a (see par. 171).

At the intersection of the lines drawn will be the centre of a circle, the circumference of which will pass through A B and the position of the observer anywhere on the circumference; A C and B C being the radius = r.

In fig. 57, from C draw C X perpendicular to A B; this will bisect A B in X. Then B C X = $\frac{1}{2}$ A C B; and since A C B = twice the observed angle, therefore B C X = a (B C X being $\frac{1}{2}$ A C B).

In Δ B C X, B C = B X . cosec B C X; that is, the radius equals $\frac{1}{2}$ A B . cosec a: this is the same law as deduced in par. 183.

The value of the cosecant changes more rapidly at small angles than in those near 90°.

Hence for a small error, when the angle is small, there will be a greater error in the radius, and consequently the arc described, than when the angle is 90°.

186. Error in Position due to the Angle of Cut of the Arcs of Reference.—Secondly, given a small error of observation

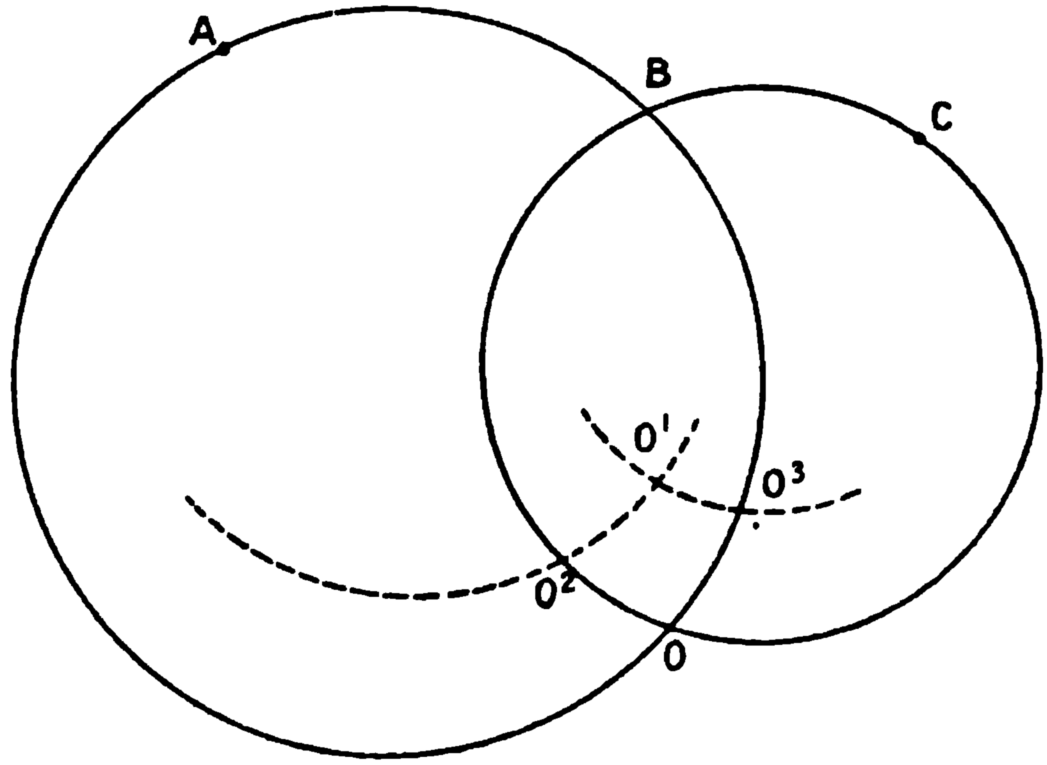


FIG. 58.

in both angles, the nearer to 90° is the receiving angle, at the position of the observer, produced by the intersecting arcs of circles, the less is the dimension of the corresponding area included in the small arcs of circles, these circles being formed by the observed angles ± the error of observation; and the amount of this area solely depends on the combination of, the magnitude of the receiving angle of the arcs at O, and the distance of the centre of each circle from the observer, *i.e.* the size of the circle (see par. 193).

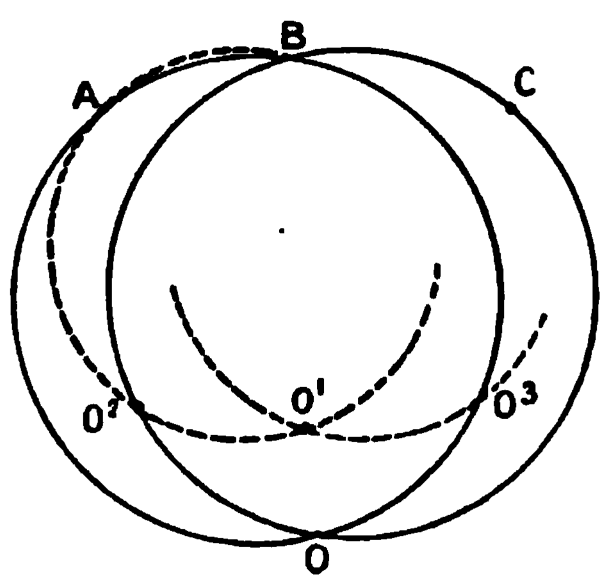


FIG. 59.

Each angle observed has a + or − error, the sign of which is unknown; each therefore creates two circles; and four circles thus existing will indicate four points of intersection, shown by O, O^1, O^2, O^3 (figs. 58 and 59).

CHAPTER XIII

STATION POINTER—*continued.*

187. Good and Bad Fixes and Total Errors in 'Angle of Observation' when Fixing by Station Pointer.—Station pointer fixing is one of the most important matters in surveying, and should be thoroughly understood: it cannot be learnt by mere statements, nor by illustrations of good and bad fixes.

The three conditions which govern the value of a 'fix' depend upon the following:—

1. Given an error of observation, in creating a circle formed by any angle between any two objects, the nearer the observed angle is to 90° the less will be the corresponding error in the circle (see par. 185).

This condition is controllable by the eye, inasmuch as it applies not only to angles but also to distances (see *Circle of Equivalent*, par. 184).

2. Given an error of observation in two angles, the corresponding maximum error of position, *i.e.*, the maximum diameter of the oblong created, is the least when the intersection of the circle is at right angles.

This is qualified by the proportion of the size of the circles to the angle of 'cut': the value of one may counteract the value of the other.

3. In using the station pointer—

(*a*) Given an error in placing the leg over the object, representing the object by a dot of say $\frac{1}{20}$ inch in diameter, the corresponding error at the centre of the instrument is least when the observed angle is 90° (see par. 190).

This condition is controllable by the eye so far as it applies to the size only of an angle.

(*b*) Given the same error as above at both legs, or at both objects, then there is the least corresponding error at the centre of the instrument when the circles cut at right angles; or, conversely, when the circles cut at right angles, a small move-

ment of the centre makes the maximum movement at any point along the legs.

This movement will depend upon the size of the circles, and the angles at which they cut; each qualifying the other. Conditions 2 and 3 (*b*) are only controllable by experience and by practice (see par. 172).

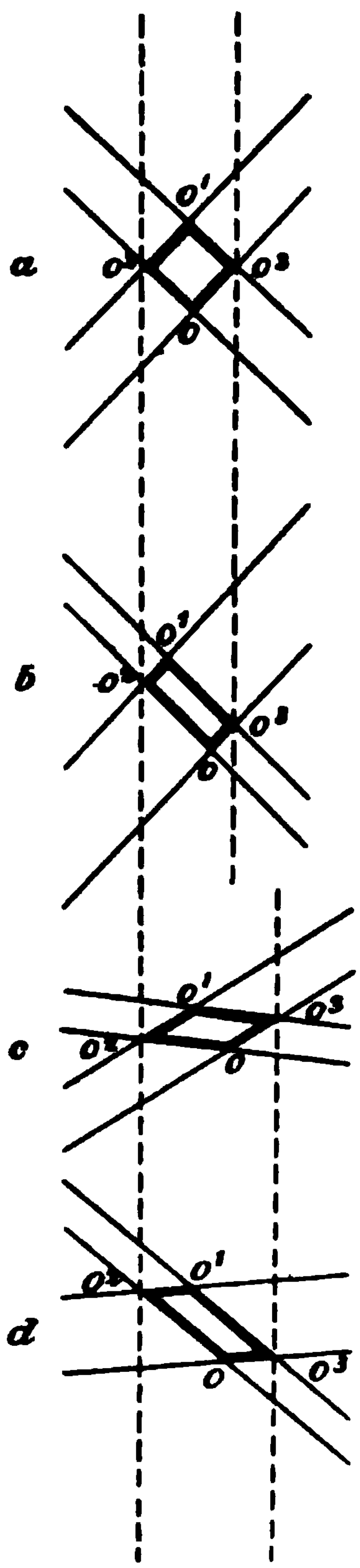


FIG. 60.

If *e* (the error) is small, we can assume these small arcs to be straight lines.

1. When the circles are of equal dimensions, and cut each other at right angles, the figure formed by the four lines will practically be a square, and the maximum error of position is either O to O^1, or from O^2 to O^3 (see fig. 60, *a*).

2. If the circles are of unequal dimensions, but still cut each other at right angles, then O to O^1, or O^2 to O^3, is the maximum error.

The areas of fig. 60, *a* and *b*, are the same.

3. Circles of equal dimensions cutting each other at an *acute* angle produce fig. 60, *c*; O^2 to O^3 now being the maximum error of position.

4. Lastly, if circles of unequal dimensions cut each other at the same acute angle as in case 2, the error of position is shown by O^2O^3 in fig. 60, *d*.

In every case, the larger the circles involved, the greater the areas representing the corresponding error of position; and hence, when using distant objects to fix with, the greatest care is required in observing the angles.

188. In taking any observation for fixing by station pointer the following errors probably arise:—

(1) Errors in the instrument ($\pm 3'$; see *ante*).

(2) Errors in reading the sextant ($\pm 1'$).

(3) Errors in the measurement of the angle ($\pm 6'$).

Total sextant and observing errors ($\pm 10'$).

(4) Error in the station pointer ($\pm 2'$; see *ante*).

(5) Error in placing the angles on the pointer ($\pm 1'$). On tracing-paper this is probably $\pm 30'$.

(6) Errors in the position of the objects.

Errors in the Position of Objects on a Published Chart.—This error is common to all published charts. It

arises from the distortion consequent on engraving, as well as on the expansion or contraction of the chart through change of temperature, and wear and tear.

It may be taken that the objects on charts are possibly $\frac{1}{8}$ inch of paper in error. The consequent error will depend directly upon their distance from the position of the observer. In fact, it is the angle subtended by $\frac{1}{8}$ inch of paper at so many inches of paper distance, and forms a part of error (8); see below.

For a place on a scale of 3 inches to a mile, at 2 miles' distance, the maximum error in the angle would amount to about $\pm 1°$; at twice that distance, or on $1\frac{1}{2}$-inch scale, the error will be halved.

189. **Angles not Taken Simultaneously.**—(7) If the angles are not taken simultaneously, the error in a boat could probably be $\pm$ 15′.

In a ship, the maximum could easily be $\pm 7°$. Suppose S (fig. 61) is the ship steering to the eastward at 15 knots. Suppose the interval of time between one angle being taken and the next is 1 minute. The ship has then traversed $\frac{1}{4}$ mile. Let A and B be two objects in transit, A being $1\frac{1}{2}$ mile and B 3 miles distant. At the instant of the first angle they were in transit; at the second, the ship was at S^1: $S S^1 = \cdot 25$ mile, $S B = 3$ miles, then $A S^1 B = 4°$: the greater the distance between A and B, the greater the error. The error No. (7) can be considered, as far as a boat is concerned, when the change of position between the angle first observed and the position at the second observation, *i.e.* the distance $S S^1$, fig. 61, to be about 30 to 50 feet, and can be included in the error of observation; but where the distance between the points where the angles were taken, such as would occur in a ship navigating along the coast, is considerable, the error becomes so large, that means should be taken either to eliminate it entirely or to reduce it to a minimum. The means of doing this are explained at par. 211.

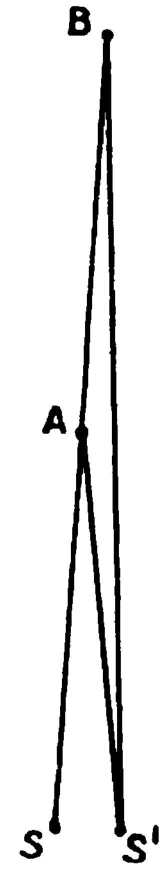


FIG. 61.

For causes up to No. (7) as enumerated above, the total error may be assumed to be $\pm$ 40′ in a boat, and 2° in a ship.

190. **Errors in Sliding the Legs.**—(8) Errors due to not placing the legs correctly over the objects. There is a double movement here: one of pivoting from the centre, the other a forward slide; in part, it resembles error (6). No value can be assigned to this, depending as it does upon the 'angle of approach' of one or of both legs, which varies with the size of the angle; and this, again, depends upon how the instrument is handled.

Let us take two extremes. Let A, B, C, be three objects from which a fix is obtained, the angles being A O B, C O B.

In fig. 62, for a movement of direct approach of the legs, $=C^1C$ or A^1A, the centre has to be moved from O^1 to O.

In this case, a small error in C^1C, makes a larger error in O^1O:

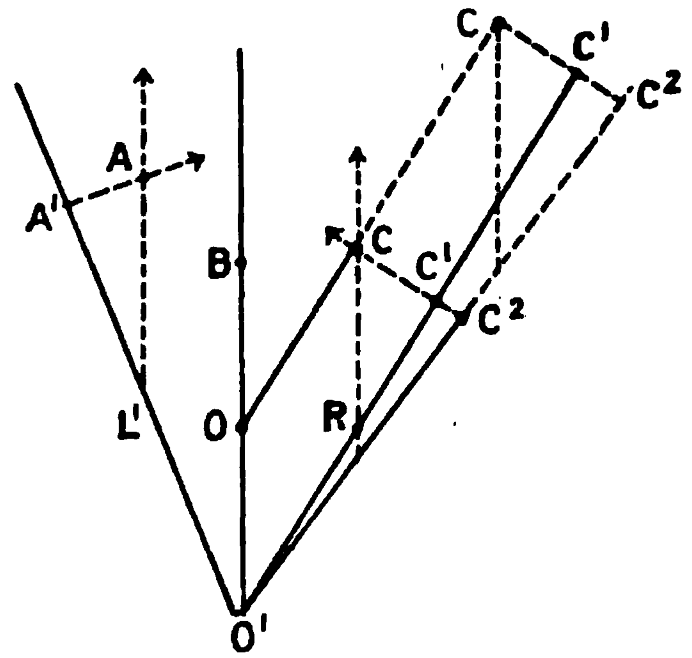


FIG. 62.

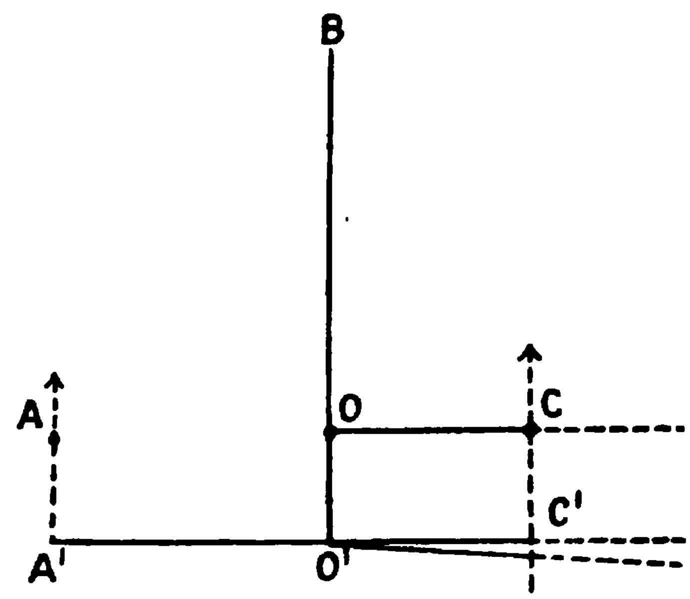


FIG. 63.

$O\,O^1 = C\,R = C\,C^1 \,.\, \sec$ observed angle. If $B\,O\,C = 20°$, then the sec of $20° = 2 \times O\,O^1 = 2\ C\,C^1$. Along $C\,C^1$ is the error in placing the legs over the object. Conversely, O^1 can be moved a small way towards O, but the change along C^1C will be inappreciable.

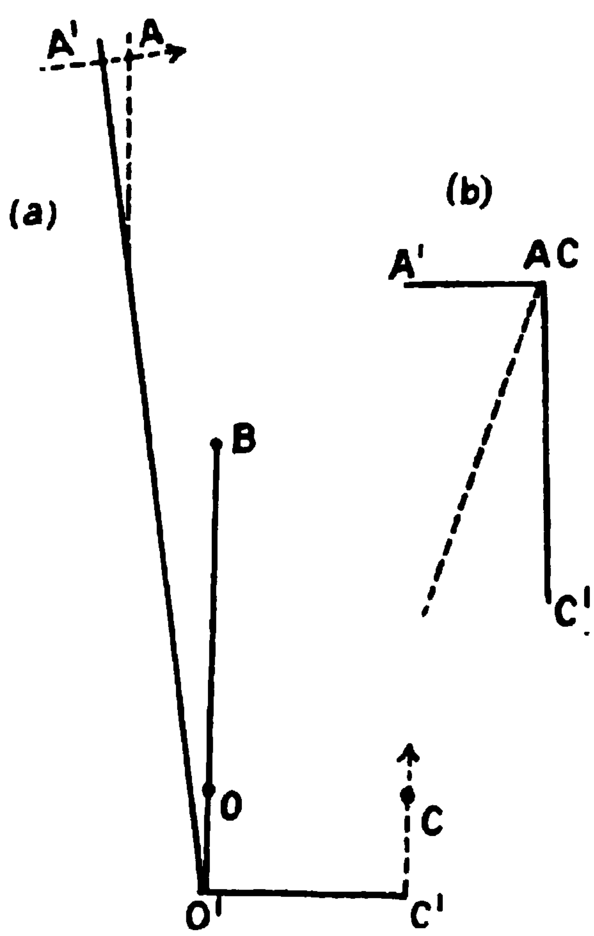


FIG. 64.

In fig. 63, where both A O B and C O B are 90°, for the same length of movement of the legs as before, *i.e.* C^1 to C, the centre has only to move over the distance O^1O.

Here the movement of O^1 to O is parallel with the movement from C^1 to C; and if one movement is evident, so will the other be. $O\,O^1 = C\,C^1 \,.\, \sec 90°$; $O\,O^1 = C\,C^1 \,.\, \sec 90°$ being infinity.

There are, however, intermediate cases, such as where one angle may be small while the other is near 90°. In this case, where the fix might read A 10 B 90 C (see fig. 64, *a*), the angle of approach, *when using both legs*, is the resultant of two movements: one from C^1 to C, and the other from A^1 to A (see fig. 64, *b*). A^1A being at right angles to O^1A^1, it may be represented by the dotted line. The smaller A^1A is, the nearer does the resultant approach to $C\,C^1$.

When A^1A, fig. 64 (*a*), $=0$, then A is in line with B, and the movement is from C^1 to C, and to A *through* B.

Now, owing to the causes enumerated, the total error of observation is 40′ in the case of a boat, and 2° in the case of a ship; that is, when the angles are not taken simultaneously.

And, according to previous demonstrations, the greater the distance and the smaller the observed angle the greater is the corresponding error of position for an error of observation.

Referring to fig. 62, as O C increases in distance with a given error of observation, the greater will $C\,C^2$ become.

For the same distance in fig. 63, when B O C is a right angle, $C\,C^1$ will increase in the same proportion as in the other case to $C\,C^1$, but there will be less consequent movement in the instrument.

The angle A O C is the angle at O between A and C, respectively the left and right objects; that is, the sum of B O C and B O A, known as the 'whole angle.'

191. Therefore the nearer each angle is to 90°, or one 90° and the other very distant and making 0°, the better is the fix as regards the 'approach' of the legs towards the objects.

An inferior approach occurs when the small displacement of the legs from the points represented by $C\,C^1$, fig. 62, makes an *increased* displacement of the centre, shown as $O\,O^1$, fig. 62.

This paragraph should be combined with the 'pivoting' of the legs, which is a separate thing (see par. 182).

192. It must not be forgotten that there is, for an 'error of observation,' excluding Nos. 6 and 8, the balancing of angles with distances—small angles with short distances as against a 90° angle with longer distances (see *Circle of Equivalent Errors*, par. 183). In practice, the circle of equivalents is not drawn; and practice alone will enable an observer to conceive it by the eye.

193. Cuts of Circles.—In connection with the preceding statements, there follows the problem of intersecting circles.

On p. 66, par. 170, it has been shown that, given the horizontal angle subtended by two objects, a circle may be drawn through these objects and the position of the observer; and since, in fixing with the station pointer, there are three objects and two angles, one object being common to both angles, there will consequently be two circles, and at their intersection is the position of the observer.

In A 40 B 50 C, B is the common object, the angles being A 40 B, B 50 C.

In $\left.\begin{array}{l}\text{A 90 C}\\ \text{B 50 C}\end{array}\right\}$ C is the common object, the angle being $\left\{\begin{array}{l}\text{A 90 C}\\ \text{B 90 C}\end{array}\right.$

A 90 C is known as the 'whole angle,' and, according to fig. 47, B 50 C is the right angle; if one be subtracted from the other, the difference derived will be the left angle, viz. 40°.

At p. 15 it has been explained how the angle between two objects may be obtained if one of them is too indistinct to reflect: the above may be a case in point. For other reasons for this method of finding the derived angle, see par. 131.

Given, then, an angle at O between A and B, a circle may be described through O, A, and B; and, similarly, given the angle between B and C, another circle can be described (see par. 171).

These two circles emanate from the right and the left angles separately; and at their intersection will be the position of the observer.

If A and C be joined, a circle may be drawn through them and position O, with the dimension of A O C, the whole angle (see par. 174).

When projected, the nearer to a right angle any two of these circles intersect at O, the better.

194. Angle of Cut not Conceivable by Eye.—When the ship is fixed by two *bearings*, the value of the fix depends upon the receiving angle at the position of the observer and the distances of the objects fixed from (see par. 183); and being *lines* of position, the degree of goodness or of badness of the fix is evident to the eye.

In the case of *circles*, the intersection of the arcs of position (the 'receiving arcs') are not evident, unless they are actually drawn.

In the first case, that of bearings, two or any number of objects may be in one line; obviously the lines of bearing superpose and there is no cut anywhere.

195. Circles Superposing.—Similar conditions may arise with the circles through any number of points without the fact being obvious; and, if they all coincide, there is no cut. For example—A, B, and D, fig. 65, are all on the same circle with the observer; if the angles B O D, B O A, be placed on respectively the right and the left legs of the pointer, the centre of the instrument can be placed at any part of the arc A O D, and the legs will still pass through A, B, and D: O may then be anywhere along the arc A O D.

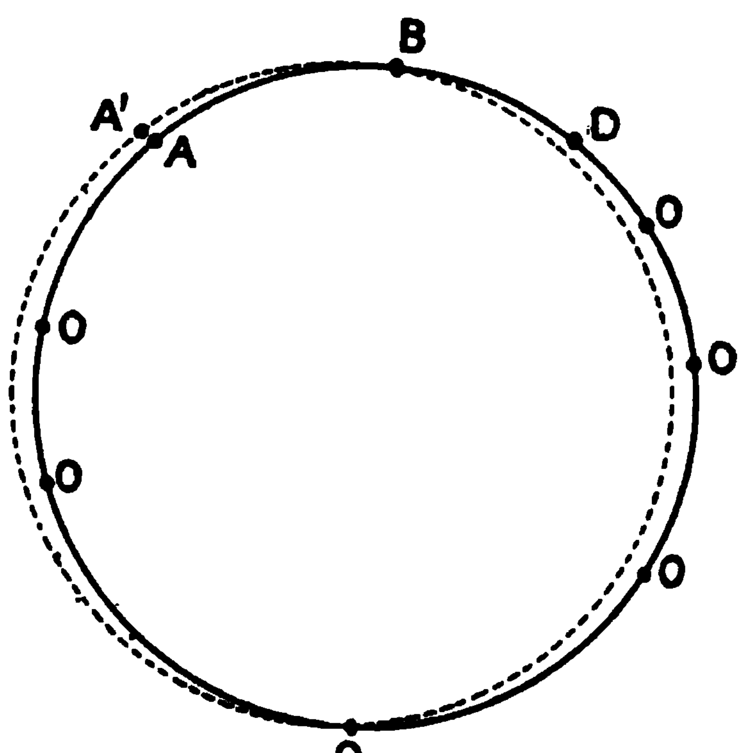


Fig. 65.

This is evidently impossible to fix.

The chances are very much against this coincidence occurring; but if A is at A^1 for instance, *i.e.*, just off the common circumference, then the intersection of the two circles at O is at a very acute angle.

And, for the error of observation, the oblong of arcs will be very elongated, leaving the points of the maximum diameter a considerable distance apart; the error of position then may be very large.

Where the choice of objects limits the observer to an evidently bad fix, such as shown, recourse must be had to a *line* of position, (a bearing) taken of one of the objects (see par. 198). Such a fix then consists of the intersection of an arc of position, with a line of position.

196. Two Equal Circles cutting at Right Angles.—Now two circles of *any* dimensions can be drawn so that they will cut each other at right angles; from one of infinite diameter to the smallest possible, each in combination with the other.

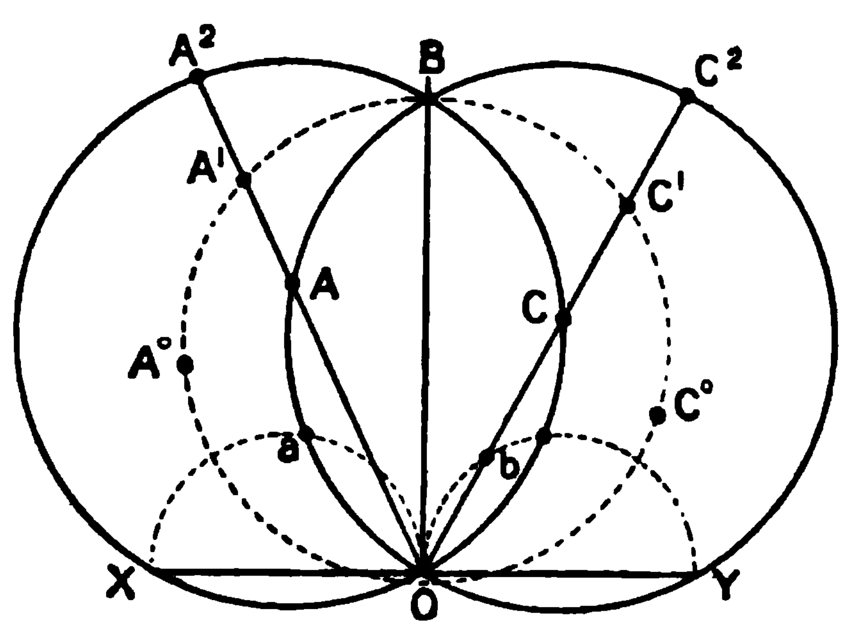


FIG. 66.

Take the case of equal circles first. In the above case (fig. 65) they are of the same diameter, and their 'cut' is an impossible one for fixing purposes.

Let them be drawn so as to cut at right angles (see fig. 66); then the circumference of each will pass through the centre of the other. Let A, B, and C represent respectively the left, middle, and right objects, and O, where the circles intersect, be the position of the observer.

Then, from the observer at O, objects placed at A, B, and C produce the best possible cut; but the angles of approach B O A and B O C, being very small, are not well conditioned, and a small error in observing the angle, or in the manipulation of the legs over the objects, would make a large or magnified error of position.

It is still, however, a very good fix, but *only* as regards cuts of circles.

197. Undesirable Fixes.—Suppose this fact admitted, and the size of the angles is put aside, there is another objection to the fix being generally adopted.

For, bisect O B, and draw a circle with OB as diameter, shown dotted in fig. 66.

It will be now noticed that, though the angles B O A and B O C remain the same, if C is at C^1, and A at A^1, the conditions of fixing with A^1, B, and C^1—all, including O, being on the one circle—are impossible, as explained by fig. 65.

In selecting such subjects as A and C, it is hardly possible that anyone, however experienced, will be able to know definitely whether A is at A or at A^1, or if C is at C^1 or not.

In the one case, the fix is the very best as regards cut; in the other, the very worst; and since there is such a small margin for mistaking one *distance* for the other, such a fix is usually let alone.

If the distances O A and O C remained as shown, and a small change was made in the angle, overcoming the objection to the small angle, it would place them at A° and C°, both $A\,O^\circ = A\,O$ and $A\,C^\circ = A\,C$, on the same circle as B and O—again in an impossible position for fixing, though not very different in their relative positions from the very best cut.

And since there is no way of exactly knowing what their true relative positions are, such a fix is not used.

198. Bad Fix using Station Pointer and Bearing.—The student is here reminded that in par. 179 it is stated that if the fix is unavoidably a bad one, there must be check angles taken, and all the angles plotted on tracing paper; and in par. 195 that in a vessel with a suitable compass, a bearing of one of these objects could be taken (see *Circle of Equivalent Errors*, par. 184). In fig. 66, C or A is the best to take the bearings of, for they fall within the circle of equivalent errors, which in this case is $B\,C^1O$ or $B\,A^1O$.

199. Middle Object Nearest.—Referring to fig. 66, if O A and O C be produced to A^2 and C^2, the circles $C^2B\,O$ and $A^2B\,O$ still intersect at O, and the cut remains the same; but the difference between these positions and A^1 and C^1 with B, is that both A^2 and C^2 are further from the observer than B; and hence while B is nearer to O than A^2 or C^2, neither of them can be on the circle described through B and O; therefore, when the middle object (B) is nearer the observer (O) than either of the other two, they cannot all four possibly be on the same circle. This is the condition necessary to avoid the impossible fix.

200. Good Cuts with Bad Angles.—Now the fixing objects are A^2, B, C^2 (suppose $B\,O\,A^2$ and $B\,O\,C^2 = 30^\circ$), and the cut is a right angle; still it is not the best fix, for the angle of approach is not good, the angles being small.

The angles being small compared with the distance, an error of observation will make a considerable error in position (see p. 190)—a greater error than if A^2 or C^2 were at A or C.

201. Good Cuts with Good Angles.—Still retaining the same circles, take two positions X and Y at *right angles* to A B.

The cuts of the circles remain the same, and the fix is now the very best possible; for the angle of approach is the best possible, on account of each being 90°.

These angles are such that an error of observation makes the least error in position, and the distances are short as compared with O B.

If now on O X and O Y respectively a circle of equivalents be drawn—that is, with diameters O X or O Y—it will be seen that they intersect the original circles through B O X and B O Y at positions *a* and *b*.

202. Good Cuts with Equivalent Positions.—These last positions *a* and *b*, are equivalent in every way to X and Y for fixing objects in connection with B; they are not used in practice, for the reason already given, *i.e.* to avoid the impossible, but are shown to illustrate the theory of equivalent angles and *distances* combined with cuts of circles.

203. Cuts of Large and Small Circles.—The next consideration is, that of circles one of which is of infinite radius, and the *other of normal* size.

Let A B O (fig. 67), the circumference of a circle A O of infinite radius, cut the circle C B O at right angles through O, the position of the observer.

A B O will evidently pass through the centre of circle C B O, and O B is the diameter of that circle.

The *cut* of the circles is the best possible with *any* position of C along B C O.

The *angle of approach* will be the best when C is at C^1, and O C^1 is at right angles to O B.

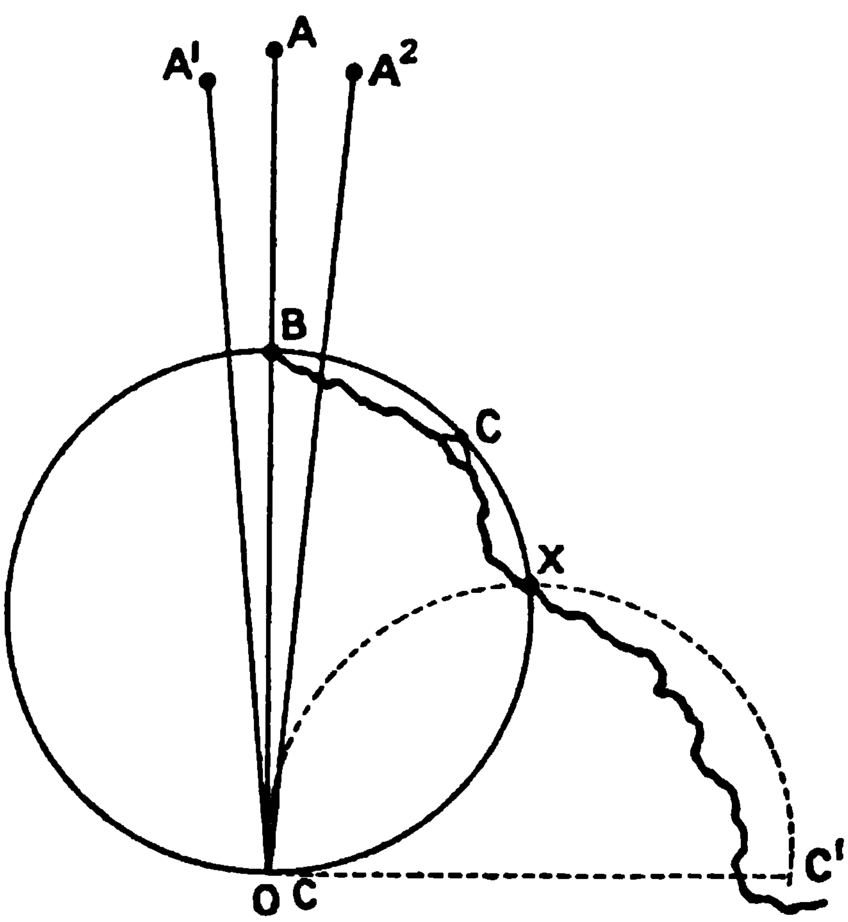


Fig. 67.

If B O C^1 is 90°, the error of position will also be at a minimum for error of observation; but a circle through B O C^1 would not cut A O at right angles; and the nearer is C^1 the less that error and the better the cut at O.

Hence, with two objects in transit, an angle of 90° to the third object, and as near as possible to the observer, shown by C on the line OC^1, gives the best possible fix.

As near as possible, requires that C shall be at least no nearer, or not much nearer than B, under *practical* conditions.

Let C^1 be the practical position, *i.e.*, the normal position along a coast-line, of the right object.

The dotted circle represents an arc of equivalents drawn with $O\,C^1$ as diameter, and where it intersects arc B C O at X gives the position of an equivalent position to C^1 on the *desired* circle B C O. X, then, will *practically* be the best object to take.

204. Objects in or near Transit.—In theory it is immaterial what the distance of A may be, but in practice the further distant A is, the more 'sensitive' is the position of O to the transit; that is, the further is A, a small error at O will make an appreciable error in the transit (see par. 189, fig. 61).

205. The Middle Object the Furthest.—Before concluding with this particular case, it may be pointed out that a small deviation from the above will necessarily also be good. If, for instance, A is at A^1 or A^2, and $B\,O\,A^1$ or $B\,O\,A^2$ very small, the circles through A^1, B, and O, or A^2, B, and O, will nearly coincide with A B O—near enough to be practically the same. Such a fix is therefore admissible, and would read A^1 5 B 75 X, or B 5 A^2 65 X. Or if A^1 or A^2 are a considerable distance off, the result is the same. In each case great care must be taken in measuring the small angle, or the angle to the distant object.

In the first case it may be noticed that in B 5 A^2 65 X, A^2, the middle object, is the furthest, and the fix does not conform to the rule about the middle object being the nearest; it must, however, be selected with great discretion, remembering that it must be *considerably* the furthest distant, or at least three times the distance of O B.

206. Between the cuts of circles of equal size, and those of infinite opposites, there are infinite intermediate cases of circles of varying sizes, and their dimensions will depend upon the angle at the position of the observer, and the relative position of the objects.

207. No Simple Law for selecting Objects for Circles to cut at Right Angles.—It is not possible to give a definite law, which can be humanly practicable, to assist an observer to select his objects, so that the circles formed will cut each other at the observer's position at right angles.

208. To Avoid the Impossible Case.—But there are more or less definite conditions which prevent the opposite, viz., the impossible case.

So long as the observer is not in the same circumference of a circle which joins the three objects, it is necessary that the middle object shall be nearer than either of the other two: nearer than one of them does not suffice.

If an observer is within the triangle formed by joining the

three objects, he cannot possibly be on the circumference of the circle joining them.

209. All Other Cases Possible.—In all other cases the circles, of varying sizes, will cut at varying angles.

The angle at which they will cut, depends entirely on the choice of objects; but how to choose them so that the cut will be nearly right angled, cannot be stated in any practical form.

It may, however, be suggested that, assuming that the middle object is the nearest to the observer, he must, by practice or intuition, conceive an infinite number of circles going through his position and the middle object; and one of them to pass through whatever object he has at his disposal on one side of the middle one, say, to the right.

210. Choosing a Circle.—In selecting this right object, he must carry in his mind the size of an angle as compared to distance, so as to take into account error of observation (see par. 185, as well as *Circle of Equivalents*, par. 184).

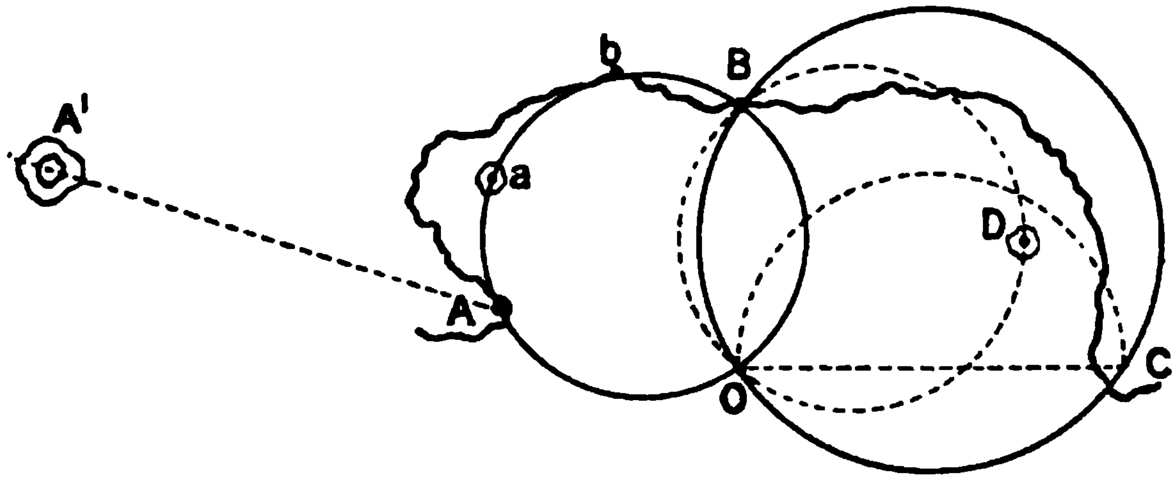


FIG. 68.

Let O (fig. 68) be an observer, B the nearest and middle object. Conceive two circles, one going through C and the other through D; conceive again a circle of equivalents from O C, shown by a circle with dotted line. Of these two circles the smaller is the better to take, because the object D falls within the semicircle on O C.

The only possible way to obtain the left circle, is to again conceive a number of circles going through B and the left objects at his disposal.

Then, with some idea of the position of the centre of the first circle, turn in a direction at right angles to it, and conceive another circle going through O B and the left object.

Let A B O represent such a circle; then, evidently, A is the best object to select.

But supposing A^1 to be further off than A, but to make the same angle B O A, then *a* or *b* would give a better fix than A^1, *a* better than *b*.

In the above fig. 68 the circles cut at right angles, the angles

are well conditioned in relation to the distances O A and O D; and the angle of approach A O B and B O D is almost the best possible; so that the fix is a very good one.

It must, however, be so much guess-work, and nothing but continuous practice and analysis of a fix, will ensure an observer picking out intuitively, after a time, the best objects.

211. Angles not Taken Simultaneously—Choosing Circles.—Heretofore it has been considered that either the angles at O are taken simultaneously, or that the observer has not shifted his position more than a few yards while taking each angle.

When, however, the angles are not taken simultaneously, and there is a considerable distance between the position of the first angle and the second, the case becomes more complicated.

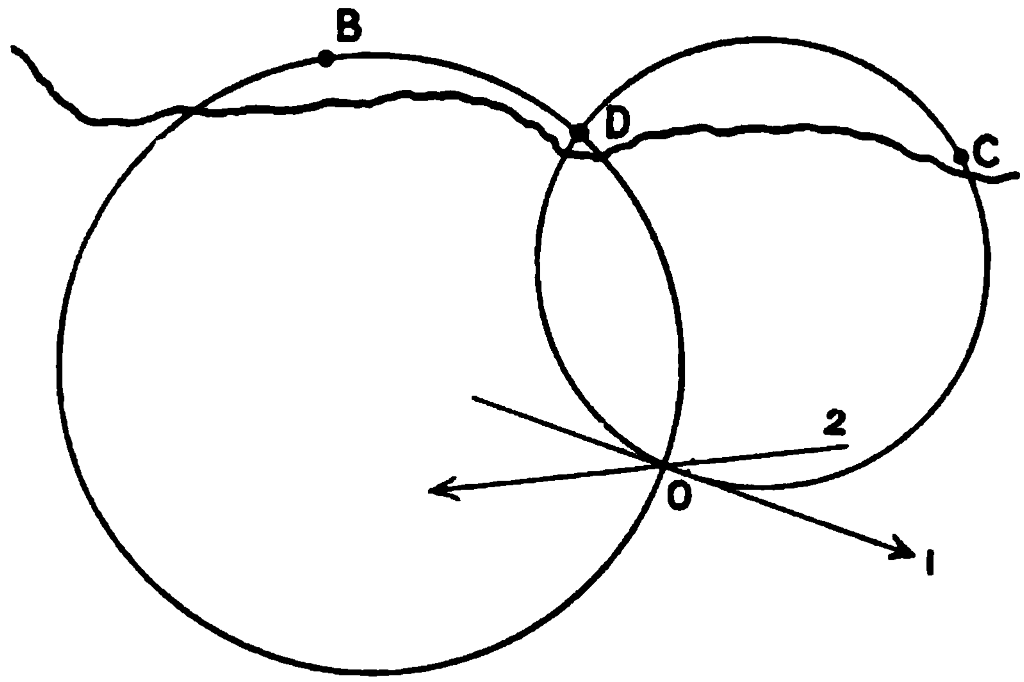


FIG. 69.

Supposing, in this case, a ship is steering south-east 15 knots, and that the interval between taking the angles is one minute; then the observer has changed his position $\frac{1}{4}$ mile from where he took one angle, to the position at which he took the other angle.

In fig. 69 the arrow 1 shows the direction of ship's course. It must now be arranged that the line of her course shall coincide with the circumference of one circle, and shall be directly towards or from the centre of the other.

In the one circle, D O C, the angle will not be changing, while in the other, B D O, it will be changing to its maximum.

It is required to conceive the *first* circle, there being given D, the nearest object, and O, near about the position of the observer: the centre evidently must lie 'abeam.'

C D O fulfils the condition; then C should be the right object. After guessing at the position of the centre of the circle, turn to a direction at right angles to it, in this case 'astern,' and conceive another circle with its centre in that direction. This will in the figure give circle B D O.

212. Order of Taking Angles.—The observer is now running along the circumference of D O C, and almost directly away from the centre of B D O.

The angle D O C is not changing, or changing as slightly as possible during a short interval, *i.e.*, small distance, while B O D is changing its maximum.

And since the position is required at the instant of the second observation, the correct way to fix is to take first D O C, then B O D.

If the ship is running east or west (see arrow 2), then both angles are changing; according to figure 69, the ship is going westward, and B O D is changing the faster and is *increasing*, while D O C is *decreasing* a slightly less amount.

In this case, and for that reason, the *sum* of the angles will be changing its minimum or not at all. Therefore the whole angle B O C is taken first, because it changes least, followed by B O D, which is changing most, while the angle C O D is derived.

It is written down thus:—

B	90°	C
	30°	D

30° is the left angle, and the difference between C to B and C to D, viz. 60°, is the derived right angle. These angles are placed in the pointers, the position fixed by the *right* and *left* angle, and the error due to change of position while observing is reduced to a minimum.

As a matter of fact, B 90 C and B 30 D are the two angles that constitute the fix; and if a circle be *drawn* through B O D and one through B O C their intersection will give position O. But in using the station pointer it is actually fixed by B O D and B O C; B O C being derived from the difference between B O C and B O D more correctly than if taken independently; so that the right and the left angles are always the two which guide the usual action of the station pointer and fix the position (par. 171).

This disposes of the error under heading (7) (par. 189) when the change of position is considerable, or when the relation of the change to the scale of a chart is considerable.

It must be looked upon as a change of so many inches of paper. On a scale of 1 inch to a mile, $\frac{1}{4}$ mile = ·25 inch; on a scale of 10 inches to a mile, ·25 inch = $\frac{1}{40}$ mile =, roughly, 50 yards.

On such a scale as this, a boat shifting her position 50 yards corresponds to a ship going 15 knots and changing her position $\frac{1}{4}$ mile on the 1-inch scale.

The corresponding error of position for a movement in the interval of taking the angles, will depend upon the cuts of circles and their magnitude and the size of the angles observed.

In the one case it may merely be an error incidental to navigation, and perhaps of not much importance; but in the case of a boat fixing, perhaps, the position of a submerged rock or of a sounding, etc., ·25 inch is a serious error, and, as is shown, may be produced by the improper fixing described.

213. Not Necessary to 'Stop' to Fix except in Large Scales.—As a rule, when a boat or ship is running on a line of soundings, it should not be necessary to stop over each position of a fix, so that there is all the more reason for knowing what suitable angles and objects to select in order to allow for the non-stop.

In fig. 70 the arrows show the direction of a vessel steering directly away from or towards the shore. Selecting objects A, B, C to fix with, she is running along or near the circumference of B O C, almost directly away from the centre of B O A (arrow 1), but still more directly from the centre of the circle through A O C (see arrow 2).

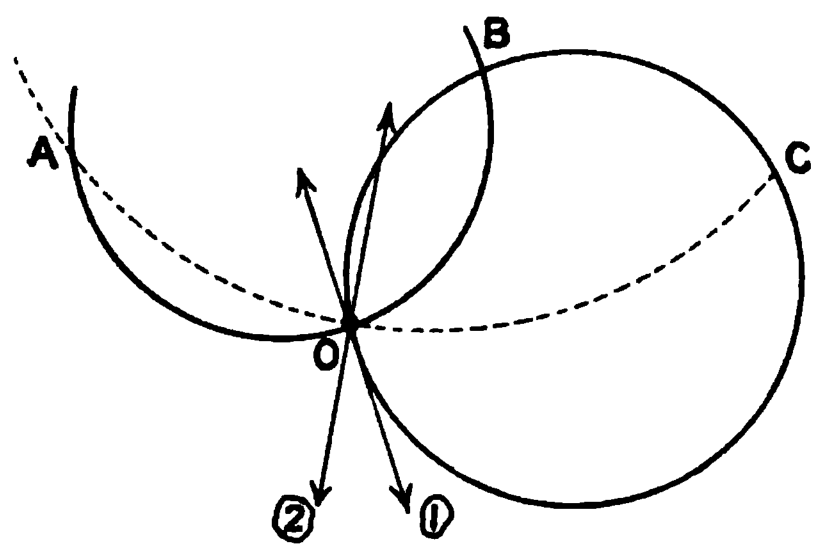


FIG. 70.

By arrow 2 the angle C O B is not changing much, while the angle A O C is changing rapidly. The fix should be

B 40 C
A 100

40° is the 'right' angle; and, consequent upon C A being 100°, A O B is the left and derived angle = 60°. With these two angles, O is fixed.

The fix (at the second instant of observation) would have been grossly in error had B to A been taken first and followed by C to B. There is sometimes a temptation to do this, because, the left angle of the previous fix having been the last taken, the sextant is in a convenient state for it to be taken first at the next fix.

214. When Necessary to Anchor to Fix.—In harbours, in a tide-way, or with a wind blowing a boat away from the first position in which the fixing angles are taken, and especially, when the scale is a large one (anything over 6 inches to the mile), it is necessary either to anchor at each fix in order to obtain the angles, or that two observers shall be present to take the angles simultaneously; because the direction of drift is not at all certain, and without this knowledge no tangent to one circle and towards or from the centre of another will avail. In very large scales, such as 25 inches to a mile, for instance, an angle

between the same two objects at each end of the boat may differ considerably, therefore simultaneous angles from one spot in the boat must be taken.

NOTE.—A device can be suggested that may meet the case for beginners, as it will enable them to pick out the best fix under the conditions before them.

215. Procure two circular horn protractors, the larger the better, and from the centre of each draw, in different coloured inks, a number of circles, each having a radius ½ inch smaller than the next outside one.

Let X and Y represent the two protractors.

On X draw a number of lines parallel to, and on Y a number at right angles to, the 0° to 180° diameter of each: as shown in fig. 71, these lines are nearly superposed.

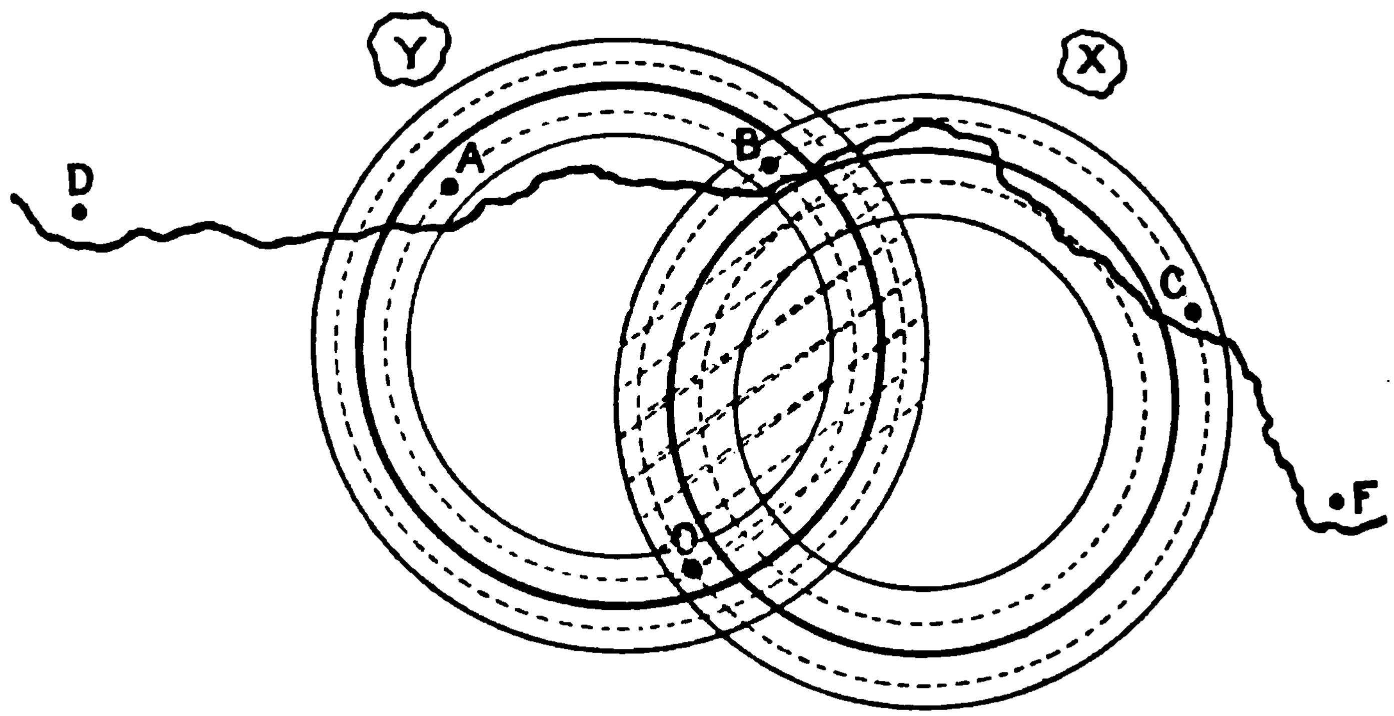


FIG. 71.

Now D, A, B, C, F represent a number of fixed points.

The observer is somewhere near O.

Required the best objects to fix with.

First, Take the nearest object B for a middle object.

Secondly, Place the protractor X until one of the circles marked on it will pass through O and B, and as near as possible through any objects that are on the chart. Suppose the most suitable circle is, as shown, the second from the rim: this passes through O, B, and C.

Thirdly, Place the protractor Y so that the lines drawn on it will lie parallel to, or overlap, those on the X protractor (fig. 71); then slide it either *along* those lines, or at *right angles* to them, until one of the circles drawn on it gives the best fit over two objects on that side and the position of the observer (the lines on each protractor must remain parallel to each other), and let the most suitable circle be as shown, fig. 71, viz., the third from the rim: this passes through A, B, and O.

Then if A on the left, and B in the middle, and C on the right are taken for the fix, their fixing circles will cut nearly at right angles, and it is the best fix under the conditions as regards cuts of circles.

B O D and B O F (fig. 71) are better angles than A O B and B O C, as the angle of approach to them is better; but D and F are further than A or C in their equivalent positions—a fact that can be demonstrated by drawing a circle of equivalents on the sides O D and O F.

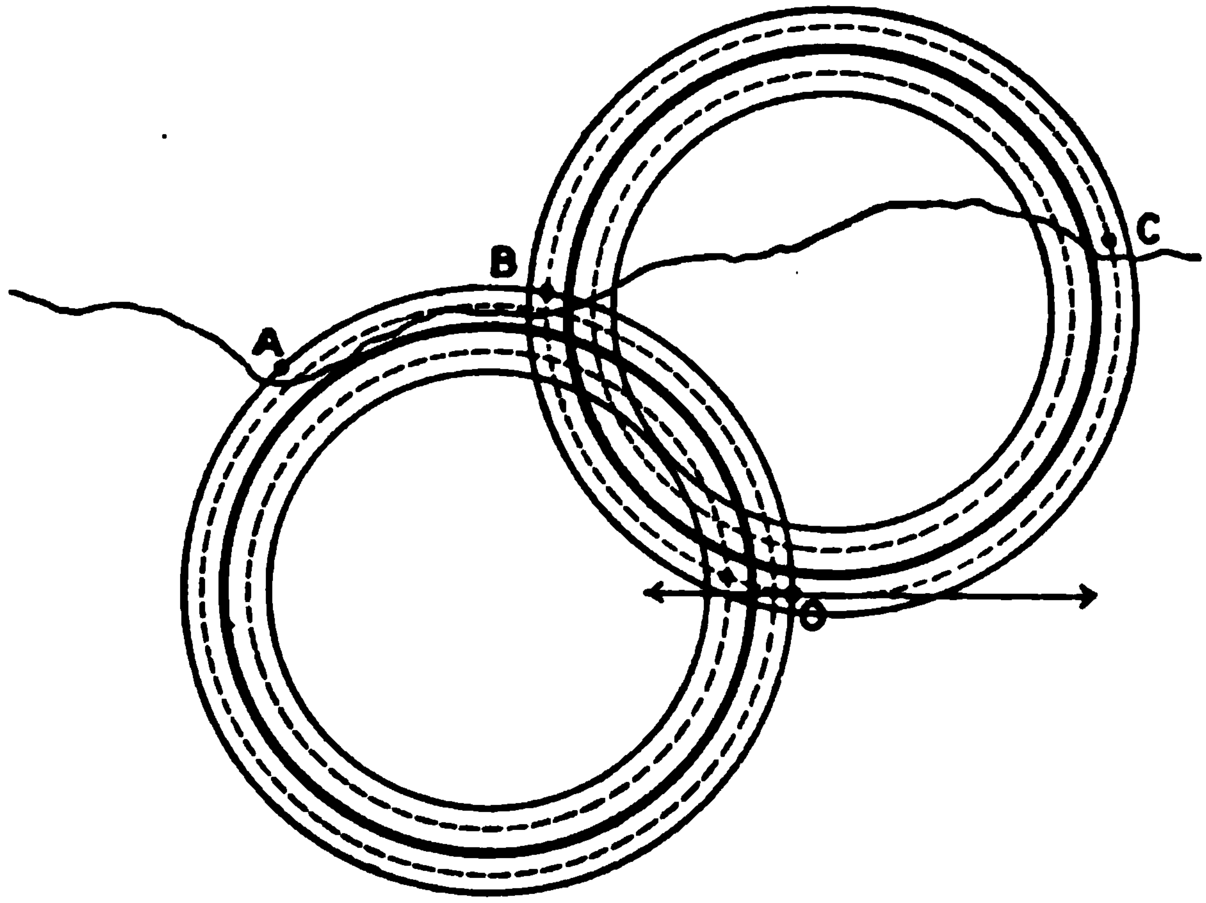


FIG. 72.

Now in the case of a ship in motion, one of the protractors must be so placed that the direction of the ship's change of position is a tangent to the circle, in which case its centre is abeam, or directly in line with the centre, *i.e.*, ahead or astern; and the other protractor will be at right angles to it. Obviously, in the case shown in fig. 72 the right angle is taken first, then the left; and, if desired, a third protractor can be utilised representing the whole angle. With the two protractors the exact and best possible fix can be picked out, and with the third is determined the order in which the angles should be taken, *i.e.*, right and left separately, or the whole followed by either, or *vice versa*.

CHAPTER XIV

ANGULAR MEASUREMENTS FOR TRUE BEARINGS.

216. For Finding True Bearing.—An angular measurement is made with a sextant between the sun or a star, and the object, for the purpose of finding a line of true bearing to that object.

Either the object is reflected to the sun, or the sun reflected to the object, depending upon which of them is on the right.

217. Using Sextant Handle Up.—If the land object is on the right, and is too dim to bear reflection, or loses its definition when reflected to the sun when conveniently shaded, the sextant may be used upside down, that is, handle up; and looking to the right object instead of the left, as is done when the proper way up, reflect the left to it: in this case it is the sun.

At the instant of contact, the time is taken with a stop-watch or a suitable watch, and from this the hour angle is found. Then, given the latitude, hour angle, and declination, by spherics the sun's azimuth or true bearing can be calculated. With the sun's true bearing, and the *horizontal* angle between the sun and the object, the true bearing of the object is deduced.

218. True Bearings (T.B.).—A 'true' bearing of any object is the angle at the observer's position between the 'true' meridian and the tangent to the great circle joining him and the object.

219. The T.B. on a Gnomonic Projection.—On a gnomonic projection (see p. 165), since all great circles are represented by straight lines, the 'True' bearing is the angle between the true meridian and the straight line joining the observer's position with the object.

220. Great Circle not a Straight Line on Mercator Chart.—The straight line joining the observer with any object on a Mercator's chart is not a great circle; though the difference between it and a great circle, when the distance is within 5 miles, is negligible in ordinary working latitudes.

221. Mercatorial Bearing.—The angle between the true

meridian and the *straight line* joining the observer with an object on a Mercator's chart, is a 'Mercatorial' bearing.

222. Difference between True and Mercatorial Bearings.—The difference between a True and a Mercatorial bearing is due to the convergency of the meridians, a fact which is not shown on a Mercator's chart; and one differs from the other by half the convergency of the meridian nearly (see p. 171).

223. Convergency Neglected on a Plan.—The distance on a Plan being less than can be practically affected by convergency, convergency is neglected.

224. Use of True Meridian on a Plan.—If, on a plan, the T.B. of one fixed point from another is determined, then the true meridian can be projected relative to the line joining the two fixed points, and consequently the Plan placed on its right slue.

225. Fixing by True Bearings.—The T.B. of any object from a fixed point, laid off from the true meridian as zero, will give a line of reference from that fixed point through the object; and, in conjunction with similar lines of reference projected from other points, will enable that object to be fixed by true bearings.

This manner of fixing, or of obtaining a line of reference, is adopted for a position some distance from land, or some distance inland, from whence two (for an absolute fix) or perhaps only one object may be visible, this latter merely giving one line of reference; as, for example, when fixing the position of a lightship, or a buoy, or for a line of reference to a point at the head of a river.

226. Finding True Meridian.—To find the line of the true meridian, the point selected for the observation should be at one extremity of the plan, and the true bearing found of some point at the other extremity (the further apart these points are the better; see fig. 85, p. 122, par. 265).

Time to Observe.—The sun's azimuth changes least when it bears about east or west, and most when the bearing is north or south; consequently an error in time, equivalent to the error in an observation, will make the minimum corresponding error in azimuth when the sun bears about east or west; therefore the best time to observe is when it is nearest that bearing—the altitude should not be less than about 20°. For an error in latitude, the corresponding error in azimuth is greatest when the sun is east or west, but as a problem in surveying the probable error in latitude will be so small as to be negligible; if it is not so, then no definite statement can be made as to the best time to observe so that the errors in H.A. and in latitude cancel each other.

227. Object to Take with it.—As regards the position of the object relative to that of the sun; at par. 131, it has been demonstrated that the difference between the horizontal angle

and the angular measurement varies as the cos altitude and distance apart, *i.e.*, the inclination, in fact. Hence, the lower the altitude the better, and the smaller the angle of inclination between the objects, the better.

Also, for an error in the observer's position, a projected line of reference making a small angle with the zero of reference, makes the least error in the line projected (see Appendix II., p. 438). Hence the nearer the zero line of reference is to the north and south line the smaller will be the angle necessary to project, which establishes the direction of the meridian line.

228. Thus, referring to fig. 73, let O be the observer, and S a point having little or no altitude. When the sun was about east, for instance, its altitude being not less than 20°, the angle is observed at O between S and the sun. Suppose it to be about 100°; the angle of inclination will be almost negligible.

After calculating the true bearing of the sun, that of S from O can be deduced.

If S is a 'fixed' point, the line O S, making a small angle with the true meridian, is the best zero line of reference from whence to plot the true meridian line.

On the other hand, if S is not a point 'fixed' in the chart, a further angle will be required to a point (A) which is 'fixed,' but which was unsuitable for the first part of the observation; this will determine the true bearing of A, and from the zero line O A the true meridian line will be established.

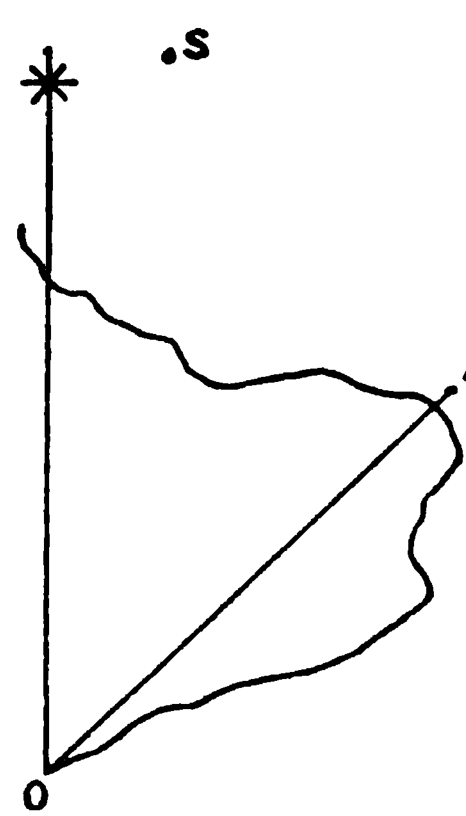


FIG. 73.

229. Solution of Spherical Triangles.—Neither the sun nor the object may be on the horizon,[1] or either or both may have an altitude. In the solution of the spherical triangles involved for the determination of the horizontal angle between the sun and the object, it will be necessary to draw a figure. As being the most complicated, let us suppose both to have an altitude.

Fig. 74 is on the plane of the horizon, Z being the zenith, and P Z S, the celestial meridian, being the great circle of the true meridian produced to infinity.

Let X be the true position of the sun, having an altitude X*a*;

[1] The sun's true centre is on the horizon when, under normal conditions of 'dip' and refraction, its apparent lower limb is about a semi-diameter above the horizon: instances of abnormal 'dip,' ranging from −18′ to +5′, are common *in all* latitudes.

and O the position of the object, having an altitude Oa. This altitude was measured with a theodolite and found to be 5° 00′.

OX^1 is an arc of a great circle joining O, and the *apparent* position of the sun; *i.e.*, aX^1 is the uncorrected altitude; and $O Z X^1$ is the horizontal angle between X^1 and O, and is the angle required. To calculate it, it is necessary to know the value of the three sides of the spherical triangle $O X^1 Z$.

The observed angle was taken at Z between the point O and one of the limbs of the sun, for choice the nearer limb; and since $O X^1$ represents the angular distance between O and the sun's centre, then, measured angle + S.D. ± I.E. = $O X^1$.

$Z O = 90° - Oa = 85°$, the altitude of O being 5°.

$Z X^1 = 90° - X^1 a$, or 90° − sun's *apparent* altitude.

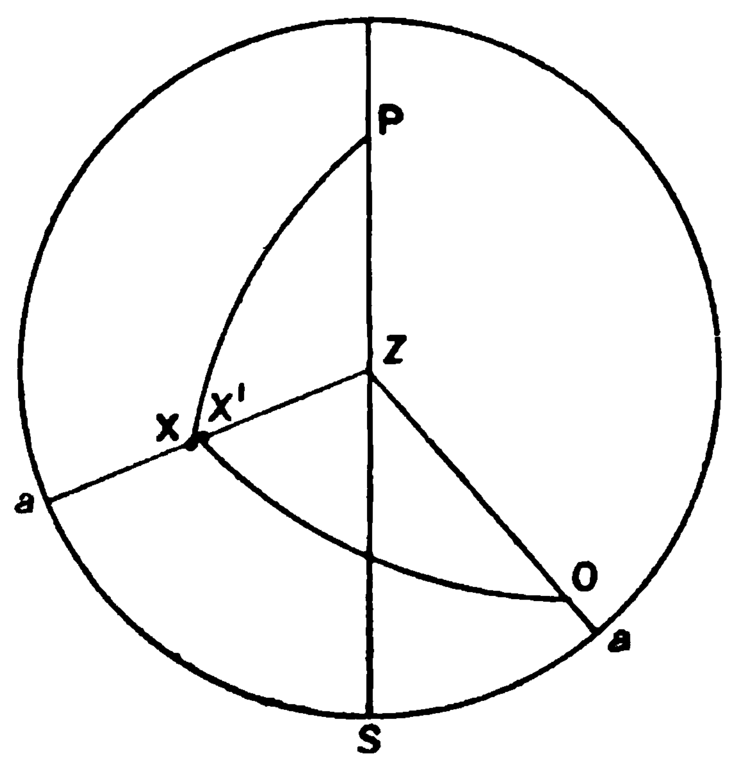


FIG. 74.

Now, either the sun's altitude was observed at the same instant as was the angle, in which case, after applying the usual corrections, except the refraction, the sun's apparent zenith distance—that is, $Z X^1$—is found.

Then in spherical triangle $Z X^1 O$ find angle $X^1 Z O$, which is the horizontal angle between O and X (given three sides of a spherical triangle, to find the angle opposite $X^1 O$).

Or, if the altitude is not observed, then it must be calculated from triangle P Z X.

In which case, P Z being the co-latitude = 90° − lat., P X is the polar distance = 90° − dec., and Z P X is the hour angle.

The hour angle is deduced in the ordinary manner from the time by watch, applying to it its error on mean time of place and the equation of time; both the declination and equation of time being corrected for Greenwich mean time.

Now having P X, P Z, and Z P X, the side Z X can be calculated

(from two sides and the included angle, find the third side). This is the *true* zenith distance; Apply refraction the opposite way, and deduce the *apparent* Z.D., *i.e.*, ZX^1; and with this quantity, as before, calculate OZX^1, that is, the horizontal angle between O and the *apparent* position of X.

It is only necessary now to find the true bearing of X, to which apply the horizontal angle between it and O, to find the true bearing of O.

In the triangle PZX; PZ, PX, and ZX (the *true* zenith distance) are known; calculate angle PZX (from three sides, to find one angle).

PZX is the true bearing of X from Z.

In fig. 74, $SZO =$ true bearing of O from Z.

$$= PSZ + OZX - 180°.$$

(See example, par. 232.)

230. To Observe an Angular Measurement.—To make the actual measurement with a sextant. First, without the telescope, make an approximate contact between the sun's limb and the object; then ship the telescope, hold the sextant with its plane near the plane joining the sun and the object, and by twisting the wrist slightly the sun is made to describe an arc (see fig. 75): the contact must be made at the tangent of the arc.

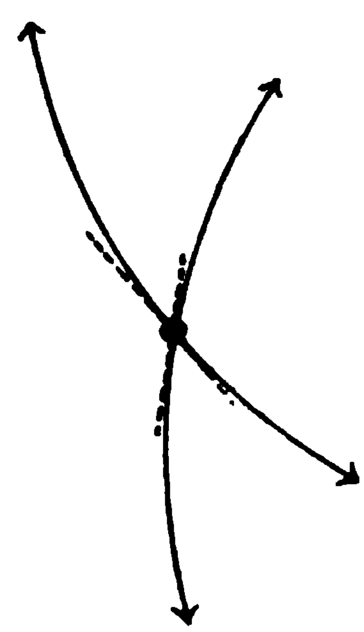


FIG. 75.

Obviously, the more vertical the arc the nearer does the angular measurement approach to an altitude; and a small error in the altitude of the object will make a magnified error in the horizontal angle; whereas if of the same altitude, or nearly so, the error in the measured angle due to an error of altitude will be a minimum.

231. Jeffers' Method of Finding a True Bearing.—The American writer Jeffers gives an ingenious method of finding the true bearing of an object.

A plummet-line suspended in exact transit between the observer and the object, the true bearing of which is required, is steadied in a bucket of water.

An artificial horizon is set up in exactly the same direction, so that the plummet-line in transit with the object is reflected through it to the observer's eye.

Now with his sextant, on a stand, at 'the ready,' he waits till each limb of the sun, in succession, touches the reflected plummet-line, both the sun and plummet-line being reflected in the artificial horizon. At each instant of contact he also reads off the sun's altitude.

With the mean altitude, and the other parts of the spherical triangle P Z X (fig. 74) the sun's true bearing is found; and, if the plummet-line was correctly in line with the zero object, then angle P Z X is the true bearing of both the sun and the object.

If the altitude of the zero object is sufficiently high that the object can be seen in the mercury of the artificial horizon, and the plummet-line is motionless as well as in exact transit with it at the time of observation, both being seen in the mercury, then the observation must be correct.

There is an almost insuperable difficulty in keeping the plummet-line perfectly steady, and at a small distance away, the transit line is a short base, when an error in position makes the maximum error in the angle observed; and it is for this reason that any angles measured by these means are not recommended. Moreover, to 'catch' the movement of the sun in altitude as well as its contact with the plummet-line is not an easy observation.

232. Calculation of True Bearing through a Calculated Altitude.—The following is an example of the calculation of a True bearing obtained at Greenwich, where one observer takes the angle between the sun and the object, while his assistant takes the time:—

Lat. 51° 28′ 56″ N., long. $1^s{\cdot}4$ W., 14th September 1905.
Watch time, $10^h\ 04^m\ 00^s{\cdot}5$. Obsy. staff, 74° 13 |☉. I.E. + 3′.
Comparison of watch on chr., $3^h\ 58^m\ 48^s{\cdot}2$ slow.
Error of chr., $1^h\ 16^m\ 4^s$ slow on G.M.T. Alt. of obs., 5° 00′.

	h m s		° ′ ″	
Watch	10 04 00·5	Dec.	3 34 31 N.	change 57·5 −
Compn.	+ 3 58 48·2	change	−0 2 00	2·1
Error of Chr.	+ 0 1 16·4		3 32 31 N.	60 | 120·75
			90 0 0	2·00
G.M.T.	2 04 05·1	P.D.	86 27 30	
			m s	
d. long.	0 0 1·4 W.	Eq. T.	4 17·9 +	change ·9 +
				2·1
M.T.P.	2 04 03·7	change	+ 0 1·9	
Eq. T.	+ 0 4 19·8			1·9
App. T.	2 08 23·5		4 19·8	

In △ P Z S (fig. 76) P Z = 90° − lat. = 38° 31′ 04′
P X = 90° − dec. = 86° 27′ 30″

$$Z P X = 2^h\ 08^m\ 23^s{\cdot}5$$

To find Z X.

	h m s	
Log hav	2 08 23·5 = 8·883205	
Log sin	38° 31′ 04″ = 9·794308	
„ sin	86 27 30 = 9·999170	
Diff.	47 56 26 8·676683	log hav = 25° 10′ 30″
Log vers.	25 10 30 – 0094986	
„ „	47 56 26 – 0330112	

Log vers. 0425098 = 54° 54′ 26″

Z S, the true Z.D. =	54° 54′ 26″	True alt. = 35° 05′ 34″
Refrn. for altitude +	0 1 23	
App. Z.D.	54 55 49 = Z X[1] in spherical triangle Z O X[1].	

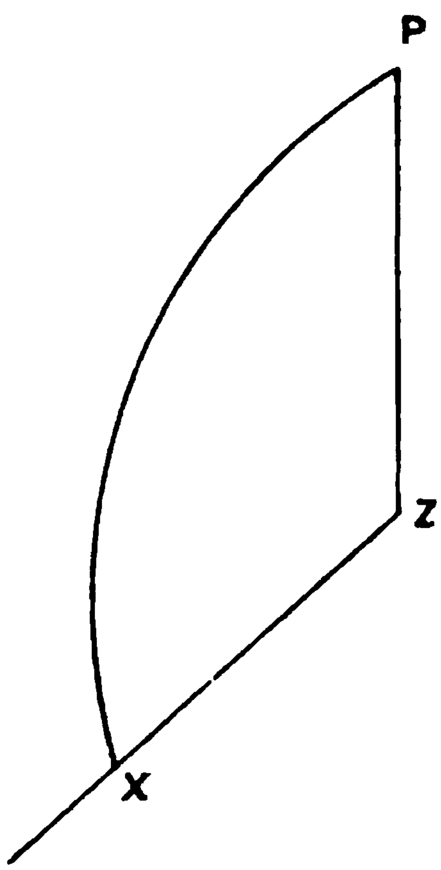


Fig. 76.

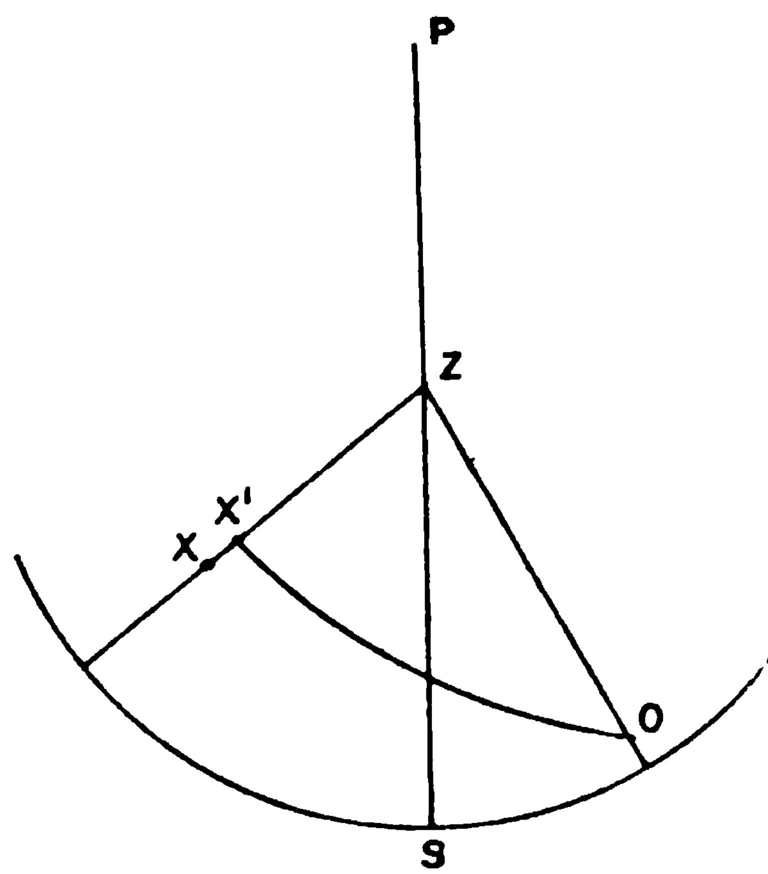


Fig. 77.

In Δ P Z X, to find P Z X.

Log sec. lat.	51° 28′ 56″	– 10·205681
Alt.	35 05 34	– 10·087123
	16 23 22	
P.D.	86 27 30	
Log ½ hav	102 50 52	– 4·893092
	70 04 08	– 4·758950

Hav 9·944846 = N. 139° 35′ W.

North, because declination is N.; and W., because the hour angle (H.A.) is P.M.

T.B. of sun's centre (⊕) N. 139° 35′ W.

In Δ Z X^1O (fig. 77),

$$
\begin{aligned}
& \quad\ \ \overset{h}{} \quad \overset{m}{} \quad \overset{s}{} \\
ZX^1 &= 54\ 55\ 49 \\
ZO &= 85°\ 00'\ 00'' \text{ (alt. of O being 5°)} \\
OX^1 &= \text{obs. angle} + \text{S.D.} \pm \text{I.E.}
\end{aligned}
$$

Obs. angle = 74° 01′, the object being to the left of the sun's left limb.

$$
\begin{array}{lrr}
 & \circ & ' \\
\text{S.D.} + & 0 & 16 \\
\text{I.E.} + & 0 & 3 \\
\hline
OX^1 = & 74 & 20,^1
\end{array}
$$
angle to centre of *apparent* sun.

In triangle Z X^1O, to find O Z X^1.

$$
\begin{array}{lrrrl}
 & \circ & ' & '' & \\
\text{Log cosec } ZO & 85 & 00 & 00 & = 10{\cdot}001656 \\
\quad ,, \quad ,, \quad ZX^1 & 54 & 55 & 49 & = 10{\cdot}087007 \\
\hline
 & 30 & 04 & 11 & \\
OX^1 & 74 & 20 & 0 & \\
\hline
\text{Log } \tfrac{1}{2} \text{ hav} & 104 & 24 & 11 & = 4{\cdot}897722 \\
\quad ,, \quad ,, & 44 & 15 & 49 & = 4{\cdot}576042 \\
\hline
 & & & \text{log hav.} & 9{\cdot}562427 = 74°\ 21'\ *
\end{array}
$$

Then O Z X^1 = 74° 21′: this is the horizontal angle between X^1 and O.

The true bearing of O is S Z O, P Z S being the true meridian through the zenith (Z).

$$SZO = PZX + X^1ZO - 180°$$

$$
\begin{array}{rrr}
 & \circ & ' \\
PZX = & 139 & 35 \\
X^1ZO = & 74 & 21 \\
\hline
 & 213 & 56 \\
 & 180 & 00 \\
\hline
SZO = S & 33 & 56\ E
\end{array}
$$

* It happens to be a very unusual coincidence that the observed angular distance is practically equal to the horizontal angle, where the altitudes are so widely different.

Example of a true bearing of an object being determined by one observer with a sextant, taking alternately the sun's altitude and its angular distance to an object (see note, p. 106).

The case is applicable when either there is no chronometer, or when its error on G.M.T. is not known accurately.

The observer starts by taking the sun's altitude in the artificial horizon; then he measures the angular distance between the sun and the object with another sextant; then again the altitude, followed by another angle, and so on, eventually finishing up with the altitude.

He should try and space his times so that the middle time of the altitudes corresponds very nearly with the middle time of the angles; for if these do not nearly agree, the altitude corresponding to the middle time of the angles will have to be interpolated, as is the case in the following example.

Example.—At Z, lat. 51° 29′ N., long. 22″ W., at about 3·30 P.M., 5th June, to find the true bearing of ⊕ object on the horizon.

The altitude was taken in an artificial horizon, and the angular measurement with a sextant.

	Watch time.	Alt. ⊖.	Angle.	Time by W.
	h m s	° ′	° ′	h m s
	3 25 46	79 40		
			⊕ 106 31 ○	3 26 32
	3 27 38	80 10		
			105 25	3 28 20
	3 29 33	80 40		
			104 40	3 30 00
	3 31 27	81 10		
			103 34	3 31 50
	3 33 23	81 40		
Mean	3 29 33	80 40	105 02	3 29 10

	h m			
Approximate M.T.	3 29			
Long.	0 00			
G.M.T.	3 29	Dec.	22° 32′ 14″ N.	+16·6
			0 0 52	3·5
			22 33 12	58·1
		P.D.	67 26·48	

Difference between the mean time of alt. and of angle = 23 secs.

The alt. changed 2° (from 79° 40′ to 81° 40′) = 120′ in $7^m\ 37^s$ (from $3^h\ 25^m\ 46^s$ to $3^h\ 33^m\ 23^s$). How much will it have changed in 23 secs? $7^m\ 37^s : 23^s :: 120' : x$; $x = \dfrac{120'\ 23''}{7^m\ 37^s}$; this equals 6′.

The middle alt. is 80° 40′ at $3^h\ 29^m\ 33^s$; therefore the alt. at $3^h\ 29^m\ 10^s$ is 80° 34′.

Obsd. alt.	80° 34′ 0″ ⊖	
I.E.	+ 6 10	
	2 | 80 40 10	
	40 20 05	
R. & P.	1 03	
T.A. ⊖	40 19 00	
Z.D.	49 41 0	
Log cosec co-lat.	38° 31′ 0″	10·205691
Log cosec Z.D.	49 41 0	10·117772
	11 10 0	
P.D.	67 26 48	
Log ½ hav	78 36 48	4·801728
Log ½ hav	56 16 48	4·673600

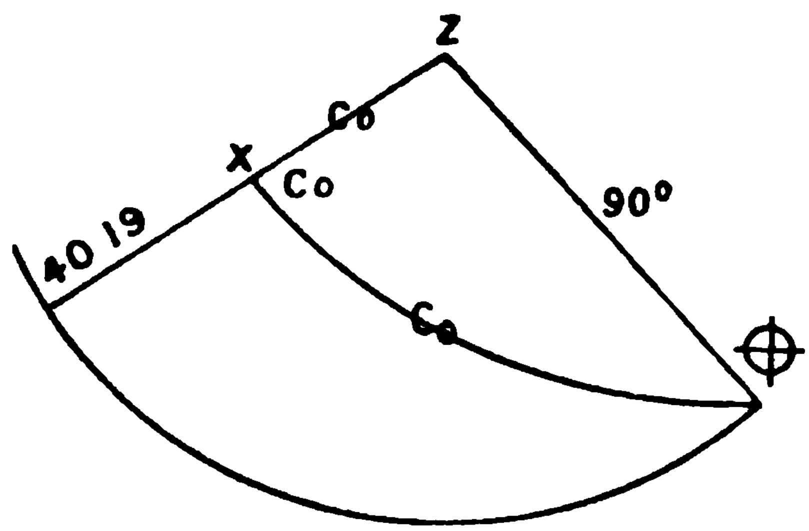


FIG. 78.

Hav 9·798791 = 104° 58′ 30″ = T.B. ⊙

In Δ X Z $\oplus$ (fig. 78), cos X $\oplus$ = cos Z . sin Z X

$-$ $\qquad$ $-$ $\qquad$ $+$

cos Z = cos X $\oplus$. cosec Z X

X $\oplus$ = obs. angle ± S.D. ± I.E.
= 105° 02′
S.D. + I.E. = 0° 16′

Angle to ◑ 105° 18′

Log cos	105° 18′	9·421395
Log cosec Z X	49° 41′	10·117772

Cos 9·539167 = 68° 45′ 15″ = Z
180 00

Since the sign of Z is −, then = 111 15 = horizontal angle
from above, T.B. ⊙ = N. 104 58 W.

∴ T.B. of $\oplus$ N. 216 13 W. = S. 36° 13′ E.

NOTE.—This form of observation can be undertaken by two observers. In such a case the altitude and angle are taken simultaneously, and there is no need to interpolate; and, if observed with 'equal altitudes' on each side of the meridian, 'errors of observation' will be reduced to a minimum: in this manner more rigid accuracy is obtained than in the 'time azimuth,' p. 102.

CHAPTER XV.

TRUE BEARING.

233. To Fix a Position by True Bearings.—When from any point the true bearing is obtained of any 'fixed' object, if this reversed bearing is projected *from the fixed object*, the line is one of reference to the point; and such true bearings to two fixed objects will give two lines of reference with which the point may be fixed.

This method of fixing a position is one of necessity, and arises in those cases where the fixed objects are both distant and inaccessible.

234. *Example.*—Let it be required to fix the position of the Tongue Lightship at the Thames entrance.

Suppose it be necessary for the observer to take his angles from the crow's-nest of the light-vessel, the objects on shore being otherwise indistinct or invisible.

From position T (fig. 79), at sunrise, the angle is taken between F (Foreland Lighthouse) and the sun: time is taken with a watch, compared with a chronometer, and so on, and the hour angle deduced.

For convenience, and also for accuracy's sake, the sun will be on the horizon (see par. 227), and the angular measurement will practically be the horizontal angle.

When the sun is on the horizon, the zenith distance $= 90°$; and the $\sin \text{Az.} = \overset{+}{\sin \text{h.a.}} \times \cos\,(\overset{\pm}{90°} \mp \text{dec.})$.

Apply the proper signs to each quantity (see Appendix III., p. 440), and $+ \times - = -$, $- \times - = +$; and if sin Az is $-$, then subtract it from 180°, as in *Example*, p. 105, par. 232.

This gives the true bearing (T.B.) of the sun's true centre.

For all practical purposes of this nature of fixing, the horizontal angle to the sun's true centre is equal to the angle to its apparent centre, when the sun is of the same altitude as the object observed.

If, then, this angle is applied to the sun's T.B., the result will be the T.B. of the object.

Then from F, a fixed point on the Mercator chart, the true bearing of T can be projected from the Mercatorial meridian (see *Convergency and Mercatorial Bearings*, pp. 98 and 172): this is a line of reference from F to T (F to T is about 10 miles). *N.B.*—Convergency has been neglected.

Without changing his position, the observer can now take the angle between F and R (Reculvers Church steeple). This angle applied to the T.B. of F gives the T.B. of R from T, and, conversely, the T.B. of T from R.

If this bearing is laid off from R, R T is the second line of reference, and at the intersection of R T and F T will be the position of T.

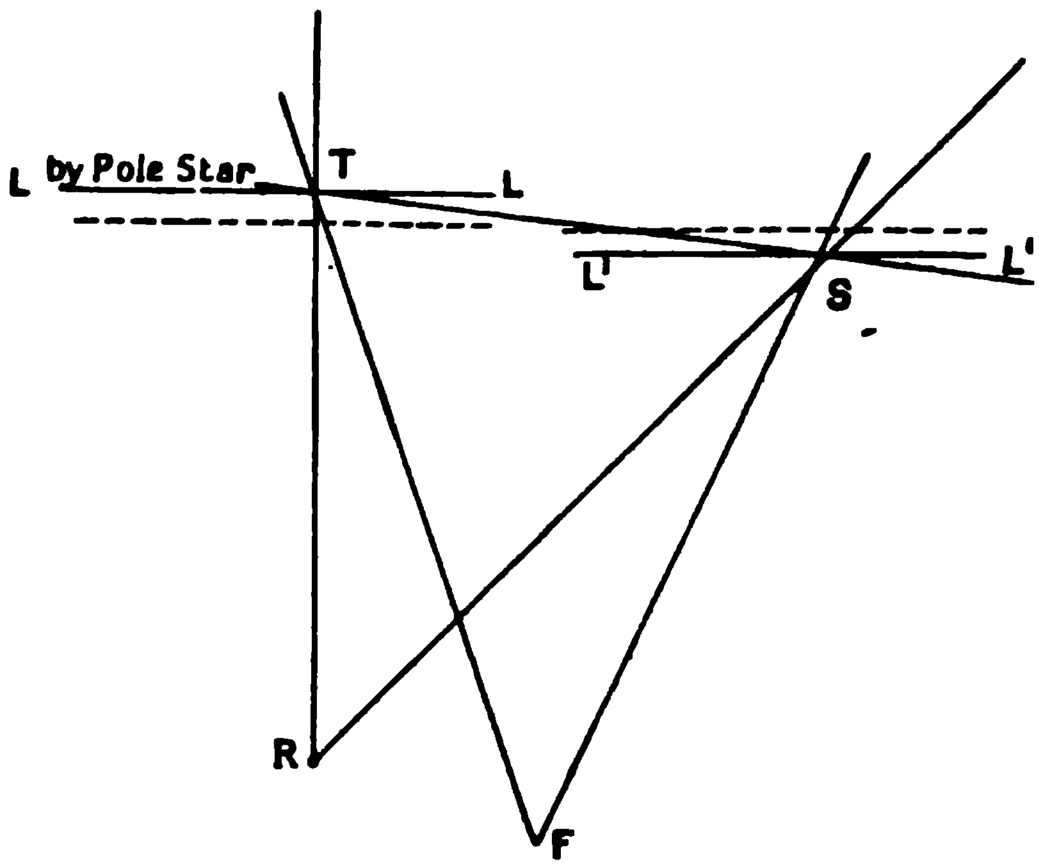


FIG. 79.

This, however, does not quite conclude the matter, because there is nothing to indicate that either, or both, of these lines of reference projected are either correct in bearing or, supposing the observation to be without error, are correctly projected; for, even given both in error, the error being small or large, the two lines must intersect somewhere.

Then it must be necessary to obtain a third line of reference as a check.

235. True Bearings with Astronomical Fixing.—The best direction of a 'check' line of reference is one at right angles to either R T or F T.

There happens to be nothing in the vicinity, so that recourse must be had to a celestial body for a line of position.

In this particular instance the Pole Star will suit admirably, and from it a latitude is obtained.

This line of latitude will be east and west, and should pass

through T, if T is correct ; but it is equally sure not to be, under the conditions (see dotted lines in fig. 79, and *Example* on p. 331).

Besides errors of observation in the true bearings, and the error in the position of R and F on a distorted chart, there is yet the correction for convergency (see p. 171), which will probably be neglected in this case ; and there is the error of observation in the Pole Star (see *Errors of Sea Horizon*, par. 123, and *Errors on Projecting the Lines of Reference*: see *Protractor*, p. 53).

The error of position will not be large; in fact, it will be inappreciable for the purposes that a light-vessel fulfils, and not more than probably her circle of swing. The example is given, simply to show how true bearings can be applied for purposes of fixing, and that a rough check can be obtained by an astronomical observation.

It is still possible to obtain more lines of position in connection with the light-vessel by means of other twilight stars, suitably selected (see works on navigation).

236. **True Bearings with a Triangulation.**—Or, abandoning the star method of checking the fix, make a small triangulation by the true bearings only, the check being an angle.

Using the same figure (79), let the ship take up a position near S so that a line of reference projected from it will cut R T or F T at nearly a right angle.

From this position of the ship, find the T.B. of F (this might be done when the sun was setting), and measure the angle between F and R ; from this angle will be deduced a T.B. of S from R.

Now from positions F and R project from the true meridian the lines of respective reversed Mercatorial bearings. Suppose them to intersect at S.

Provided the errors of observation are consistent in both sets of true bearings, and the lines are not projected in error, then positions T and S are both correct relative to F and R, whatever the error in the positions of F and R are.

The line checking the accuracy of the work will emanate from angle T S R ; if T S R is observed and projected at S from the line S R ; then if this line of reference, which cuts R T nearly at a right angle, passes exactly through T as plotted from R and F, then both S and T are correctly 'fixed' relative to R, F, *and* S. R to F practically being the 'plotting side': R to F represents the scale, inasmuch as their distance apart on the paper is measured by the scale of the chart.

If their stated distance differs from their true distance apart, then the scale of the work is not on the same scale as that of the chart, but on an arbitrary scale, depending upon the true length of R F. For purposes of plotting, should the scale of the chart be too small, the line R F can be drawn any length, and S and T plotted as before, whatever the scale may be (see *Enlarging*, p. 341).

The relative positions of R, F, S, and T are correct; but the distance between them will not be as measured from the scale on the chart, but by a scale in which R F in inches of paper represents their true distance apart in miles or yards or feet.

237. Difference between Astronomical Position and Triangulation in Short Distances.—If, it may be suggested, astronomical lines of position are introduced in addition to the triangulation, the following will occur:—

The Pole Star was used for an east and west line of position at T; if the same body is used at S, though both observations will separately be in error, the difference between the results should give a *difference* of latitude to within possibly one mile.

If the same twilight star or stars, bearing as near as possible east or west or both, are used for the determination of longitude in both positions, S and T, then the error in the *difference* of longitude deduced between S and T should also be about one mile.

The total result is an *astronomical* d. latitude and d. longitude, combined with a d. latitude and d. longitude by *triangulation.*

Now it must not be forgotten that errors in astronomical observations are minutes of arc and mean miles; while the accumulated errors incidental to a triangulation, though minutes of arc, produce only errors in fractions of a mile: for instance, in a distance of 10 miles, an error 10′ of arc in projecting a line 10 inches long = about 200 feet; hence the error in difference of latitude and of longitude astronomically is really out of all proportion to the error in two positions *a few miles apart* as found by true bearings and angles; therefore astronomical lines of position do not assist as much as one would expect in such a case as this.

238. Astronomical Position necessary when only One Line of Bearing.—But supposing only one shore object visible, whose correct position is known, then a true bearing obtained of it gives one line of reference; and, astronomical lines of position combined with this line are unavoidable to fix the position of the ship or buoy.

In such a case, with well-selected celestial bodies, the position will be more accurately fixed than any form of practical navigation can find in error.

239. A Rough Line of True Bearing is a Line of Reference.—Another purpose that a true-bearing line serves is best illustrated by the following fact.

A pearl-shell lugger off the north coast of Queensland had tumbled upon a small rocky patch, on which she anchored. In the attempt to define its position, the master was only able to afford the information, that, from his anchorage the top of the mast and the light of a light-vessel in the vicinity were just visible, and that the sun set about 2 inches to the right of it.

Given the day of the month and the year, it was possible to calculate the sun's true bearing from an approximate position of the lugger: allowing 2 inches to be perhaps 5°, this angle applied to the true bearing of the sun gave the T.B. of the light-vessel, and when laid off from her position, gave an approximate line of reference to the shoal within perhaps 10°: allowing a margin of a mile on either side, would make the distance between 8 and 10 miles since the height of the light-vessel's mast was about 40 feet.

Courses were steered on lines converging to the light-vessel, spread over 10° each way of the supposed bearing, for a distance ranging from 5 to 10 miles. The whereabouts of the shoal was located exactly, and eventually fixed by the intersection of one line of true bearing from a distant island, and numerous *astronomical* lines of position.

In justice to the master of the lugger, great credit is due to him for the unconventional, though unconsciously resourceful, manner of his observations; and it was a far more correct report of the position of a shoal than the hundreds of others which followed, and which in most cases depended on most accurately observed compass bearings, given to a quarter of a point, of the most distant land visible—irrespective of the state of the tide or state of the compass—or of, usually, what was called the 'end of the land'; while islands, beacons, and well-fixed summits were dotted in profusion round and *near* them.

240. Fixing the Head of a River.—On arriving at the head of a river, the absence of objects suitable to fix by, is a parallel case to that of a ship having only one or perhaps two distant objects, or even none at all, to fix by (see pars. 237 and 238).

As in the case of a ship, if there is nothing in sight, astronomical observations by artificial horizon must be adopted; but on shore, the errors of observation being less than that with a sea horizon, the results will be more accurate (see *Latitude by Ex. Meridians* (Appendix IV.) *and Longitude depending upon 'Absolute' Altitudes* (p. 38, par. 114), *or by Meridian Distances with Equal Altitudes* (Chapter XII., p. 415).

If from any position up a river one fixed object in the distance can be sighted, or the converse, then a line of true bearing can be found, which will give a line of reference from one position to the other. This will be a 'check' in *direction* of the position of the observer found by the triangulation up the river; for, in the triangulation, the errors in the triangles and of plotting are accumulating.

If the ascertained true-bearing line is at right angles to the general trend of the river, it will serve as a check to the total *distance* from its mouth, and each triangulated distance from

point to point can be readjusted for the whole error (see *Triangulation of a River*, p. 244, par. 443). Should the ascertained line of true bearing be nearly parallel with the general direction of the river, it checks the slue, and the *angles* of the triangulation might require readjusting.

In addition to the line of true bearing, it may be necessary to obtain astronomical lines of position; and that for latitude is probably simpler and easier to obtain correctly than that for longitude; for an error in the hour angle in the one makes a less corresponding error in latitude than the same error could make in longitude; hence an east and west astronomical line would probably be easier to take, and more accurate when taken, than a north and south line of position; consequently a north and south line of *true bearing* would be of the greatest value.

241. By Two True Bearings.—Where two fixed objects are visible, the true bearing of one may be observed and the true bearing of the other deduced from an angle between them.

These two true bearings, with, if necessary, the convergency correction applied, projected at the fixed objects from the Mercatorial meridian, *i.e.*, the north and south line drawn on the paper, will give the position of the observer.

The value of such a fix will depend upon the receiving angle at the observer's position and the distance of the objects (see par. 183), just the same as was shown in *Example*, par. 234.

CHAPTER XVI.

HEIGHTS.

242. Vertical Sextant Angles: 'Spherical' Sides.—The vertical angle of measurement of any object is its altitude, *i.e.*, the angle between the horizontal plane and the line joining the observer's position with the object. Dealing with a terrestrial object, such an angle is known as its angle of elevation. In fig. 80 the altitude of the sun, X, above the horizontal plane of the sea horizon is the spherical length of the arc X H, part of a great circle Z H; H H^1 being the celestial horizon, *i.e.*, the terrestrial horizon produced to infinity.

If O in fig. 81 be an observer, V the visible horizon, O V produced to infinity will be the celestial horizon.

In the celestial concave, Z P and Z H (fig. 80) are great circles passing through Z at a practically infinite distance from the observer.

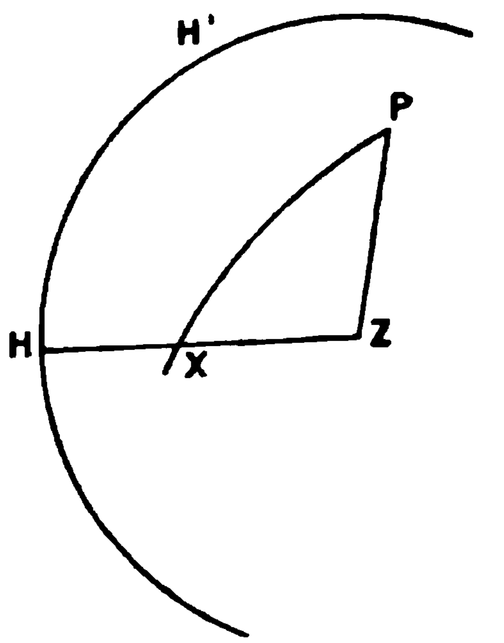


FIG. 80.

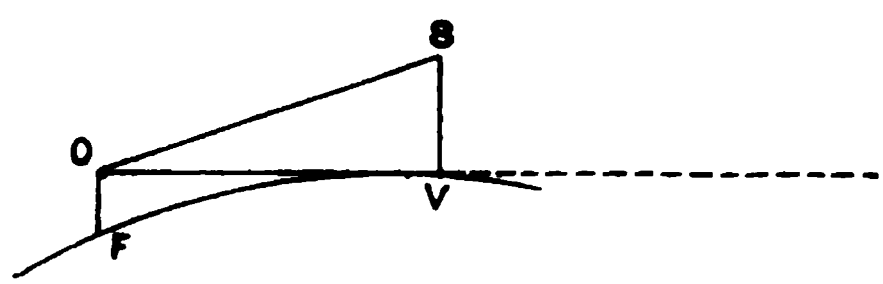


FIG. 81.

All the triangles in fig. 80 are spherical triangles, while those in fig. 81 are plane triangles.

243. Vertical Angles in Plane Triangles.—But when the object is on the surface of the earth, and the view is bounded by the visible horizon, then the altitude of an object is considered to be the angle of elevation in a plane right-angled triangle, having for base the distance from the observer to the base of the object, and for the 'perpendicular' its height.

In triangle S O V (fig. 81), S O V is the angle of elevation of S above the horizon line O V, and O V is the distance of O from V as plotted on the chart; and S V (the height in feet) = O V (the distance in feet) . tan S O V.

The *horizontal* line O V is then the zero line for elevations, and when using a sextant it is necessary that this imaginary line shall be accessible without interruption for its whole distance in order to measure the angle S O V.

The horizontal distance of V from O varies with the height of the eye above the surface. If, for instance, O F is 5 feet, OV = 2·5 miles (see *Inman's Tables*).

In fig. 81, taking O F as 5 feet and O V $2\frac{1}{2}$ miles, then V is exactly on the zero line O V, and the angle S O V can be observed.

244. Shore Horizon.—But if, as in fig. 82, V^1 is the point to which S is reflected, the horizon O V being interrupted, then S O V^1 is the angle observed, while S O V is the angle of elevation.

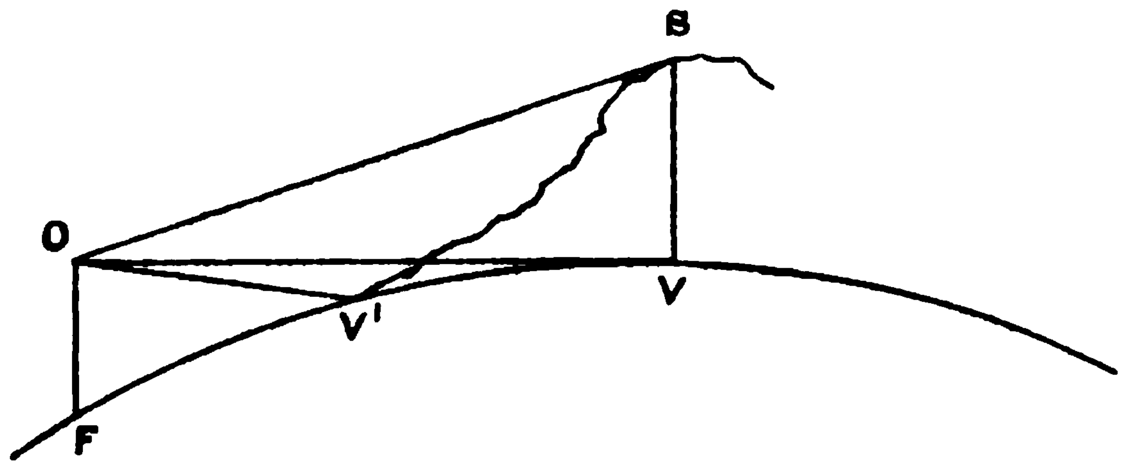


FIG. 82.

This point V^1 represents the nearest point on the shore in line with the summit S; hence the angle is measured to a substituted zero line O V^1, known as a 'shore horizon' line; and the angle is too large.

The amount of the correction, V O V^1, will depend upon O V, which varies as O F, and upon O V^1 the distance of the shore; hence necessity for the use of the table of the 'dip for shore horizon' (see *Inman's Tables*, and Appendix IX., p. 461).

The correction changes very rapidly when O V or O F are large and O V^1 is small; and an error in either O F or O V^1, when O F is large and O V^1 small, will make a large error in the correction to the observed angle, and in the consequent error in the height. S V = O V . tan S O V; the tangent of small angles changing very rapidly, the error will increase inversely as the size of the angle.

O F is the height of O above the water-line, and O V^1 must be measured on the chart from O to the point at the water's edge, where the shore meets it. Evidently the correctness of this distance will depend upon the accuracy of the coast at V, as plotted on the chart, at the point where the water-line meets the line O V^1. The correction taken from *Inman's Tables* subtracted from the observed angle will bring it to an horizontal angle S O V.

S V is the height of S above O, and O V is the level of the sea at the time of the observation.

245. Reducing Heights to H.W.S. Datum.—All heights

on the chart are shown as measured from high-water springs; hence the position of O relative to H.W.S. must be deduced.

For example, if $OF = 30$ feet, and it is half-tide at the time of observation, and if spring tides rise 20 feet, then at half-tide the tide is 10 feet below the level of H.W.S.

If the observer is 30 feet above half-tide, and half-tide is 10 feet below H.W.S., then the observation is taken at a level 20 feet above H.W.S., and that quantity will be added to S V to indicate its height above H.W.S.

246. Heights by Sea Horizon: Correction for Curvature.—The second case is where the zero line O V (fig. 83) produced cuts off a part of the height, *i.e.*, where the base of the hill is beyond the distance of the visible sea horizon shown by O V. Here S O X is the observed angle of visible elevation, while SOV^1 is the angle subtended by SV^1; and

$$SOV^1 = SOX + \text{correction } (VOV^1).$$

VOV^1 being small,

$$SV^1 = OV^1 . \tan SOV^1 \text{ very nearly} = OV^1 . \tan (SOX + \text{correction}).$$

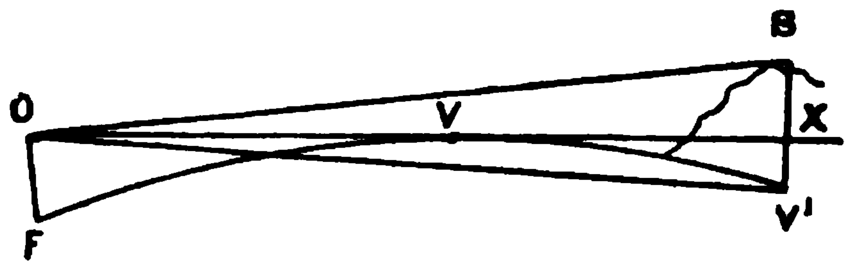


FIG. 83.

The correction, including refraction, is 25″ for every mile in distance OV^1, and is always +: or see footnote.

The necessary addition must be made to SV^1 to find its height above H.W.S. where the observation is made from a position afloat (as described in par. 245).

247. Heights on Charts.—All charts give the height above high-water ordinary springs of prominent mountains, hills, hillocks, cliffs, summits of islands, lighthouses, and rocks.

High-water ordinary springs (H.W.O.S.) is adopted in the datum for heights as the zero, because for a given height the least distance it is visible is at H.W.O.S.; at any other state of the tide the objects will be visible at a greater distance: obviously so, because their height relative to the level of the observer is higher.

In the case of lighthouses, summits of islets, or rocks the height is stated with all possible accuracy; this height being used in navigation for determining the distance and for fixing.[1]

[1] When the angle of elevation is small (under 7°).—Let θ'' be the angle.

$$\text{Circular measure of an angle } \theta'' = \frac{\text{arc it subtends}}{\text{radius of the arc}}.$$

[*Continued overleaf.*

248. Use of Vertical Danger Angle.—The angle of elevations of these may be used if required as a 'vertical danger angle'; for, given the height and the distance it is deemed safe to navigate from the point, the corresponding angle of elevation can be found: $\frac{\text{height in feet}}{\text{dist. in feet}} = \tan$ elev., or elev. in secs. of arc $= \frac{\text{height in feet} \times 34}{\text{dist. in miles}}$. (See footnote, pp. 115, 116.)

This angle is that subtended by the height stated at a given point on the sea-level of H.W.O.S.; and this point or position is consequently on an arc of a vertical circle passing through the position of the observer, the object, and a point *vertically* beneath it. To increase the angle is to bring the arc of position passing through the observer nearer the shore; and, conversely, to reduce it is to recede from the land. The height of objects likely to be used for such a purpose should be well determined.

The calculated positions will always be nearer the shore than the true, except at H.W.O.S.; if, on the other hand, the heights were given from L.W.O.S., then the true position would always be nearer the shore than the observed. Evidently the H.W. datum is a safer one, and is consistently used for all heights. If the correction for dip is omitted, the margin of safety is still greater.

249. When Heights need only be Approximately Correct.—The use of heights of hills other than that for finding an arc of position is chiefly to enable one hill to be distinguished from another as seen from any point of view, and a matter of a few feet in error is of no vital importance so long as the heights are shown relatively correct.

In fig. 36 on page 50 the heights were merely used as distinguishing numbers.

When there is no room in the land position of a chart to insert the height, as in the case of islets or rocks, the height is written in *upright* figures and enclosed in brackets to distinguish it from a sounding. (For heights by theodolite angles, see *Theodolite*, par. 280.)

Circular unit = angle whose circular measure is unity

$$= \frac{360°}{\pi} = 206,267'';$$

then
$$\frac{\theta''}{206,267'} = \frac{\text{circular measure of } \theta''}{1}$$
$$= \frac{\text{arc}}{\text{radius}} = \frac{\text{chord}}{\text{radius}} \text{ (when } \theta'' \text{ is small).}$$
$$\therefore \theta'' = 206,267'' \times \frac{\text{feet subtended by } \theta'' \text{ (height)}}{\text{dist. in miles} \times 6080}$$

(1) $$\therefore \text{height} = \frac{\text{dist. in miles} \times 6080}{206,267} \times \theta'' = \frac{\text{dist. in miles} \times \theta''}{34 \text{ nearly}}$$

(2) $$\theta'' = \frac{\text{height in feet} \times 34}{\text{dist. in miles}}$$

(3) $$\text{dist. in miles} = \frac{\text{height in feet} \times 34}{\text{elev. in seconds of arc}}.$$

CHAPTER XVII.

LANDING COMPASS AND THEODOLITE.

250. Principle of.—If a circular horn protractor is placed over a sheet of paper, on a fairly level stand, with the centre over any spot in the paper, and a ruler is placed on the top of it so that one edge passing over the centre of the protractor is directed to any object, the line of this edge, or line of sight, drawn on the paper will give a line of reference to that object; the angle indicated by the ruler at the edge of the protractor may read anything.

Without in any way moving the protractor, if the ruler is kept over the centre, and is made to point to any other object, this will again be a line of reference to the other object: the ruler will indicate the angle at the edge of the protractor.

The difference between the first reading shown on the protractor, and the second, will, roughly, be the angle between the two objects.

Now, this is but a rough measurement of an angle in the plane of the protractor, and not in the plane joining the two objects; and is, therefore, neither a horizontal angle nor an angular measurement.

But, supposing there be means of making the protractor *horizontal*, and of securing it in that position; that there are improved means of obtaining the line of sight by a vertical line at the end of the ruler, then the difference between the first reading of direction and the second, will be the *horizontal* angle between the two objects.

251. Admiralty Landing Compass.—The next step in advance of the rough protractor idea, is the Admiralty landing compass.

Like most large compasses, it possesses a graduated ring or 'verge' on the rim of the glass cover; this, then, will now represent the protractor, but, instead of resting on a fixed plane depending upon the table or stand, it is fitted with gimbals like any ship's

compass, and therefore swings freely. Theoretically it should 'find' the horizontal, but does not do so exactly in practice, because the points of suspension are not placed quite symmetrically at the end of each diameter; moreover, if disturbed, it does not return to its initial position, owing to friction. Its plane, then, may be said to be vaguely horizontal when first 'set up,' and irregularly in error when handled.

252. **Line of Sight.**—Its line of sight is attached to a ring, nearly the size of the graduated verge, revolving from the same centre; and consists of a slit at one end of the diameter, which is used as a 'back sight' where the eye is placed, and a thread stretched on a vertical frame at the other, for a 'fore-sight'; so long as these are truly at the ends of the diameter of the graduated arc, whatever error there is in the foresight—*i.e.* whether truly vertical to the plane of the graduated circle or not—if the objects observed are exactly on the same level, the angle measured between them is unaffected when the plane of the 'verge' is the same throughout; but if the fore thread or the hind slit are not true—that is, leaning to one side or the other—and the objects observed are at different levels, and the plane of the 'verge' when handled is irregular, then the angles are inconsistently in error.

253. **Index and Vernier.**—Attached to the ring which carries the sight line, there is an index with a vernier. If, then, an object is sighted on the fore-thread and through the back slit, followed by another similar observation to another object, the *difference* between each of the readings will give the more or less horizontal angle between the objects, at the centre of the instrument.

This involves constant subtraction; for example, looking at an object A, that being the initial line of reference, suppose the reading is $12° 22'$; turning the line of sight round to B, the reading is $56° 37'$, towards C it is $91° 55'$, and so on; then the angle between A and B is $56° 37' - 12° 22' = 44° 15'$; that between A and C is $91° 55' - 12° 22' = 79° 33'$.

254. **Setting of Reading when Pointing to the Zero.**—This system is cumbrous and is not practised, except for special purposes; it would evidently be generally simpler if the reading at the initial line of reference was $0° 0'$; to obtain this, the *graduated plate* would have to be turned round until its reading was $0° 0'$ by the index, while the line of sight was pointing to the initial object.

Or, if the index is set at $0° 0'$ as nearly as possible, there being no clamping nor tangent screw to assist in the operation, and the *whole compass bowl* is turned round until the line of sight is over the initial object, and clamped, in that position, to the plate on the tripod, by the one screw underneath it, there being again no

tangent screw to assist, then the one end is attained; for, now, *the index reads* 0° 0′, the graduated plate is a fixture, and the line of sight is pointing to A; turning the line of sight round, the reading is taken of B, C, D, etc., in succession, and the readings therefore are also the angles from the initial line A.

255. Centring Error Eliminated by Opposite Indexes.—Besides the errors already enumerated in the instrument, there is the centring error, that is, when the centre of the ring carrying the line of sight does not coincide with the centre of the graduated verge; this is eliminated by the addition of another index and vernier, each vernier being assumed to be at the end of the diameter of the upper ring.

The mean of the readings of these two indexes is the correct one; vernier errors and coarse cutting are additional sources of error, and the general results with this instrument must be accepted for what they are worth.

Graduations 0° to 360°.—The graduation on the 'verge,' beginning at 360° 00′ or 0° 00′, continues to the right through the quadrants to 100°, 200°, and so on to 360° 00′.

256. Zero.—The initial line of reference is one specially selected for that purpose; the object in that line being termed the zero or ⊕; it will be the zero line, from which all the other angles are referred; and since, though preferable, yet optional, the reading of the ⊕ is usually 0° 0′, *i.e.* 360° 00′ (*vide* par. 254), it is necessary to state what the reading of the ⊕ actually was; thus an observer will record as follows:—

At 0 ⊕ A 360° 00′. 180° 04′ (this is the reading of the opposite index); then follow the other angles as observed (for further explanation see *Theodolite*), which are written down in the same manner as when using that instrument.

By a gradual and general process of improvement the present theodolite has been evolved.

257. Evolution from Landing Compass to Theodolite.—It also has a circular plate, which varies in size from a diameter of 3 inches to one of more than 12 inches, the graduation being the same as on the compass verge. The degree of graduation will usually depend upon the diameter; some are graduated to 30′, some to 20′, and some to 10′ of arc.

Under this there are a pair of thick metal plates of smaller diameter than the graduated arc; they are separated by three, or in some by four, thick mill-headed screws.

258. Levelling Plates.—The lower of these plates screws on to the tripod, and becomes a temporary fixture; by means of the mill-headed screws which bear on it, the *graduated* plate is levelled; the levels for this purpose being placed on it, as shown by L L, fig. 84.

The 'line of sight' is supplied by a telescope resting in Y's, which are at the ends of a graduated arc, working in a vertical plane; the centre of this arc is vertically over the centre of the graduated arc; the whole is supported by pillars on an upper or 'index' plate, which rests on and revolves horizontally from the same centre as the graduated arc.

When the lower levelling plate is screwed on to the tripod, and the graduated arc is made level, and the levelling screws tightened up, there exists a truly horizontal arc, graduated from 0° to 360°, with a telescope capable of a horizontal and vertical movement to guide the line of sight.

A clamping thumb screw is attached to a 'sleeve' extending from the graduated plate; when this is tightened up the plate is immovable.

259. Horizontal Angles.—We now have an immovable, graduated plate, which is truly horizontal.

If, as before, the line of sight which is attached to the upper or index plate, is pointed to an object A (see fig. 84), and afterwards to an object B, the difference of the readings of the angles to each will be the horizontal angle between them; no matter what their respective altitudes may be.

260. Position of the Index.—But it would evidently be most inconvenient to read off these angles by the actual line of sight, *i.e.* by an index immediately under and in line with the telescope, as the head would constantly knock against the telescope. The same results will be obtained if each reading was taken from the same *relative* position of the line of sight; hence the index with its vernier is placed abreast of the telescope. (See V, fig. 84.)

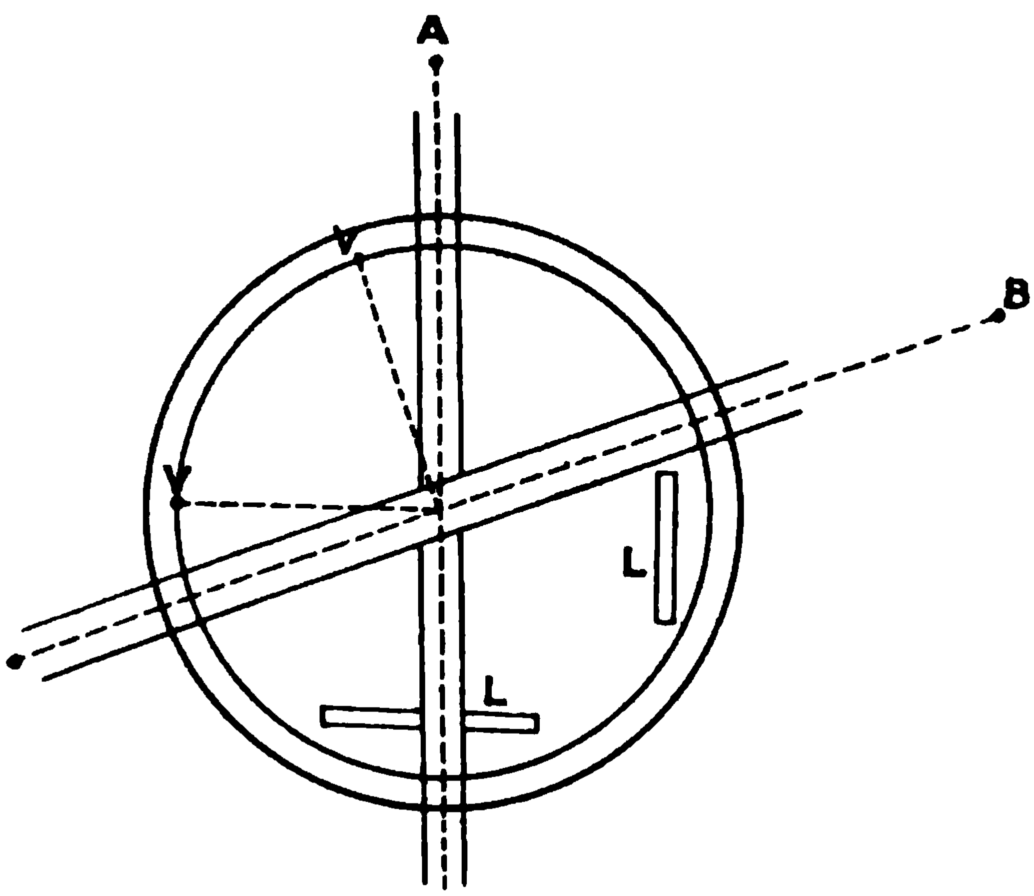


FIG. 84.

For the same reason that the landing compass has them, there are two indexes, each with its attached vernier, while, in addition, a microscope is also provided to allow of a more accurate reading.

The mean of the reading of these indexes will eliminate the error in the position of the centre (centring error) of the index plate carrying the telescope.

261. Errors in Vernier and Centring and Graduations.—A theodolite has vernier errors consequent on centring errors, just as a sextant has; it also has errors in the graduation, but the total errors are not so pronounced as those in a compass, because the arc works from an axis instead of being suspended from the ends of its diameter, and therefore the instrument is more accurate in these respects.

For the same purpose as when using the compass ring (see par. 254), one index is set at 0° 0′, *i.e.* 360° 0′; but there is the additional assistance of a *tangent* screw for accurate setting, when the graduated plate is turned to the direction of the zero line of reference ⊕, or for a more refined observation and greater accuracy in taking the angles; as well as a microscope for reading off. For these reasons also the theodolite has a great advantage over the compass.

When the instrument is level, and the line of the telescope points over the zero object, the reading being 0° 0′ on one vernier, state the following:—

At 0 ⊕ A 360° 00′ 179° 58′.

262. Position of the Zero Readings.—Now, there will be two columns of angles; those under the 360° 00′ (the left column) belong to that zero reading. It is written on the left, because it is customary in European countries to write from left to right, and the lowest number on the left; therefore, for uniformity's sake, this zero reading would be to the left of the observer when he is looking through the telescope.

Hence, in setting the index, the 0° 0′ or 360° 00′ reading should be made with the left vernier; and then the reading of the right vernier be taken. This is the rule which common sense dictates; if it is not carried out, utter chaos ensues, though it is not absolutely erroneous to set the readings in the opposite order.

As each angle is taken, the left vernier should be read off first, then the right; if not taken in this order (sometimes it is handier not to), and the observer has an assistant, he must call out, 'right vernier' (right column of angles), or 'left vernier' (left column), as he reads off each.

263. Method of Writing Angles.—Then a continuation of the notebook reads:—

		Left column.	Right column.
At 0 ⊕	A	360° 00′	179° 58′
	B	44 15	224 14
	C	79 33	259 33,

and so on.

264. Necessity of Revolving the Line of Sight in one Direction.—In handling the instrument the angles should always be taken by revolving the line of sight, which in this case is the telescope, round in the same continuous direction either way. In the above readings the telescope must have been turned round to the right, because the reading to the first object is the least; had it been turned to the left, the angle immediately below 360° 00′ might have been 327° 52′, and the next 300° 12′, and so on.

The graduated plate is supposed to be more or less firmly fixed; and the legs to be more or less immovable; but in practice it is very easy to move both of them slightly; and, however gently the telescope is moved round, something will be shifted so as to disturb the graduated plate or to throw the instrument out of level.

If the movement of the telescope has been continuous in one direction, some reasonable value can be assigned to the correction for each angle, by its proportion to the whole error caused by the disturbance of level and the shifting of the graduated plate; but if the movement of the telescope has been first to one side of zero, then to the other, not only will the error of *level* caused be greater, but no consistency can be traced in the error in the change of position of the plate; in fact, it may show no resulting error, while as a matter of fact the instrument has moved.

265. Selecting the Zero.—Now this error of movement introduces the method of its deduction. Suppose an observer at O, fig. 85, selects the line O A as a zero line; after taking a number of angles to the right, he refers the instrument to its initial reading of 0° 0′; then any disarrangement of the instrument will be indicated by the error A*a*, the line of sight now pointing to '*a*' instead of A as originally set; but had he taken O X, then the error of movement is shown by X*x*, a much more *evident* error. Hence, to detect the error of movement, the further O X is the better; in other words, the further the zero selected the better.

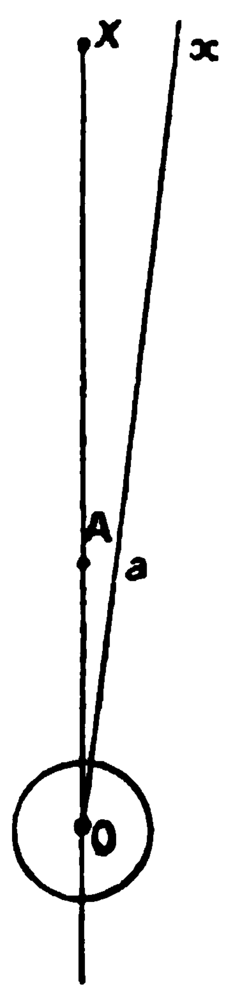


Fig. 85.

In practice, the order is reversed; the centre of the cross wires is referred back to the zero object, the reading of which was 360° 0′, and the error in *reading* is indicated. In the above case, when referred back to A, the reading would have been 359° 55′ instead of 360° 00′, showing the instrument had shifted 5′ during the whole revolution.

266. Z.O.K.—To detect the error of movement, whether large or small, it is therefore essential to refer to the zero line O X; and if the reading now is different from what it was originally set at, the zero is said not to be correct, and the reading it returns to is stated, such as 359° 57′; and conversely, if

it does return correctly, the zero is said to be O.K., written Z.O.K.

267. Necessity to refer back to Zero.—This practice is essentially a part of the value of any angle, or angles, taken with the theodolite; and without it the observations are incomplete, and perhaps absolutely useless.

Whatever the reading of the zero is, when referred back, should be stated; and if 359° 57′, for example, it might be near enough for the particular angles that are required, when, for instance, the angles do not form part of the very important main triangles.

268. Repeating the Angles if Necessary.—Part of the error might be discovered by repeating the angles at or near every 90° of the readings; but, if not checked in this manner, the statement with reference to the ⊕ should remain; it is only cheating one's self to write O.K. when it is not correct.

Then, after the last angle of the round or series is taken, the statement should be Z.O.K or Z 359° 59′, or whatever it is.

269. Readings in Parts of the Arc to Reduce Errors to Graduations: Changing the Readings of the Zero.—When dealing with the principal angles of a triangulation 359° 56′ will not do; the angles should be repeated.

Where the greatest possible accuracy is required, some means should also be adopted of reducing the error in the graduation of the instrument.

For this purpose, different readings for the zero can be set on the vernier.

For example, B, C, D being main stations, it is required to obtain a number of measurements for their angles.

						Angle by right vernier.
Then at 0′ ⊕	A	360°	00′	179°	58′	
						44° 16′
	B	44	15	124	14	
						79 35
	C	79	33	159	33	
						119 55
	D	119	54	299	53	
				Z.O.K.		

Now shift the reading on the index of the arc to 100° 0′, and (without unclamping the plates) point the telescope to the zero object.

						Angle by right vernier.
Then at 0′ ⊕	A	100°	00′	279°	59′	
						44° 16′
unclamping the index plate	B	144	16	324	15	
						79 36
	C	179	34	359	35	
						119 54
	D	219	53	35	53	
			Z.O.K.			

If desirable, the operation may be repeated once again, starting with the index at 200° 0′.

						Angle by right vernier.
Then at 0′ ⊕	A	200°	0′	20°	01′	
						44° 15′
	B	244	16	64	16	
						79 35
	C	279	35	99	36	
						119 54
	D	319	55	139	55	
			Z.O.K.			

By this means there are three sets of observations, with two readings to each object; and the six measured angles on different parts of the arc are meaned; hence the error of the graduation will be minimised.

1st observation	B	44°	15′	by right vernier
		44	16	„ right „
2nd observation	B	44	16	„ left „
		44	16	„ right „
3rd observation	B	44	16	„ left „
		44	15	„ right „
			34′	

Mean 44° 15′ 40″

And so on for C and D, etc.

There is yet another method, a method of continuation, when one angle only is required, and all possible accuracy sought.

The following example will explain :—

Starting with ; at 0 ⊕	A	360°	00′	180°	01′	
						44° 16′
	B	44	15	224	17	

Leaving the index clamped at this last reading, *unclamp the graduated arc or lower plate*, and turn it so that the line of sight

points to the $\oplus$; then clamp again, and with *its* tangent screw make exact contact.

Now at 0 $\oplus$	A	44° 15′		224° 17′	
			44° 16′		44° 14′
then unclamp the index, and	B	88 31		268 31	
Repeat the operation:					
Then at 0′ $\oplus$	A	88 31		268 31	
			44 16		44 15
	B	132 47		312 46	

And this may be continued as often as desired.

Finally, refer back to the zero by the last observation, and Z.O.K.

By this means six readings of the arc are obtained for the measurement of one angle.

The instrument must be in very good working order, and in perfect adjustment, since the reference to the zero *after each measurement* is omitted, and the operation is a very delicate one to carry out without disturbing the level of the graduated plate.

270. Telescope. Eye-piece fitted with Coloured Shades.—The telescope is, in the larger size instruments, usually supplied with two eye-pieces, one of which, the shorter, inverts the objects; while the longer is fitted with shades, for use with the sun.

271. 'Wires' or 'Webs.'—Inside the telescope and beyond the eye-piece, stretched across a diaphragm or ring, there are two cross 'wires' or 'webs' usually spider webs; and, by means of screws, whose heads usually project on the outside of the tube of the telescope, this diaphragm can be adjusted, in case the wires do not cross in the optical centre.

These wires cannot usually be seen without first focussing the eye-piece: this must be the first operation when using the telescope.

272. Focussing Screw.—There is a focussing screw attached, by which either the object-glass, or the hind part, is moved for *focussing the object* observed: this is the next step necessary.

273. Means of Holding the Telescope.—The telescope fits into supports called Y's—so called because they resemble the capital letter Y—and is held there by rings closing over it, and which are pinned down; there should be no 'play' in the telescope, and usually two bits of cork take the place of springs to grip it; the Y's and ring should fit the telescope exactly, but not so tightly as to prevent it being revolved if desired.

274. Centre of Cross Wires over the Object.—When an object is observed through the telescope, the centre of the cross wires should cover it; the line from the centre of the eye-piece to the centre of the cross wires is the zero line of the telescope.

In theory, it is better that the zero line of the telescope should be in its optical centre; in practice, with a 4-, 5-, or 6-inch theodolite telescope, used in the ordinary work of a small survey, a small optical error in this position, *if the telescope does not move* during a round of angles, does not affect the results.

275. **Collimation Error, to Adjust** (see p. 149, par. 319).—But the telescope may be moved by accident; and, since there are the means of doing so, the zero line can be adjusted to the centre of the telescope; fig. 86 illustrates an exaggerated form of the error.

C is the optical centre of the telescope; C^1 the intersection of the cross wires, which cut each other at right angles.

The T's round the diaphragm show the position of the adjusting screws, but they do not always project.

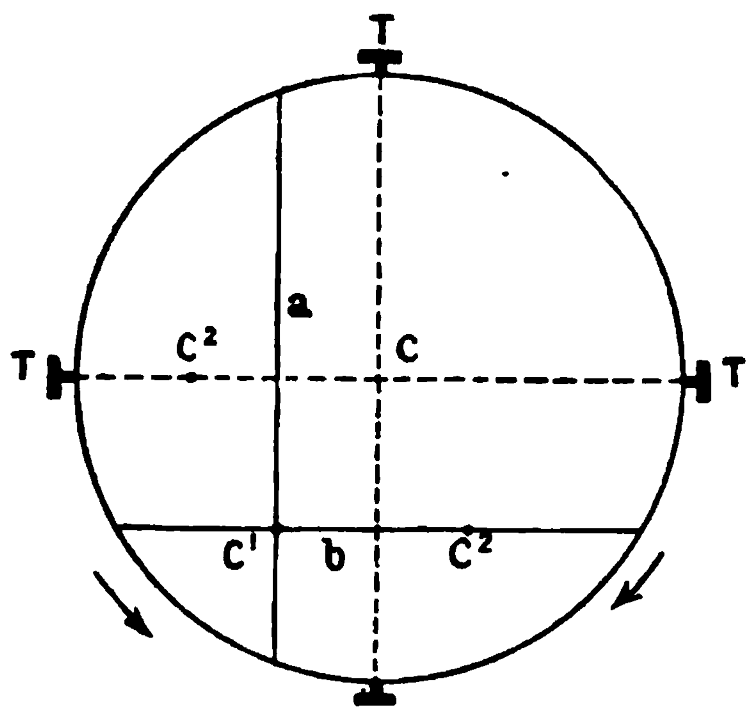


FIG. 86.

The telescope is focussed and pointed to any object; after adjusting the eye-piece for the 'wires,' and the wire C^1C^2 being roughly horizontal (see par. 278), the plate is then clamped in the position when C^1 covers the object.

If, now, the telescope is revolved in the direction shown by the left arrow, C^1 will also revolve in a circle whose radius is CC^1 (if *the telescope is untouched* while taking angles, the position of C^1 is, of course, constant, and the error in its position does not affect the angles measured).

It is required to put the zero line (one end of which is the centre of the cross wires) into such a position that, if the telescope be revolved, the line is unaffected; this position will be C. It is now at C^1.

If the telescope is turned a quarter of a revolution, C^1 will arrive at C^2; the wire previously perpendicular will now be horizontal.

In this position the centre of the cross wires now at C^2 covers a certain point in the field of view, whereas, before, when at C^1, it covered a different object.

Half-way between C^1 and C^2 gives the vertical line through the optical centre of the telescope; and by the following method C^1 can be moved to that line by screwing up on the left adjusting screw attached to the diaphragm, and screwing back on the right.

If the object which C^1 covered, is covered again by C^2, by means

of a movement of the *horizontal* tangent screw on the arc, half the difference between the *reading* on the horizontal arc, when looking at C[1] and that at C[2], gives the exact reading on the line of C; set this reading on the arc, and move the erroneous C[1] on to it by the diaphragm adjusting screws. (See *Theodolite Adjustment*, p. 149.)

This will now place the vertical wire *b* through the optical centre of the telescope: if the telescope is turned a quarter of a revolution back, as shown by the right arrow, whatever error there is shown by C[2] not covering the same object will be removed by means of the other pair of diaphragm screws, and, finally, the cross wires are in the optical centre of the telescope.

This operation is called adjusting for collimation. (See *Theodolite Adjustment*, p. 149.)

276. Micrometer Wires.—In the theodolite telescopes used by engineers, there are two additional horizontal wires placed with reference to the focal length of the telescope (see fig. 87).

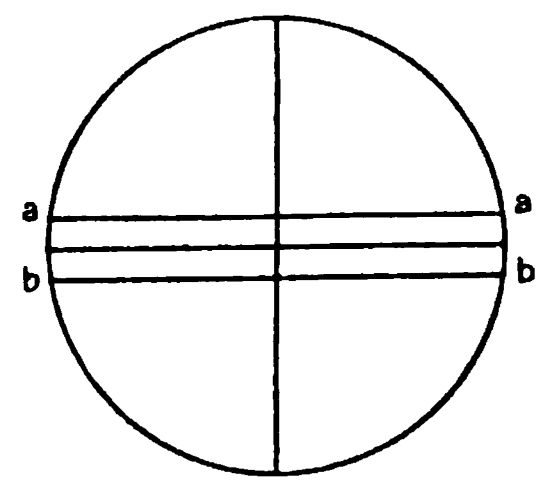


Fig. 87.

They are 'micrometer' wires, and used for the purpose of measuring distances in combination with a staff; this staff is graduated in feet, tenths, and $\frac{1}{100}$ths. (See *Levelling Staff*, p. 154.)

The distance in feet, measured on the staff, held perpendicular to the line of sight, between the wire *a* and wire *b* (see fig. 88) is the distance per cent. in feet, of the observer from the position of the staff; where 1 foot is 100 per cent.

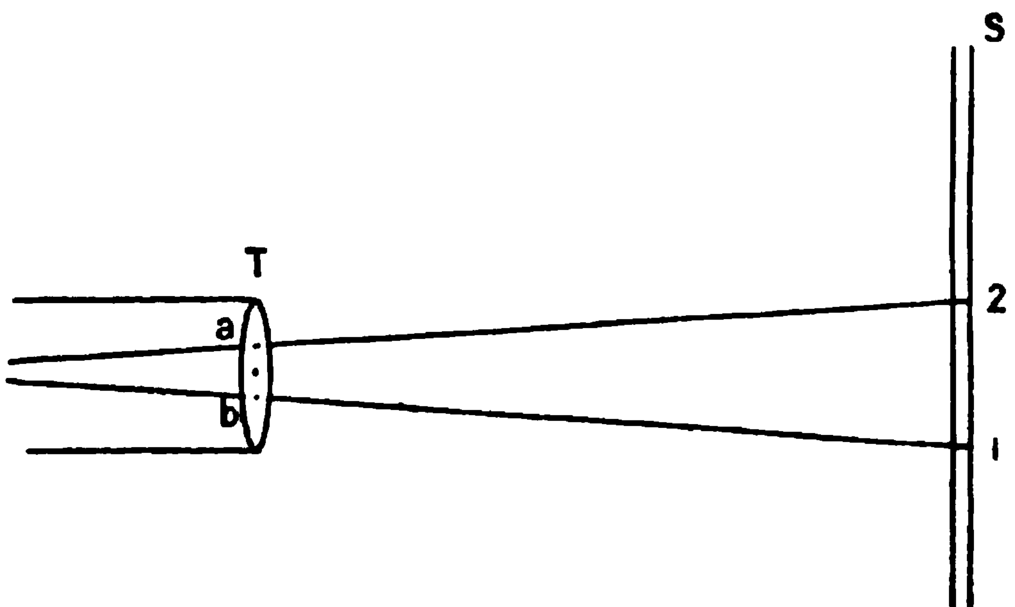


Fig. 88.

Example.—T is the position of the telescope, *a* and *b* the horizontal wires.

From the eye, the lower wire *b* reads 1 foot, and the upper wire reads 2 feet.

The distance between the readings is, therefore, 1 foot.

In this case, from T to S is 100 feet.

If the difference of readings had been ·75, then the distance from T to S is 75 feet.

277. Vertical Arc.—The movement of the line of sight horizontally is a movement in azimuth, but that term only applies to theodolites specially fitted for observation of celestial bodies in azimuth and in altitude.

The movement vertically is that in altitude or elevation.

An ordinary theodolite has a movement in elevation by means of a vertical arc, see fig. 89, the centre of which is vertically above the centre of the horizontal arc. As its purpose implies, it is not only for directing the telescope to an object above or below the horizontal, but also it is graduated for measuring an elevation.

The radius is usually shorter than that of the horizontal arc.

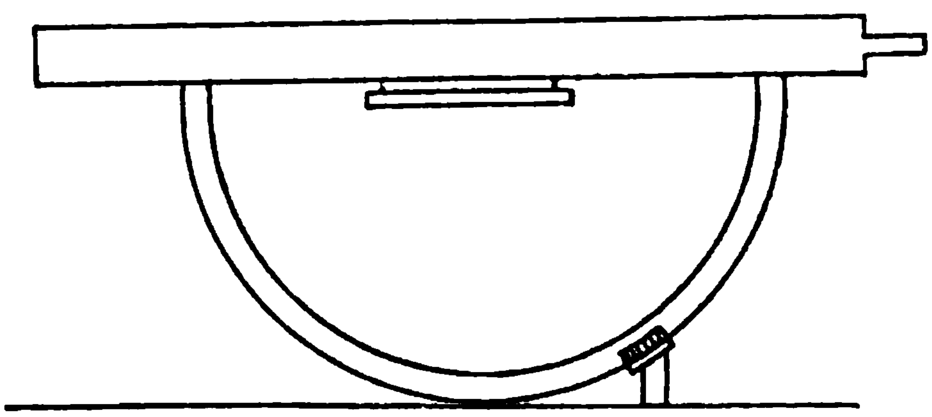


FIG. 89.

In the explanation given of the telescope, it is stated that it is merely held in its place on the Y's by a circular clip, and that it was liable to derangement by a light touch.

If it were required to take the altitude of the sun with a theodolite it is obviously necessary that the horizontal wire, over which the altitude is measured, shall be exactly horizontal and *shall remain so*; but, as there are no means of setting it exactly horizontal, and, moreover, it is liable to be moved, because not a fixture, it is unserviceable for sun altitudes, and is merely used for elevations.

But in the case of *relative* heights (see *Heights*, par. 249), it serves the purpose of measuring small elevations, practically near enough for ordinary uses.

An error in the vertical plane, due to faulty construction (par. 324), is a serious error in a sun altitude, or in any observation of the sun (see *True Bearings*, p. 138); whereas in finding a height, the corresponding error in an elevation is practically harmless.

The vertical arc, then, is for the purpose of directing the telescope to objects of different levels when measuring horizontal angles, and also for measuring the elevations of hills for the ultimate purpose of determining their heights.

Both in measuring the horizontal angle to, and the vertical angle

of, a summit which is not actually a pyramid, from various points of observation, both angles must be taken to the highest point, as that will be the only common point viewed from the different places.

The line of the horizontal wire horizontally sweeping the summit, guides the eye to the highest point, and here for this purpose only it is required to be horizontal.

278. Making Wires Horizontal.—After the telescope has been placed in the Y's and pinned there, and the instrument levelled, turn the telescope to a distant well-defined object, a steady plummet line or the side of a building.

Place one *end* of the vertical wire over the object, then, by means of the tangent screw to the vertical arc, sweep the wire up and down the object. If the object remains on the wire from one end of it to the other, then the wire is truly vertical (always providing the vertical movement of the arc is true); if not, revolve the telescope, until by the same movement as before the wire does not deviate off the object.

When that wire is vertical, then the horizontal wire may be assumed to be horizontal.

The same result can be obtained by means of the sea horizon with the horizontal wire.

279. Bubble Error.—The angle of elevation or of depression, is the angle between the horizontal plane at the position of the eye, and the elevation or depression of the horizontal wire.

Zero for Elevations.—The reading of the angle on the vertical arc, when the telescope is levelled, by means of the level attached to it, is the zero for elevation or depression.

It is the same story over again of the index error of a sextant, but here it is known as the 'bubble' error.

If, then, the telescope is levelled by its own level, this is the horizontal plane, and the reading on the vertical arc is the 'bubble' error.

Angles of elevation are measured on the left of the 0° on the vertical arc, while those of depression are measured on the right.

The bubble error takes the same sign as a similar error does on a sextant, that is, − if its reading is on the same side of the 0° as the measured angle, and + if on the opposite side; so that a bubble error reading 2′ 40″ on the left of the 0°, would be − to elevations, and + to depressions.

Now this error is due to a combination of, an error in the vertical arc, faulty adjustment of telescope level, with an error in the level of the instrument in the plane in which the telescope is being directed. Suppose, for instance, the plane in line with the telescope to be the whole source of error, as shown in fig. 90, H L being the horizontal arc of the theodolite out of level.

Then the 0° of the vernier will appear to the left of the index, and the reading at V will be the bubble error.

Now supposing that the plane of H L, at right angles to that shown, is correctly level, then, when the telescope is pointing along that plane there will be no bubble error, providing that the vertical arc is correctly placed on the instrument. (See par. 324.)

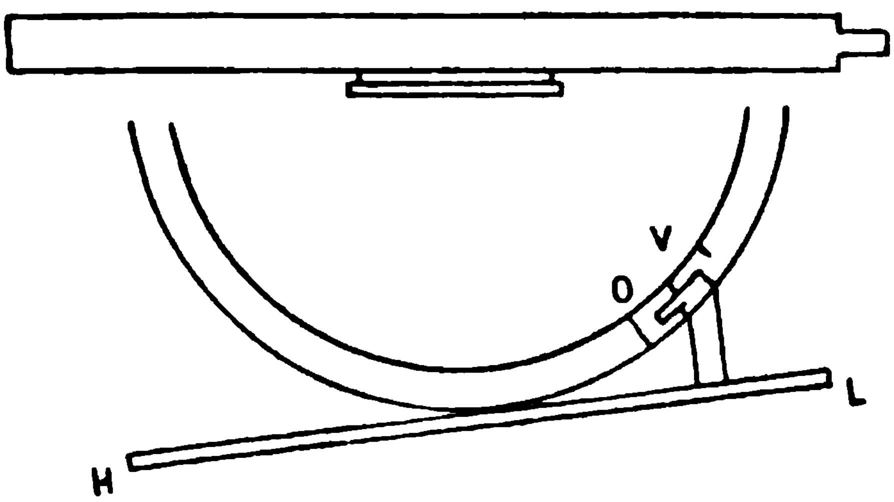


Fig. 90.

Therefore the bubble error is a varying quantity, and is different in different directions.

From fig. 90, it will be noted that when the telescope is pointing round in the opposite direction, the size of the bubble error will be the same, but its reading will be on the opposite side of the 0°.

Therefore, providing there is no constant error in the position of the vertical arc, and the level attached to the telescope is in adjustment, the bubble error will be $\pm$ in one semicircle, and $\mp$ of the same value in the other; if found at each quadrant of the horizontal angle, by interpolation, the error must be applied to each elevation, according to its horizontal angle.

Example.—The bubble when the horizontal arc reading is

0°	is	+	2′ 30″	to angle of elevation.	
90°	,,	−	30″	,,	,,
180°	,,	−	1′ 30″	,,	,,
270°	,,	+	1′ 30″	,,	,,

There is no occasion in the ordinary theodolite to separate the error of levelling from the error of the arc, so that, it is accepted as a whole error.

If, now, the angle of elevation to an object reading 50° is 1° 16′ 20″, then by interpolation the correction is + 1′ 40″ (50° = $\frac{5}{9}$. 3′ = 1′ 40″), and the correct elevation is 1° 18′.

Reversing Telescope for Elevation.—Again, just as with the sextant, if an angle is measured to the right of the 0°, and, with the same objects, then to the left, the total error of the index is cancelled, and includes the error of levelling.

So that, if, after taking the elevation, the position of the telescope is reversed, end for end, the object-glass is towards the observer: by turning the index plate horizontally, it will be made to point

to the same object as before, and the vertical angle is measured again; in the first case the angle was measured to the left of the 0°, and in the second it will be to the right.

Example.—Right way round elev. 1° 16′ 20″
reversed „ elev. 1 19 40

mean reading „ elev. 1° 18′ as before (see above).

This process is more lengthy, but more accurate, because the levelling for bubble error in the single observation depends also upon the *adjustment* of the level under the telescope; hence, by whichever method an elevation is corrected must depend upon the accuracy desired.

280. Difference of Level measured by a Theodolite.—From an angle of elevation or of depression with a theodolite is deduced the *difference* of level.

An observer at O, fig. 91, measures the angle A O H, and the height A H is deduced from it.

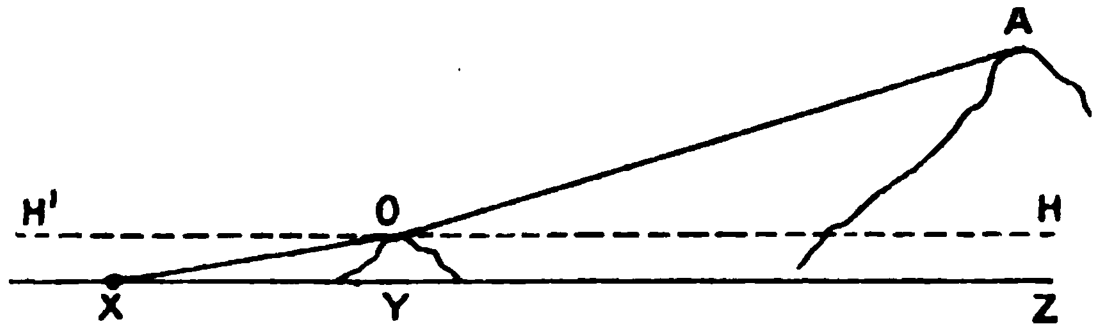


FIG. 91.

A H = O H . tan A O H (O H is the distance from O to the *plotted* position of A on the chart).

A H is therefore the difference of level between O and A.

If O has a height of his own, he can, by an angle of depression to a fixed object X on the high-water line, determine this height; the object X may be in any direction; or in several directions in order to reduce the error of observation, or the error due to an error in distance between O and X, or several X's; though this does not eliminate the bubble error.

The fixed position X being on the water-line, O Y = Y X tan H'O X; and O Y + A H = A Z, the height of A above the level of X.

Example of finding the height with the theodolite angle (fig. 91).

At A the angle of elevation of a summit B is 2° 27′ 10″; bubble error is 2′ 30″ left, the height of O's theodolite not being taken into account.

The angle of depression to X on the H.W.S. line is 0° 15′ 30″; bubble error, 2′ 30″ right.

The distance of B is 24·8 inches of paper.

The distance of X measured on the chart is 4·8 inches.

The scale of the chart is 3·1 inches = 1 mile = 6077·5 feet.

Required the height of B above H.W.S.

1. To calculate the height of A—

A's height in feet = distance to X in feet . tan. depn. to X;

distance to X in feet $= \dfrac{4\cdot8 \times 6077\cdot5}{3\cdot1}$;

distance of X in miles $= \dfrac{4\cdot8}{3\cdot1}$;

correction for curvature, etc. $= \dfrac{4\cdot8}{3\cdot1} \,.\, 25'' = +40''$;[1]

correction for bubble error is $-2'\ 30''$;
total correction to observed angle $= -1'\ 50''\ (-2'\ 30'' + 40'')$;
corrected angle $= 0^\circ\ 15'\ 30'' - 1'\ 50'' = 0^\circ\ 13'\ 40''$.

Then, height $= \dfrac{4\cdot8 \times 6077\cdot5}{3\cdot1} \,.\, \tan 0^\circ\ 13'\ 40''$; or see footnote p. 115.

log 6077·5	3·783725	
log 3·1	·491362	
	3·292363	; this will be a constant for all heights measured on this particular chart.
log 4·8	·681241	
log tan 0° 13′ 40″	7·599375	
	1·572979	= 37·4 feet = A's height above H.W.S.

2. B in feet above A $= \dfrac{24\cdot8 \times 6077\cdot5}{3\cdot1} \,.\, \tan\ (2^\circ\ 27'\ 10'' + 2'\ 30''$ + curvature correction); see footnote p. 115.

curvature correction $= \dfrac{24\cdot8}{3\cdot1} \,.\, 25'' = +3'\ 20''$;[1]

corrected elevation $2^\circ\ 33'$;

$\log \dfrac{6077\cdot5}{3\cdot1}$	3·292363	(see above).
log tan 2° 33′	8·648704	
	1·941067	
log 24·8	·394452	
	log 2·335519	= 2165 feet above A.
	A is	37 feet above H.W.S.
	Therefore B is	2202 feet above H.W.S.

[1] When A is beyond O's visible horizon, the correction for curvature, including refraction, must be applied.

281. Finding the Height Independent of the Horizontal Distance.—If O is on the water-line he can find the height of

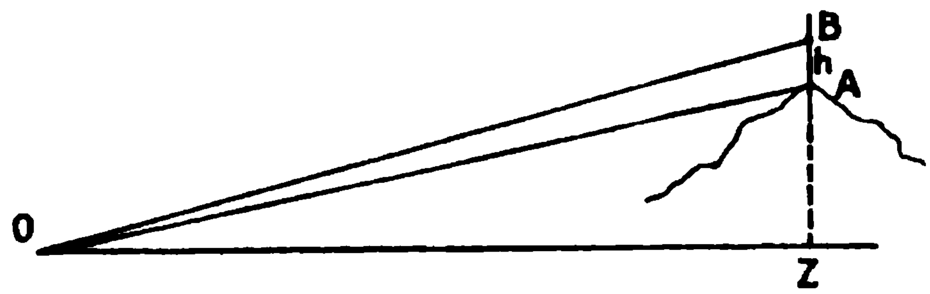


FIG. 92.

A Z (fig. 92) independently of the length O Z, if a staff A B of known length is held vertically at A. Let A B = h (10-ft. pole);

B O Z = elevation of B;
A O Z = elevation of A;
O A Z = 90° − A O Z,
B O A = B O Z − A O Z.

$$AZ = OA \cdot \sin AOZ \quad . \quad . \quad . \quad (1)$$

$$OA : AB : \sin OBA : \sin BOA \; (OBA = 90° - BOZ).$$

$$\therefore \quad OA = AB \cdot \sin OBA \cdot \operatorname{cosec} BOA \quad . \quad . \quad (2)$$

and since A Z = O A . sin A O Z,

then A Z = A B . cos B O Z (sin O B A) . cosec B O A . sin A O Z . . . . (3)

= h . cos elevation B . cosec diff. of elevations . sin elevn. A.

By a similar investigation, A Z can be found in fig. 93.

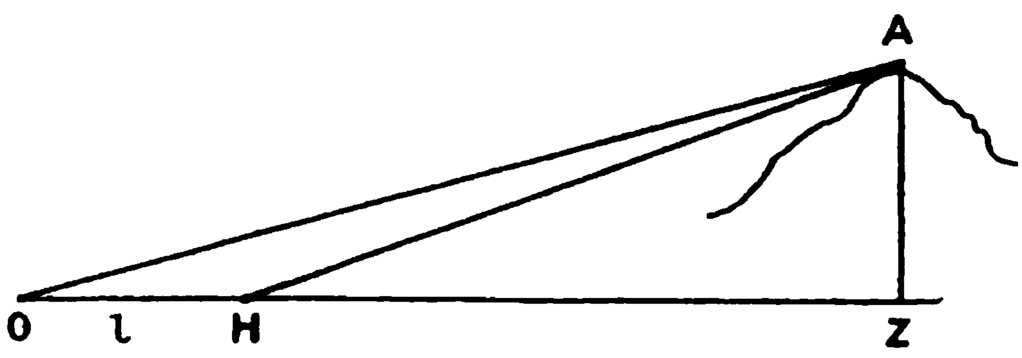


FIG. 93.

O is the first position, observing A O Z;
H is the second position, observing A H Z;
OH is a measured distance on the line OZ = l (see *Micrometer and Levelling Staff*, p. 127);
and A Z = O H . sin A H Z . sin A O Z cosec O A H;
= l sin elev. A from H . sin elev. from O . cosec diff. elev.
Now, referring to fig. 92.
If A B is placed at right angles to O A, then O A = A B . cot B O A, and for a given value of A B in tens of feet and of angles B O A, a table can be constructed giving the corresponding

distance of O A; if, for example, A B = 10 and B O H = 1°, then O A = 572·9 feet (10 . cot 1°). (See natural cotangent tables.)

282. 'Difference between Hypotenuse and Base.'—If the angle O A Z is observed, the readings on the *other side* of the vertical arc of the theodolite, give the 'difference between O A and O Z'; thus if O A Z = 26°, O A – O Z reads 10 (*per cent.*) (see *Readings on Vertical Arc of Theodolite*, '*Difference between Hypotenuse and Base*'), ∴ O Z = 515, *i.e.* 572·9 – 57·3 (10 per cent.); hence if O A is found from the first table, subtract the 'reading' per cent. = O Z; which is the distance of O from Z. Or, referring to explanation of theodolite telescope, par. 276, by means of the additional micrometer cross wires, and of a levelling staff, the distance O A is found without any calculation.

In the above example, the distance subtended by the cross wires would be 5·73 feet, ∴ A O = 573 feet.

The vertical arc reading alluded to above, being a percentage difference between the hypotenuse and base, if read off, then the horizontal distance O Z can be found; in the above case the reading was 10 (per cent.); and since the levelling staff read 5·72 feet, therefore hypotenuse – base = 573 feet.

The readings on a levelling staff, with the ordinary theodolite, being hardly discernible beyond 500 feet, the special purpose of the micrometer wires is only for chaining along an uneven surface. The difference of their readings gives the *percentage* difference between the length measured along an incline, represented by O A, for instance, and the horizontal distance O Z; but Z O A will, in practice, probably be much smaller than 26°, and the distance under 500 feet.

The same principles govern the action in all theodolites.

283. Everest or Transit Theodolites.—In the 'Everest' pattern, the telescope is *fixed* on pillars, and can only be moved in altitude, and the 'stride' level provides special means for levelling the horizontal wire of the telescope, hence the instrument may be used for the measurement of altitudes; it is graduated more closely than the ordinary theodolite on both the horizontal and vertical arcs, and can therefore be used for 'transit observations.'

Since the Everest theodolite can be used for taking angles in azimuth, also serving as a 'transit' instrument, and for measuring altitudes, it gradually rises to the dignity of a 'portable transit' and 'altazimuth' combined; but in the course of its development in 'observatories,' it has become a separate instrument from the 'transit,' and is known as an 'altazimuth.'

284. To 'Set up' a Theodolite.—Every theodolite has its own tripod legs. In H.M. Naval Service, for some at present inexplicable reason, the theodolites and legs are not interchangeable with each other, though they are so in other Departments.

285. Care of Tripod Legs.—It is therefore most important that the 'cap' of the tripod should not be lost, for, if the thread which it covers is in any way burred or damaged, the particular theodolite belonging to it is equally *hors de combat.*

286. Centre of Tripod Plumb the Spot.—The centre of the tripod legs must plumb the spot of observation (see *False Station*, p. 214, and p. 228), and there is a plummet supplied for that purpose, which is intended to be placed over the hook under the head of the tripod.

287. Angle of Legs.—At the time of 'setting up' the tripod, it is as well to have the legs separated at about equal angles; and the angle of spread should be as large as possible, consistent with observing; both for the safety of the theodolite, and for its immobility; always, of course, consistent with other requirements.

One more point necessary is, that the plate at the head of the tripod should be roughly horizontal when the tripod legs are set up.

288. To take the Theodolite out of the Box and 'Ship' it.—With the left-hand fingers under, seize the uppermost pillars which support the Y's.

With the right hand, grip the levelling plates; then lift both hands together, and turn the instrument upright. Convey the theodolite to the tripod, and, still holding securely with the left hand, screw the base on to the receptacle for it at the head of the tripod. Be quite sure it is screwed 'home,' then leave hold. Remove the telescope from the box, and, if required, change the eye-piece. Care should be taken in doing this, to keep a strong wind from breaking the 'wires' which will be exposed in the process. Raise the rings over the Y's, place the telescope, close the rings and pin them. There is a right and wrong 'end on' for the telescope.

289. Placing the Telescope.—The object-glass should be over the N. marked end of the compass bowl; or, what amounts to the same thing, the tangent screws for the horizontal arc and for the vertical arc should be on the observing side of the instrument; and the eye-piece of the telescope should also be on that side.

290. Focusing the 'Wires.'—The telescope being in its place, right way in and right way up according to the level attached to it, focus the cross wires by pulling forward the eye-piece until the wires are sharply defined. (See par. 271.)

291. To Level the Theodolite.—The levelling thumb-screws are between the levelling plates. Turn them all in succession, to the right or left according as they are left- or right-handed screws, so as to 'harden them up' 'hand taut' only.

If there are three or four levelling screws, according to the instrument, turn the horizontal arc round, until one of the levels is directly over one of the levelling screws.

Taking the case of the four screws, B, fig. 94.

(*a*) is levelled by screws 3 and 4 ; (*b*) is levelled by 1 and 2. If the bubble of the level is required to go up, or towards a certain screw, that side of the instrument would be screwed 'up' to raise it. In doing this, the opposite screw must be screwed 'back'; the simultaneous motion is either both inwards or both outwards, until the bubbles are in the centre of their 'runs.' After being told how to place the levels in relation to the screws,

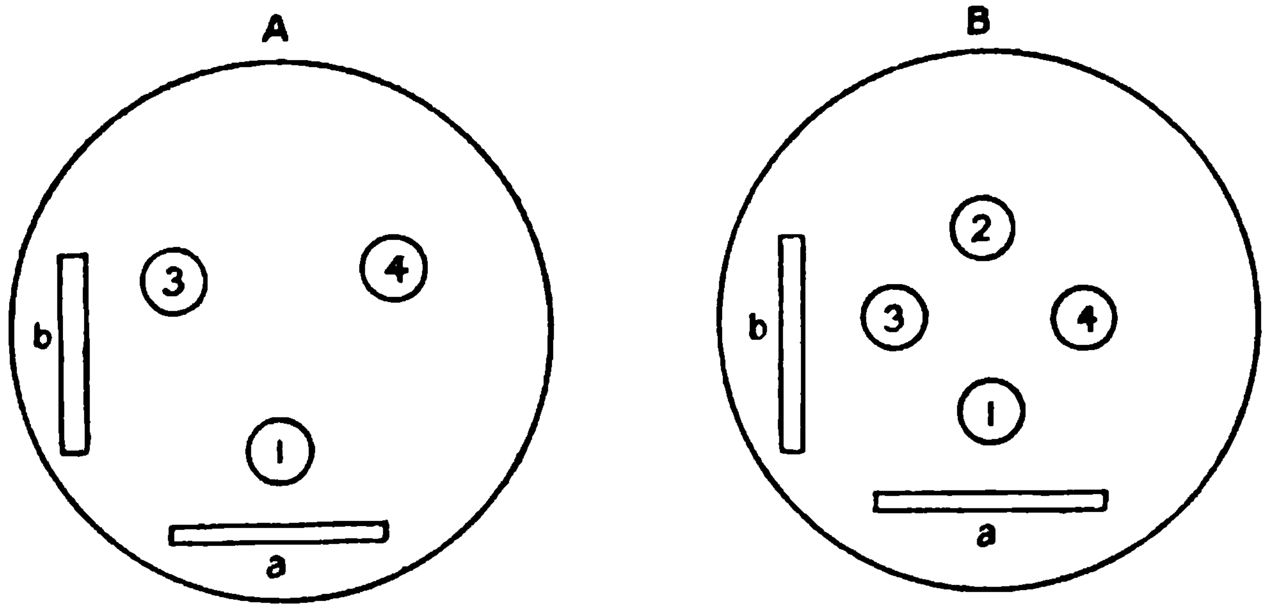


FIG. 94.

only ordinary common sense and practice will dictate how each level is adjusted in turn.

The instrument is now level, though not rigid. The axis of the horizontal arc, works on its base like a 'ball and socket'; it may be truly vertical, but the ball is not bearing up against the top of the socket. Therefore tighten up all the screws, still maintaining the bubble in the centre of the levels, until they are *reasonably* tight. If correctly carried out, the bubbles of the levels will remain in the centre of their runs while the graduated plate is turned a complete revolution.

292. Lower Levelling Plates should be nearly Horizontal.—Now, in this levelling process, the lower plate of the theodolite, which screws on parallel with the head of the tripod, is a base plate, on which the theodolite is levelled; and, if not approximately level to start with, the upper levelling plate may have to take up a considerable angle with it. Since the heads of the levelling screws are partly imbedded in one of the plates, the fact of raising one end of one plate considerably above that of the other, is liable to, and often enough does, bend or cripple or unevenly wear the threads of the screws. Hence the necessity of originally setting up the tripod legs with the top plate nearly horizontal.

In the theodolites fitted with three levelling screws, the process is precisely the same screws Nos. 3 and 4 work the level (*a*) that

lies parallel to the line joining them; while No. 1 screw is used in combination with Nos. 3 and 4 (see A, fig. 94), each level in turn being placed over one of the screws.

293. Placing the Telescope to make the Wires Horizontal and Perpendicular.—The next step is to adjust the telescope so as to make the wires horizontal and vertical (see par. 278).

Usually a zero reading of 0° 0′ serves most general purposes (see par. 269), therefore turn the index of the left vernier—that is, the one which will be on the left of the observing position—to the reading of 360° 0′, and clamp it to the arc. (See *Setting Vernier*, p. 7, par. 19.)

294. Setting the Zero Reading.—In some inferior instruments the act of clamping the index plate to the arc, may sensibly disturb the level of the index plate. If it is a perfect fit, and free from dirt, its plane should remain parallel with the horizontal arc. The error, when it arises, must be admitted as part of the error of observation in horizontal angles, though in the case of vertical angles it is eliminated (see par. 279). This particular clamping screw alluded to should not be screwed up too tightly, but only sufficiently to grip the arc: the motion of unscrewing it, if set up like a vice, is liable to disturb the level of the instrument.

295. Directing the Telescope on the Zero Object.—With the index plate and graduated plate clamped to each other, turn them both round, until the telescope points near to the zero object (see *Zeros*, p. 122, pars. 264, 265), clamp it there, and readjust the focus of the telescope for this object. Use the tangent screw, and place the centre of the cross wires exactly over the object (see *Errors of Tangent Screws*, p. 9, par. 23). The instrument will now be levelled, the centre of the cross wires over the zero object, while the reading by the left index will be 360° 0′, and by the right index 180° or thereabouts. Note this thus: At O ⊕ A – 360° 00′ 179° 59′.

296. Setting the Tangent Screws.—It may be hinted that it is very undesirable for the tangent screws of both the index plate and of the horizontal plate to be near the end of their 'runs'; for, if the mechanism of the instrument is looked into, it will be noticed that both the screws which hold the tangent screw to its guide plate, must necessarily have a certain amount of play for the change of movement and direction of the tangent screw; for the pivoting point of the tangent screw can only possibly be at a tangent to the arc at one spot, and the further from that pivot, the greater is the leverage required to displace it. There is bound to be a slight movement, which no amount of screwing up of the pivot will overcome: in fact, it is that very screwing up which makes the mischief worse.

This has been pointed out as a partial explanation for the

small 'play' in the instrument, and for the error when referring back to the zero, which is often perplexing.

The instrument is now in readiness for measuring horizontal angles.

297. Taking Horizontal Angles and Reference to Zero.—Unclamp the index plate, turn the telescope to each object in succession; if it is turned to the right, continue in that direction for all the objects observed; place the centre of the cross wires nearly over each, clamp at that position, use the tangent screw, and read off both verniers; finally, refer back to the zero object, to see whether any movement has taken place in the horizontal arc. (See par. 265.)

298. Observing with one Eye and Reading off with the other.—For those who use the right eye to observe with, it may be convenient to 'read off' with the left, but it is not so evident to the left-eyed observer that he should read off with the right. In both cases, however, there may be a rest to the observing eye.

299. Eye Tube for Sun Observations.—When an angle to the sun is required, there is usually a supplementary eye-piece with coloured shades, which is shipped when required for the purpose; through it the images are non-inverted.

300. True Bearings with Theodolite.—An angle between the zero object and the sun is required for the purpose of deducing the true bearing of the *object.* For, if the sun's true bearing can be calculated at any instant, and the angle between it and the object observed at that instant, then the true bearing of the object can be deduced.

It is not practicable to take an angle to the sun's centre; but if the vertical wire is made to touch either the right or left limb of the sun, at the centre of the cross wires, the semi-diameter applied to the reading will give the horizontal angle to the sun's centre.

301. Important that it is Level.—It is most important that the instrument be level (see par. 277). Supposing it to be so, there is still the error in the vertical wire; for the angle must be the tangent of the sun at the end of its horizontal diameter; and any single observation would contain this error, if the wire is not truly vertical.

302. Eliminating Error of Wire.—But if the wire has a small inclination,—in the process of handling the telescope while directing it to the sun such an error easily creeps in,—and an angle is taken to both limbs of the sun—in other words, the sun placed on each side of the wire—then the mean of the readings is the angle to the centre. As in the case of sun observations with a sextant, the observer waits for contact, the horizontal movement being set in advance, while with the tangent screw of

the vertical arc, the altitude is followed: in one case the sun is seen closing to the line, in the other separating from it.

303. Method and Order of Observing.—In theory the observation taken as in fig. 95 would suffice; but better results can be obtained if the suns are placed in *each* quadrant, when there need be no guessing as to the position of the centre in altitude, as arises in fig. 95; and there are, moreover, four observations for one result. The mean of the four, is the angle to the centre. Each observation is taken as shown in (*b*) fig. 96. In (*b*) (1) the left and lower limbs are first taken, in (2) the right and upper limbs.

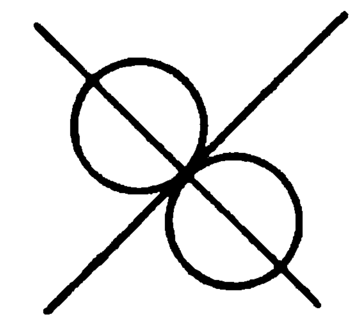


FIG. 95.

If the telescope inverts, the image, (1) will really be the sun's right limb and upper limb; the next, (2), would be the left and lower; and if the observation were taken A.M., the movement of the sun after the first angle, would be in the direction of the second—*i.e.* rising from upper to lower limb, and towards the meridian, from S.E. to S. in the example; but the direction of the change in azimuth would depend upon the latitude and declina-

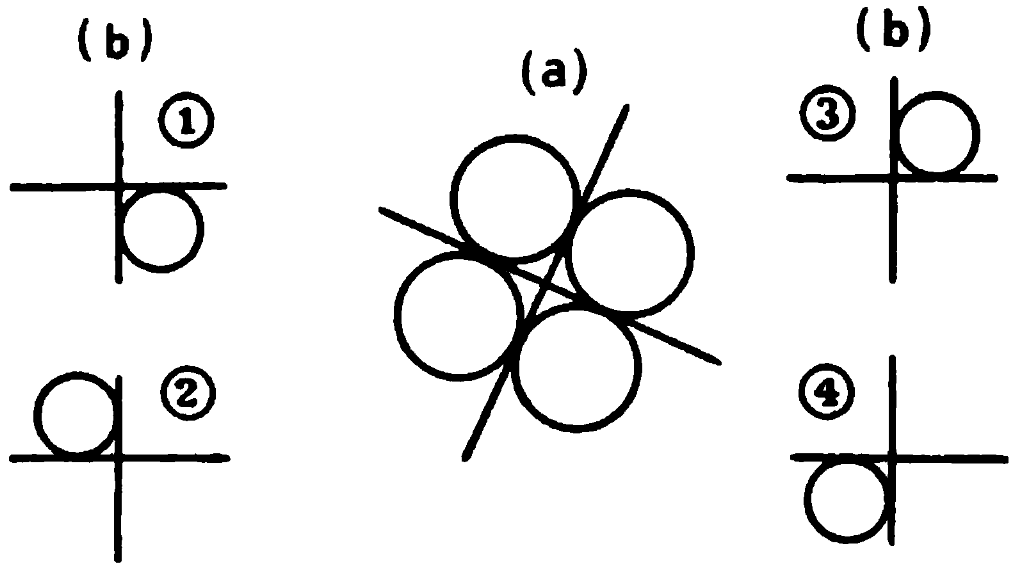


FIG. 96.

tion. Therefore the position of the first of any pair of sights is adapted to the geographical position of the observer, as well as, of course, whether A.M. or P.M., so that its movement shall be towards the next position required.

Example of finding the sun's true bearing by azimuth and from thence deducing the true bearing of the 'zero.'

The following details have been supplied by a pupil:—

(1) Set up the tripod for the theodolite at the given spot, and centre it correctly over the mark by means of the plummet. Set the cap of the tripod as nearly horizontal as possible by anything in the vicinity.

(2) Remove the theodolite from its box, holding it by the pillars with the left hand, and round the levelling plates with the right;

still retaining the hold with the left hand, screw the lower levelling plate on to the head of the tripod; make quite sure that it is screwed 'home.'

(3) 'Ship' the telescope with the object-glass over the N. end of the compass bowl, which is in the centre of the theodolite.

(4) Put in the securing pins.

(5) Turn the horizontal arc, so that one of the levels shall be over one of the levelling screws; and 'level up' the instrument; each level being taken in turn and levelled by the two levelling screws parallel with it.

(6) When level, tighten up all the levelling screws.

(7) Focus the eye-piece of the telescope for the 'wires'; then focus and turn the telescope so as to point to the side of a building, or to a plummet line, or to a distant object, or to the horizon line; and adjust it so as to make either the vertical wire perpendicular, or the other wire horizontal.

(8) Adjust and clamp the reading of the index by the left vernier so that it will read 0° 0′ 0″; this should be firmly clamped without being made too tight.

(9) Turn both plates round, and point the telescope to the zero object adopted, placing the centre of the cross wires over a well-defined spot on it; you may have to re-focus; clamp it there, and with the aid of the tangent screw, make the contact as exact as you can. If the short eye-piece has no coloured shades the longer eye-piece should be 'shipped.'

(10) Then release the upper plate, and move it round, using one hand on each side of it, towards the direction of the sun; then unclamp the vertical arc and move the telescope in altitude until, by both movements, the sun is visible in the field of the telescope. Clamp both the horizontal and vertical arcs.

(11) With the tangent screws of both move the cross wires of the telescope, so that the vertical wire will be in advance of the movement of the sun in bearing; re-focus the telescope. As soon as the sun appears in one of the quadrants, give notice to your assistant which quadrant you are going to begin at; that is, call 'right lower' or 'left lower' or 'left upper' or 'right upper'; and, following the movement in altitude by the tangent screw of the vertical arc, wait for the contact with the vertical wire, *i.e.* the movement in azimuth.

(12) When the sun's limbs are touching both wires, sing out 'stop.' The assistant takes the time with a watch, and writes it down opposite the quadrant stated; and you read off the angle.

Repeat this for the four quadrants; and in conclusion refer back to your zero, and say Z.O.K.

(13) Do not steady yourself by holding on to the legs of the tripod; and be careful, when changing the eye-piece, to readjust the telescope for the wires; and, when directing the telescope to

the sun, not to throw the instrument out of level, because in this observation it is very important that it shall remain quite level.

(14) The assistant compares the watch with the chronometer as soon as he is able.

(15) To return the theodolite to its box, remove the telescope, replace the short eye-piece, ease up all the clamping and levelling screws, take hold of it in the same way as when taking it out of the box, unscrew with the right hand, lift off, and coax it into the bed made for it.

Close the lid of the box gently, and if there is an unmistakable thud, open and try again.

(16) Replace the plummet, and screw on the top of the tripod.

June 5, 1908. $3^h\ 30^m$ P.M. Lat. 51° 29′. Long. 21″ E.

	h m s	° ′	
	3 27 15·6	327 43	
	3 30 16·8	329 06	
	3 31 26·8	329 23	
	3 33 13·6	329 05	
mean . . .	3 30 33·2	328 49	
comp. of watch .	6 17·		
chro. time . .	3 36 50·2	Dec. 22° 32′ 14″ N + 16·6	
chro. fast . .	7 09	+ 58	3·5
G.M.T. . . .	3 29 41·2	22 33 12	58·10
long. . . .	1·4		
mean time . .	3 29 42·6	Eq. T. + 1m 47s·5	− ·4
Eq. T. . . .	1 46·1	1·4	3·5
Hour-angle .	3 31 28·7	1 46·1	1·4

In Δ P Z X (fig. 97), PX = P. dist., P Z = co. lat., XPZ = hour-angle.

log hav. h.a.	$3^h\ 31^m\ 29^s$	9·297088	
log sin P Z	38° 31′ 00″	9·794308	
log sin P X	67° 26′ 48″	9·965440	
diff.	28° 55′ 48″	9·056836	hav. = 39° 27′ 46″
log vers.	39° 27′ 46″	0227961	
log vers.	28° 55′ 48″	0124785	
	log vers.	0352746 = 49° 39 57″ = ZX.	

In P Z X to find PZX.

log cosec PZ	38° 31′ 00″	10·205693
log cosec ZX	49 39 57	10·117880
	11 08 57	
PX	67 26 48	
log ½ hav.	78 35 45	4·801645
log ½ hav.	56 17 51	4·673721
hav.		9·798938 = 105° 00′ 03″.

Then the true bearing of ☉ (fig. 98) was N. 105° 00′ W.
The mean reading to ☉ by the theodolite was 328° 49′.

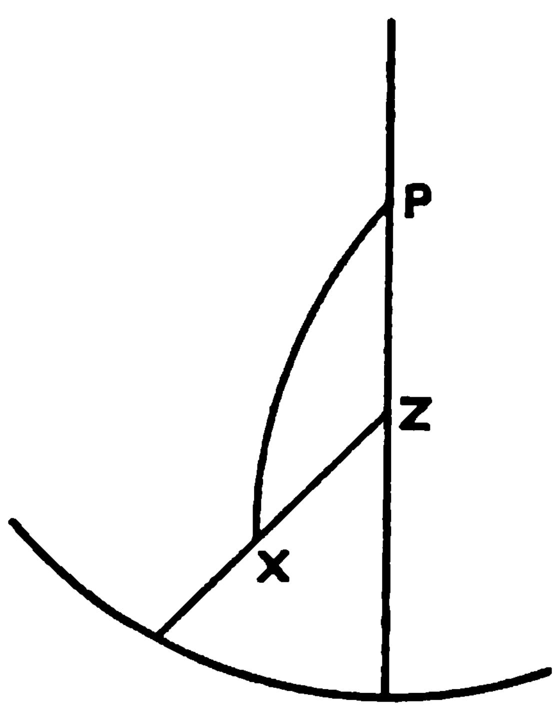


Fig. 97.

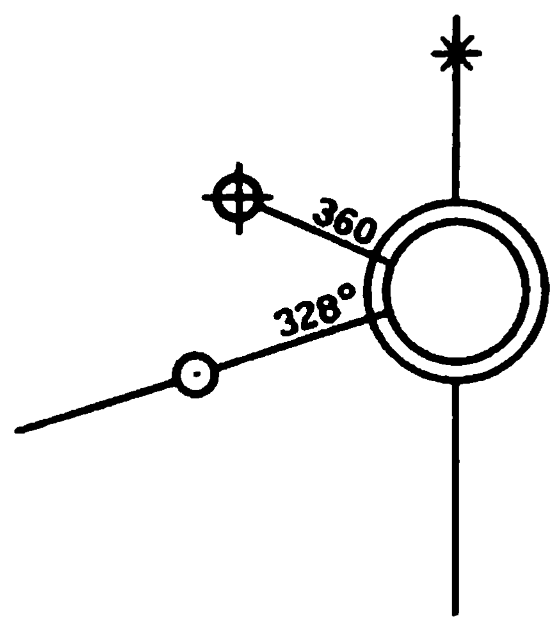


Fig. 98.

Therefore the 360° 00′ or ⌖ was 360° − 328° 49′ to the right of the sun's T.B. (See fig. 98.)

i.e. 31° 11′ to the right of N. 105° 00′ W.

31° 11′

N. 73° 49′ W. = T.B. of ⌖

304. Best Time to Observe.—It would appear, then, that if the telescope wire is vertical, the best time to take a true bearing would be when the sun's movement in azimuth is at right angles to the wire, the moment of contact being more sensible than if, for instance, the sun were rising almost parallel with the wire; and the actual error in the observation is certainly less in the first case. But the sun's movement is most rapid in azimuth when it is moving across the meridian, or the vertical line; hence, a small error in observation at that time, would make a very large difference in the sun's true bearing calculated from it; whereas,

when it is moving nearly vertically, a large error of observation will make the minimum error in azimuth.

Moreover, since the sun's altitude is at its maximum at the meridian, the error of the level of the instrument will then produce its maximum error.

Therefore, as regards the sun's position for taking true bearings its change in azimuth should be a minimum, in order to eliminate the error in observation or in the hour angle; and its altitude a minimum, on account of error in the level of the theodolite.

305. True Bearing of Sun obtained by Altitude.—If the sun's true bearing is calculated from its altitude, then another observer is necessary to observe the altitude; and since an error of altitude makes least error in azimuth when the sun is in the prime vertical, when the sun is on a bearing of east or west is the best time to observe; and finally, if it is desired to eliminate errors of observation in altitude, then both, theodolite angles in azimuth, and sextant angles for altitudes, should be taken on each side of the meridian. (See *True Bearing by Sextant Angles*, p. 102; note, p. 106, and par. 226.)

The position of the zero in reference to the sun's position is investigated on p. 99, and applies specially here for reason there given.

306. True North as a Zero.—The true bearing of the zero object having been determined, its angle from north can be used as a reading of the zero for the true bearings of any other object.

Supposing at O, the true bearing of A is found to be S. 34° 28′ E., that is, N. 145° 32′ E.

If now the index is set on the arc at 145° 32′, and the lower plate adjusted with the cross wires over A, then the reading of 360° 0′ on the arc will be the direction of true north; and all the consequent readings taken with ⊕ A 145° 32′ will give the true bearings of each object observed, all being from true north. Thus, if B reads 215° 17′, that is its angle from north; in other words, north is now zero, 360° 00′.

If C reads 17° 50′, its bearing is N. 17° 50′ E., and so on.

307. Triangulating by Bearings.—In this form of taking angles and bearings they are usually plotted by ordinates, their value or length being laid or 'set off' along both the north and south line, and the east and west line, instead of the direct method by chords.

This form of 'sets off,' however, is more adapted to land surveying in the sub-measurements of areas and allotments.

308. Theodolite Compass. — But here the connection suggests itself between the position of the telescope and the north and south line of the compass bowl; for if the telescope is on that line, then the north end of the bowl is carried round with the line of the telescope, and the angle between the magnetic north end

of the compass needle, and the position of the marked point of north, is evidently the magnetic bearing of the object. The compass bowl is graduated through 360°.

By these means then, may be obtained both the true and *magnetic* bearing of any object; the difference between the two being the variation; but the graduations on the compass bowl are only marked to degrees, and the accuracy of reading will not be reliable within half a degree.

VARIATION.

309. **Variation.**—The variation will always be required to something nearer than half a degree, and a larger compass than that attached to a theodolite is necessary in order to obtain graduations of smaller values.

The landing compass (see p. 117) serves the purpose admirably; for the compass card is graduated to 20′ of arc, and by interpolation an approximate reading of 5′ can be obtained; any compass, however, will do.

If the true bearings of a number of points are obtained first, through angles taken with the theodolite, and, following this, the bearings of the same points are taken with a compass placed over the exact spot that the theodolite occupied, then the difference between the true and compass bearing is the variation. (See *Example*, p. 147.)

310. **Where not to Observe from.**—Of course, the place of observation for variation should be free from local disturbance; that above ground is evident; but the chemical properties of the soil are usually undetermined, and it is hard to say when one is quite free from the influence of a neighbouring mass of magnetic mineral.

It may be necessary to remind an observer that keys, knives, steel cigarette cases, and steel-cased watches are local disturbances, and, if it is wise to do so on account of the safety of the articles, should be placed on one side, or else be strung up under the centre of the compass, where they will do the least possible harm.

311. **Objects to Observe for Variation.**—In selecting the objects for the purposes of finding variation, it must be borne in mind, that the compass verge has not the same efficiency in its line of sight as the telescope; it is, in fact, merely a thick-looking thread, held vertical by a frame more or less correct, viewed at 9 or 10 inches off through a slit.

Therefore objects suitably near to distinguish them properly are more adaptable for the compass: again, objects of different elevations would not be suitable to the compass, because of the rough arrangement for the vertical plane of the foresight; they

should be all on about the same level as the top of the compass, and just visible above it.

312. To Eliminate Centring Error of Card. — Then again, there is the existence of centring error in the compass card, of the same nature as in the sextant and theodolite, and probably an exaggerated form of it, as well as an error in the graduations of the compass card; the only way to eliminate both these errors is to take the bearing of a number of surrounding objects, and on nearly opposite points of the compass; eight bearings, approximately 45° apart, should give good results.

Error in Level.—Again, if the line of sight, *i.e.* the verge, does not work in a horizontal plane parallel to that of the compass card, there is still another error in the readings, which cannot be eradicated, unless the error of the inclination is a constant throughout the observations: but, caused by handling, it is in fact irregular, and comes under the head of an error of observation in the bearing, the total of which may be $\pm 10'$.

It is found the most convenient, in regulating the height and distance of the objects, as well as their positions in bearing, if a number of temporary staves are erected, surrounding the compass at a distance of a few hundred yards; the nearer they are to the observer, the greater the importance of the compass being plumbed over the exact spot that the theodolite occupied.

312*a*. To Observe for Variation.—With, first, the theodolite, the true bearing of a distant object is found (see *Example*, pp. 141 and 147); then from the angles observed between the object and the temporary marks, their true bearings are deduced. If the T.B. of $\oplus$ is S. 34° 30′ E. and the reading by theodolite to (*a*) is 62° 10′, then T.B. of *a* is S. 27° 40′ W.: or, as before described (par. 306), the zero reading may be set at 34° 30′, and the reading to *a*, *b*, and *c* will be their true bearing. (See *Example*, p. 147.)

The compass is then placed over the same spot, and the magnetic bearing of each object observed.

The difference between the true bearing and the magnetic bearing of each staff gives the variation; there being eight observations, if the staves are erected at about 45° apart.

The mean of these gives the resulting variation (see *Example*, p. 147), free from centring error.

313. Variations without using the Theodolite.—If a compass is fitted with the *graduated* verge, variation may be found with it, either wholly or partly, without the assistance of the theodolite, and without the assistance of the supplementary marks.

For, first set it up *as a theodolite*, pointing to the zero, 360° 00′ on the left vernier, and the reading, whatever it is, on the right; the bowl is now a fixture. The line of sight can then

be turned to the sun; and, by means of the ebonite reflector attached to the foresight, an angle, as indicated on the graduated verge, is obtained between the zero object and the sun's limbs, each limb taken in turn—and the time taken by watch. Just as on a theodolite, the angle is read off on the graduated 'verge' or arc, on both verniers, with the aid of a reading glass or microscope, and again, as usual, referred back to the zero. (See *Example*, p. 148.)

314. Results not so Accurate as Theodolite.—For obvious reasons, this observation will not be so accurate as that obtained with a theodolite; the means of observing are not so efficient. The conditions for the best time to observe, and positions of the sun and object—*i.e.* as to the position of the sun in altitude, in bearing, and in inclination relative to the zero object—are identical with these when taken with a theodolite or a sextant (see par. 304).

315. To Calculate and to Observe.—From the recorded time of the observation, first the true bearing of the sun is calculated, and hence the true bearing of the zero object is deduced; but these calculations are not required until all the operations for finding the variation are concluded.

The graduated arc is now set at 360° 00′, the line of sight is pointing to the zero object, and the bowl clamped; without a tangent screw, this is a rough operation.

Now, move the line of sight round to the right, until the reading on the verge, by the left index, is exactly 60° 0′, and read off the right vernier. The line of sight is now pointing to no particular object, but the *direction* is, the mean of the left and right readings, to the right of the zero. (See *Example*, p. 148.)

If, then, the true bearing of the zero is known; then the true bearing of the direction of the line of sight placed at any angle can be deduced: in this case the angle was about 60° 0′, and if, as follows, the compass bearing is taken of that direction, indicated by the sight vane, then the difference between the true and the magnetic bearing gives the variation. Practically this is correcting the centring error of the card by the graduated arc: when using the theodolite, that instrument was employed for this purpose (see par. 312).

Continue the same process at 120°, at 180°, 240°, 300°, and finally refer back to the zero, and take its compass bearing also. This produces (see *Example*, p. 148) six observations for variation: centring error will be eliminated, both of the verge, by its two readings, and of the compass card; but the inconstant position of the plane of the verge (see par. 251) remains an unknown error.

316. Example to find Variation with the Assistance of Theodolite—Azimuth calculated from 'Time.'—To find the variation, with the aid of a theodolite, without special staves, and through the azimuth *calculated from 'time.'*

Continuing the example from p. 142.

The T.B. there deduced of ⊙ was N. 73° 49′ W.

The index on the upper plate of the theodolite was now set to 286° 11′ (*i.e.* 360° − 73° 49′), and both plates turned so that the telescope was over the ⊕ object.

Unclamp the upper plate, and observe the readings to the following objects :—

		T.B.	C.B. (observed).	Variation.
	° ′	° ′	° ′	′ ″
At J ⊕	286 11	N. 73 49 W.	N. 56 40 W.	17 09 W.
dome	342 56	17 04 W.	N. 0 10 E.	17 04 W.
rail	45 06	N. 45 06 E.	N. 62 26 E.	17 20 W.
Spot (1)	107 42	N. 107 42 E.	S. 54 50 E.	17 18 W.
Spot (2)	162 34	162 34 E.	S. 0 20 E.	17 06 W.
Spot (3)	224 20	S. 44 20 W.	S. 61 30 E.	17 10 W.
		Mean variation	. .	17 13 W.

The T.B. (true bearings) are the theodolite readings converted into angles from north and south (see par. 306).

The theodolite was removed, the compass being substituted in its place, over exactly the same spot, and the C.B. (compass bearings) read off from the compass card.

The variation column is the difference of the true and compass bearings; and the mean, without applying the error of the card, was 17′ 13″ W.

317. To find Variation with Compass only—and Sextant for T.B. by Altitude Azimuth.—*Example.*—To find the variation using the 'landing compass' only; the sun's altitude being taken with a sextant, in an artificial horizon, and the sun's T.B. derived therefrom.

Set up the compass over the observation spot with the plummet; the top of the tripod should be roughly horizontal.

Set the left index of the graduated arc at 360° 00′, then turn bowl and all round, pointing the line of sight to the zero object; sight it through the slit in line with the fore wire, and clamp there.

Then move the upper plate round to the sun; be careful not to disturb the level of the bowl more than is possible.

Raise the black ebonite reflector that is attached to the fore sight, just so as to catch the image of the sun, when looking through the hind slit; raise it sufficiently to allow for advancing movement in altitude; put up any necessary coloured shade.

Then 'advance' the reading on the graduated verge, allowing for the movement of the sun in bearing, and read off.

Wait till the limb of the sun, seen in the reflector, touches the vertical wire of the fore sight, then 'stop.'

Do not touch the reading; wait till the other limb is in contact, then 'stop' again.

In the interval which elapses between the contact of the sun's limbs, take the sun's altitude in the artificial horizon, as also the time.

	Left Index.	Right Index.	T.B. (calculated).	C.B. (observed).	Variation.
Thus: at J ⊕ R	360° 00′	180° 05′	S. 12° 24′ E.	S. 4° 50′ W.	17′ 14″ W.
$3^h\ 27^m\ 10^s$ ◯|	87 00	267 15			
$3^h\ 31^m\ 26^s$ |◯	87 00	267 15			
$3^h\ 29^m\ 10^s$ alt. ⊙	80 40		T.B. (derived).		
These are not definite objects, but letters only, distinguishing each set angle. *a*	60 00	240 10	S. 47 38 W.	S. 64 50 W.	17 12 W.
b	120 00	300 12	N. 72 19 W.	N. 55 00 W.	17 19 W.
c	180 00	00 07	N. 12 23 W.	N. 4 40 E.	17 03 W.
d	240 00	59 50	N. 47 28 E.	N. 64 50 E.	17 23 W.
e	300 00	120 00	S. 72 21 E.	S. 55 10 E.	17 11 W.
	Z.O.K.		Mean variation .	.	17 13½ W.

	h m s		° ′ ″	
Mean of the times by watch	3 29 18	Dec.	22 32 14 N.	16·6 +
Error of watch fast . .	0 1 0	change +	0 58	3·5
	3 28 0		22 33 12	58·1
Long.	0 1 0 W.		90 0 0	
G.M.T.	3 29 0	P.D.	67 26 48	

Obs. alt.	80° 40′ 0″	log sec lat.	51° 29′	10·205691
I.E.	0 5 30	log sec alt.	40 32	10·119170
	2|80 34 30		10 57	
	40 17 15	P.D.	67 27	
R − P	0 1 02	log ½ hav.	78 24	4·800737
	40 16 13	log ½ hav.	56 30	4·675155
S.D. +	0 15 49		hav.	9·800753 N. 105° 19′ W.
Tr. alt.	40 32 02		T.B. ⊙	N. 105° 19′ W.

The angle between $\oplus$ and $\odot$ at
$3^h\ 29^m\ 18^s$ (mean of O| and |O times)
by left vernier is . . 87° 00′
Do. by right vernier is . . 87° 10′

Mean reading to $\odot$ . . 87·05 — T.B. $\odot$ N.105° 19′ W.
87 05

N. 192 24 W. (see fig. 99)
calculated T.B. $\oplus$ S. 12 24 E.

This bearing is transferred to the third column.

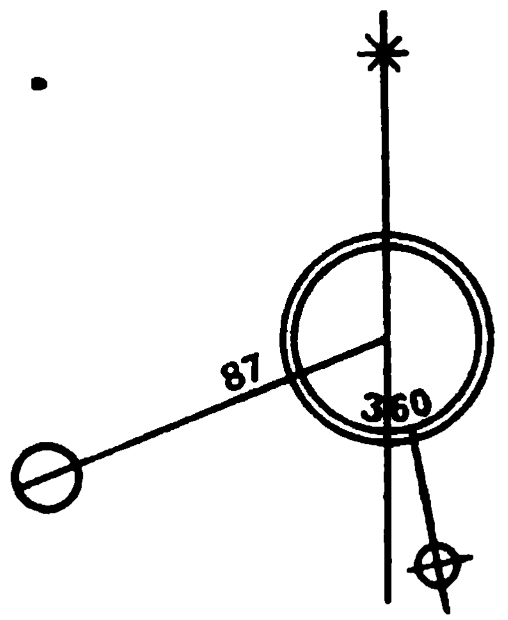


FIG. 99.

After this has been found, the columns above are filled in as shown. Since the left index reads 60° to a, and by the right index it is the difference between 180° 05′ and 240° 10′ = 60° 05′ (this disagreement with the left index being due to centring error), accept the mean of the two as the correct angle to a = 60° 02′.

Apply 60° 02′ to the right of S. 12° 24′ E. is S. 47° 38′ W.; in a similar way can the T.B. of b, c, d, and e be deduced and entered in the column; and, as before, the difference between the true and compass bearings is the variation, apart from the error of the card.

THEODOLITE ADJUSTMENT.

318. Two lines at right angles to each other, drawn on a sheet of paper, which is pinned on a wall, so that the vertical line is on the same line as the plumb, will constitute a rough collimator. This can be set up on a wall.

1. Correct for collimation.

319. Adjustment for Collimation.—The theodolite is set up roughly levelled, and the telescope is set with its centre pointing to the centre of the collimator, which, of course, will have

to be adjusted in height. The telescope is then turned on its longitudinal axis, and if the centre of the cross wires deviates from the centre of the collimator, the wires require adjusting.

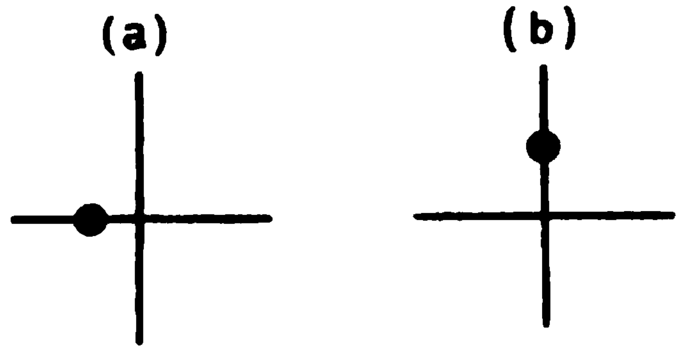


FIG. 100.

Turn the telescope so that the centre of the cross wires, if in error, is brought on to the horizontal line of the collimator (that is, turn it a quadrant; see fig. 100). What error there is is removed, half by the horizontal arc tangent screw of the theodolite, and half by the screws on the diaphragm in the line of the error (see fig. 100, *a*). Next, turn the telescope a further quadrant, and what error there is, is removed, one-half by the vertical arc tangent screw, and half by the diaphragm screws (see fig. 100, *b*).

2. The next adjustment is for 'roll.'

320. Adjustment for 'Roll.'—The telescope is levelled, and 'rolled' through an equal angle of about 45°, and if the bubble does not remain in its place, the adjustment is made to the *side* screws attached to the level—practically, this is to get the level in the same vertical plane as the axis of the telescope.

The error is of the description shown in fig. 101 (*a*). L is

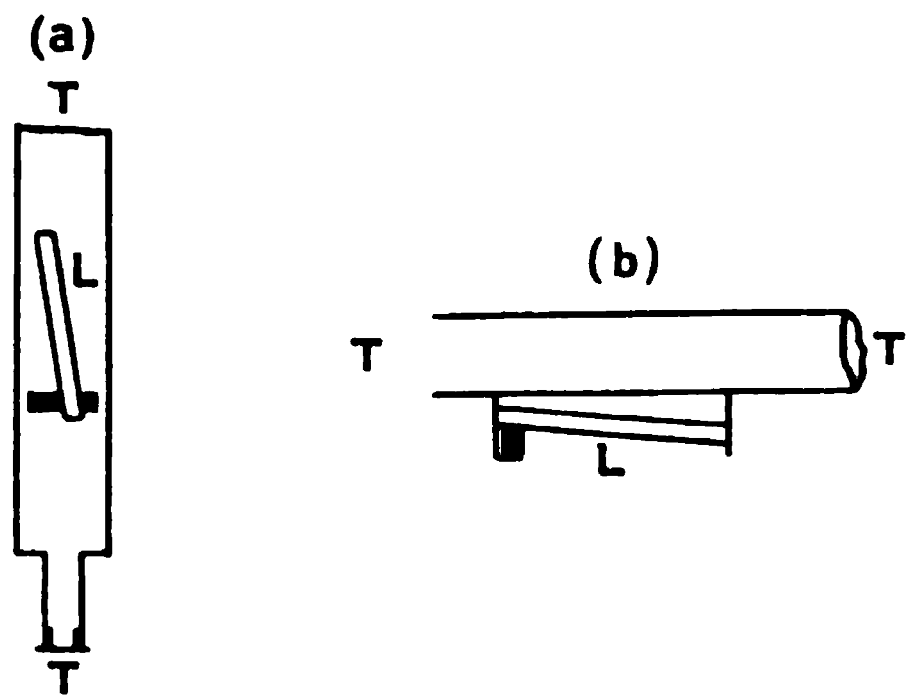


FIG. 101.

required to be parallel to T T. The adjusting screws are shown by large 'dots' which are at one end of the level only.

3. Adjustment for reversion.

321. Adjusting for 'Reversion.'—Level the theodolite, place the long axis of the telescope over two levelling screws, and level it by the level attached, half by these screws and half by the vertical arc screws. Reverse the telescope end for end; if the bubble does not remain central, then the level is not parallel with

the axis of the telescope. Half the error is removed by the vertical arc tangent screw, the other half by the adjusting screws of the level.

The error is as shown in fig. 101 (*b*).

The adjusting screw is shown by large dot; it is at one end only of the level.

4. To adjust the index plate levels.

322. Adjusting Horizontal Plate Levels.—The instrument being roughly levelled, then *unclamp the index from the graduated plate*, and *level it by means of the level on the telescope.*

Place the long axis of the telescope over a pair of the levelling screws, and in this position, bring the bubble of the *telescope level* to its centre, equally, by the levelling screws and the tangent screw of the vertical arc; turn it through 180°, and do the same there; also at 90°, and at 270°. The index plate will now be level, and the *internal* axis vertical. If the bubbles of the levels on the index plate are not now in the centre of their runs, they must be placed there, by adjusting each level; screw up one end, and back the other by an equal amount.

4 (*a*) **323. Testing the Parallelism of the Plates.**—Now direct the cross wires of the telescope to the centre of the collimator; keeping the telescope directed to this centre, turn the *graduated plate* round. If the centre of the cross wires is disturbed, up or down, then the index and graduated plates are not parallel; or, which is the same thing, the internal and external axes are inaccurate, and their common centre is not identical. Only a maker can rectify this. In the absence of a home-made collimator, up to a certain stage, a plummet line would serve almost as well; but for this last test a collimator is necessary.

4 (*b*) **324. Testing the Vertical Plane.**—The index plate being level, the vertical movement of the telescope should be in a true vertical plane. Turn the telescope so that its vertical wire is truly vertical; this will have to be done with a plumb-line. Then, using the same plumb-line, move the vertical arc so that the wire of the telescope travels up and down it. If correct, the wire will remain on the line. In the ordinary theodolite there is no adjustment for the error if it exists; but in some of them, and in transit instruments, there is a means of moving the lateral axis of the telescope.

If the error exists, and is unadjustable, the resulting error is the same as if the whole instrument were out of level; and the cause of the error is probably the same that produced error 4*a* as above described—that is, being roughly handled when removed from or replaced in the box.

CHAPTER XVIII.

THE CHAIN; THE TEN-FOOT POLE AND LEVELLING STAFF.

325. Pole held Vertically.—If a pole T B, in fig. 102, of known length, is held vertically at B, and the angle it subtends, T O B, is measured with a sextant (see *Parallax*, par. 5) or with a theodolite, then B O, the distance from the base of the pole to O, the observer, = T B . cot T O B, T B O being a right angle.

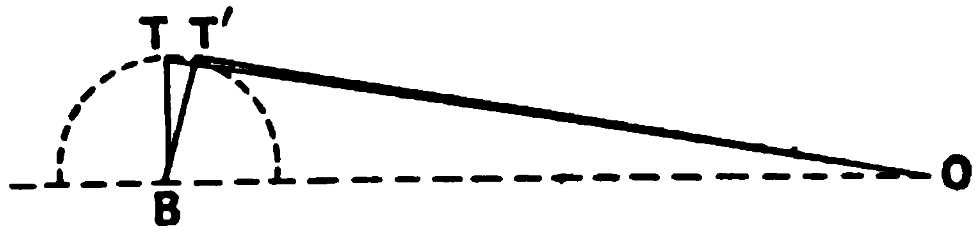


FIG. 102.

This is conditional on B T being truly vertical, and this can only be obtained accurately with a plummet. If an arc of a circle be described with centre B and a radius B T, then the maximum angle subtended by B T or B T′ is when O T′ is a tangent to B T′, and B T′ O is the right angle; then B O = B T′. cosec B O T′. The position B T′ can be obtained by moving the pole in a vertical circular movement, pivoted at B, B O T′ being the maximum angle subtended by B T.

When B O T is small there is no appreciable difference between the cosec. and the cot; and if B T is also small, then there is practically no difference between B T cot B O T and B T′. cosec B O T′.

If B T is 10 feet, ensuing calculation is simplified.

Example.—B O T is 1°; B T = 10 feet, required B O. The natural cotangent and cosec of 1° is 57·29. Multiply by 10, makes O B = 572·9.

326. Distance by 10-foot Pole. Pole held Horizontally.—Hence the natural cotangent table (see *Inman's*

Tables) is practically a distance table for a 10-foot pole, by merely shifting the decimal point.

In a similar way, if the pole is held horizontally at B (fig. 103), the angle measured at O is not the maximum angle; but

$$O\,A = A\,T \cot T\,O\,A$$
$$= \frac{T\,B}{2} \cot \frac{T\,O\,B}{2}$$

If $T\,O\,B$ is small, not greater than $2°$, then $\cot \frac{T\,O\,B}{2} = 2 \cot T\,O\,B$; therefore $O\,A = \frac{T\,B}{2} . 2 \cot T\,O\,B = T\,B \cot T\,O\,B$.

If the maximum angle $T'\,O\,B'$ is observed, obtained by the action of swinging $B\,T$ horizontally, then, by the same analysis as before, $B\,A = B\,T \operatorname{cosec} T\,O\,B$; and, using the same argument, if both $B\,T$ and angle $T\,O\,B$ are small, there is no appreciable difference between $B\,T \cot T\,O\,B$ and $B'\,T \operatorname{cosec} T'\,O\,B''$.

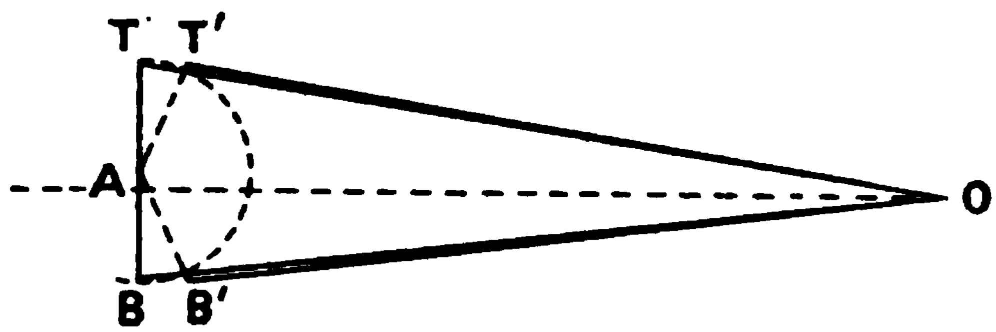


FIG. 103.

327. Errors in the Distance Obtained by 10-foot Pole.—The same method is then adopted to obtain O A when the pole is held horizontally to that when held vertically. It must not be forgotten that both the cotangent and cosecant increase very rapidly when the angles are small, and therefore a small error in observation in small angles will make a very great difference to the distance O A or O B; an erroneous position of B T, *i.e.* if B T is not nearly perpendicular to O A, will produce an error of observation; and when it is swung to and fro, by which the maximum angle is obtained, the corresponding distance will be slightly too short (see fig. 103); but as the angle subtended gets smaller, so also will *this source of error* be reduced; the correctness of the result will therefore depend almost entirely upon the observation. In practice, the actual observation is not easy to make without error, when using a sextant, and, moreover, there is the parallax of the sextant (see p. 3); it cannot therefore be said to be definitely accurate.

328. Greater Accuracy when Angles Measured by Theodolite.—With a theodolite, however, the results can be very satisfactory; knowing this, there will be a time and place where such extra accuracy is desired. It will serve, for instance,

to measure the distance from high- to low-water line (see *Example*, p. 260), or to put in the coast-line on parts where other means fail, such as creeks; or to measure the width of a river in parts, or the distance of a buoy from a certain point could be measured; and see p. 238.

329. Ten-foot Pole Construction.—The 10-foot pole is essentially the implement of a hydrographic surveyor, and is *generally* coupled with a sextant; it is simply constructed. Take any pole a little more than 10 feet in length, and nail at each end a cross piece with a white streak on a black ground, or a black streak on a white ground, so that the distance from centre to centre of the streaks is exactly 10 feet: of course, the 10-foot pole is only the means of obtaining a distance; its relative position to the observer in bearing is obtained by a small prismatic compass, or, if possible, from a sextant angle to or from another *fixed* object.

THE LEVELLING STAFF.

330. The most elaborate form of levelling staff consists of three batons of hard wood, so constructed that two of them telescope into the third, or up and down each other. Each part is marked in feet, tenths and hundredths; and when the three parts are pulled out end on to each other, form a continuous reading from the 0 up to 15 feet; any staff, however, marked in feet and inches will serve. It is always used in combination with a theodolite, or with a level, and therefore more adapted to engineers and land surveyors than generally on hydrographic work. Its range of action with most theodolite telescopes is limited to about 500 feet; beyond that the marks on it cannot be read off distinctly. Its primary purpose is for measuring the difference of level.

331. To Obtain the Difference of Level.—Let a theodolite be 'set up' at A, fig. 104, and the levelling staff held at X, at

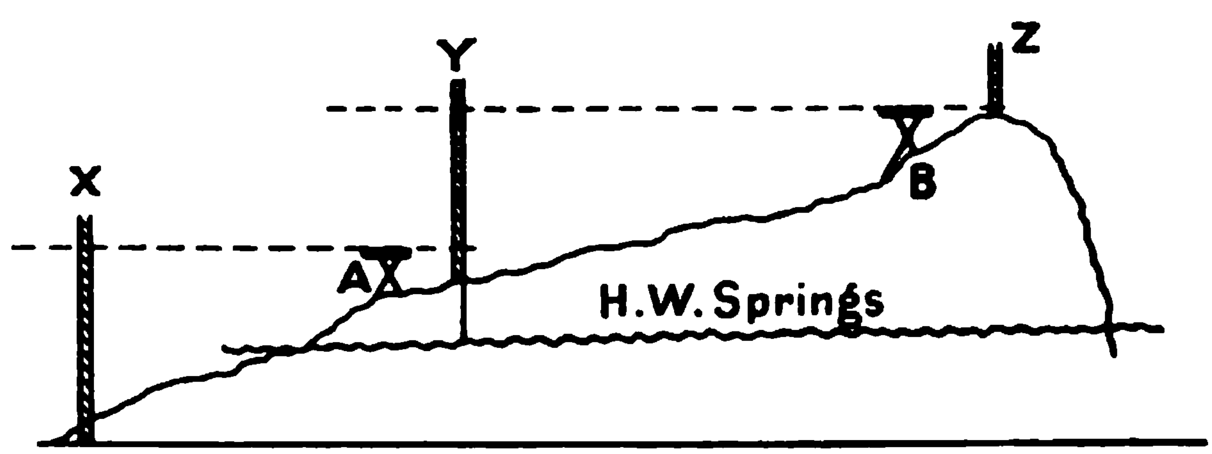


Fig. 104.

H.W. line level, on the same level or close to A; then the height of A's eye above the base of X can be read off X. Suppose

when the telescope is level the reading on the staff is 5·36 feet; in taking the readings on the levelling staff two observations are made—one with the telescope levelled the ordinary way of use, the other with the telescope levelled, but reversed end for end; by this means, the error in the adjustment in the level attached to the telescope is eliminated.

If the telescope is turned to the same levelling staff, now advanced to Y, and the reading there is 2·75 feet, then the difference of level between the foot of the theodolite at A, or the foot of the staff at X, and the base of the staff at Y due to the incline is 5·36 − 2·75 = 2·61 feet; or it is the difference between the 'back' reading and the 'fore' reading. Suppose the theodolite moved to B and the 'back' reading on Y staff to be 14·92 feet, and again the 'fore' reading on the staff at Y to be 1·3 feet, then the difference of level between the base of the staff at X and the top of the rise at Z = 'back' reading at X (5·36) − 'fore' reading at Y (2·61) + 'back' reading at Y (14·92) − 'fore' reading at Z (1·3), and so on, for any number of 'back' and 'fore' readings; therefore, the difference of height = sum of 'back' readings (20·28) − sum of 'fore' readings (3·91) = 16·37 feet. If the 'back' readings are greater than the 'fore' the work is being done going uphill, and, conversely, if the 'fore' readings are the greater, the observer is working downhill. By this means, an accurate difference of level can be obtained. The positions A, B, etc., need not be directly up the slope; they may zigzag, or be placed where practicable.

332. Finding a Horizontal Distance by Levelling Staff.—When a *horizontal* distance between any two points is required, the levelling staff again comes into use; for by it can be ascertained the distance, up or down the slope, between the observer and the staff; and by the graduations on the 'off' side of the vertical arc of the theodolite, the *difference per cent. between the measured and the horizontal distances is directly read off* (see *Theodolite*, p. 134, par. 282); by means of this reading the horizontal distance is deduced. If the upper and lower horizontal 'wires' (micrometer wires) of the telescope are used for measuring the distance, then, in fig. 105, let A to X be 5·36 feet, the levelling staff being close to A; the centre wire of the telescope is made to point to the 5·36-foot mark on the staff at S. Then the upper wire will point to U, and the lower to L, and the difference of U and L is the distance per cent. between the base of X and the base of S. Suppose this difference to be 1·36 feet, then the distance between X and S is 136 feet along the slope.

As the telescope is elevated to make it read at S the same as at X, the difference per cent., on the 'off side' of the vertical arc is read off; suppose this to be 2; then the difference between X to S (hypotenuse) and A to Y (base) is $136 \cdot \frac{2}{100} = 2{\cdot}72$ feet; and

since X to S = 136 feet, then AY = 136 − 2·72 = 133·28. Here, then, it will be noticed no direct measurement is made. Failing a levelling staff, the 10-foot pole could have been used at S, and by the angle subtended by its length, the distance X to S obtained. But the elevation of S is still required to reduce the distance to a horizontal one.

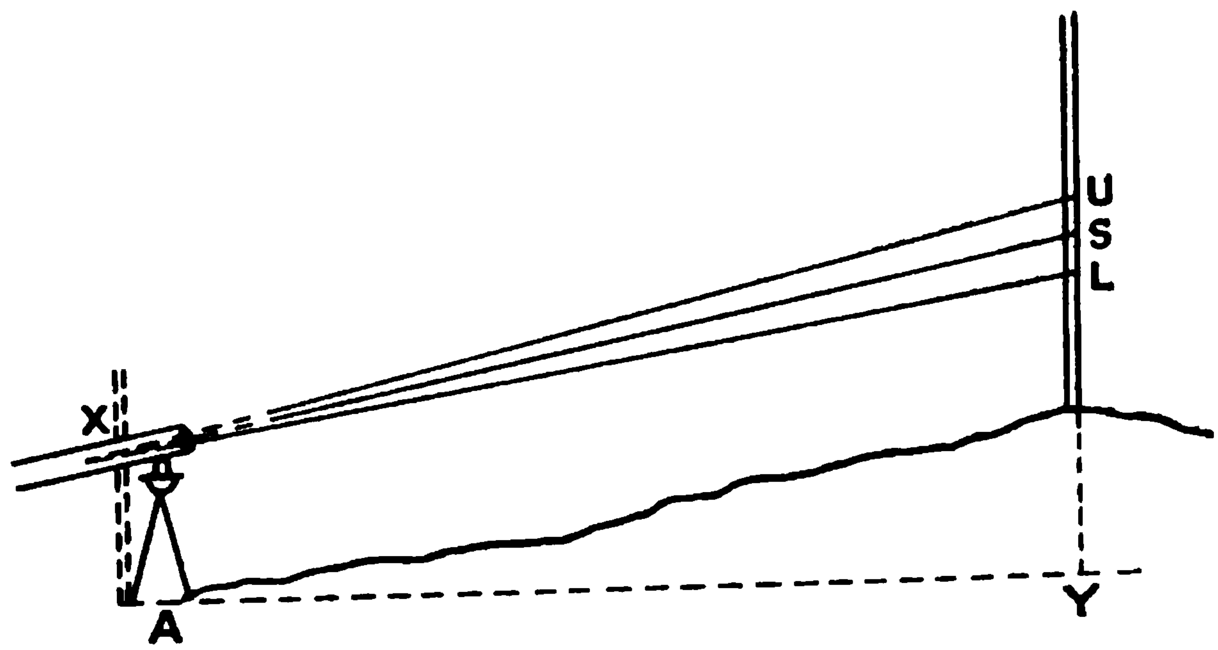


FIG. 105.

333. Example of Levelling Bench Mark to Tide Pole.— The levelling staff is also used for referring the level of the 'bench' mark to a reading on the tide pole. The 'bench' mark is a mark made on a permanent structure, with reference to the ordinary height of high- or of low-water springs (see p. 277,

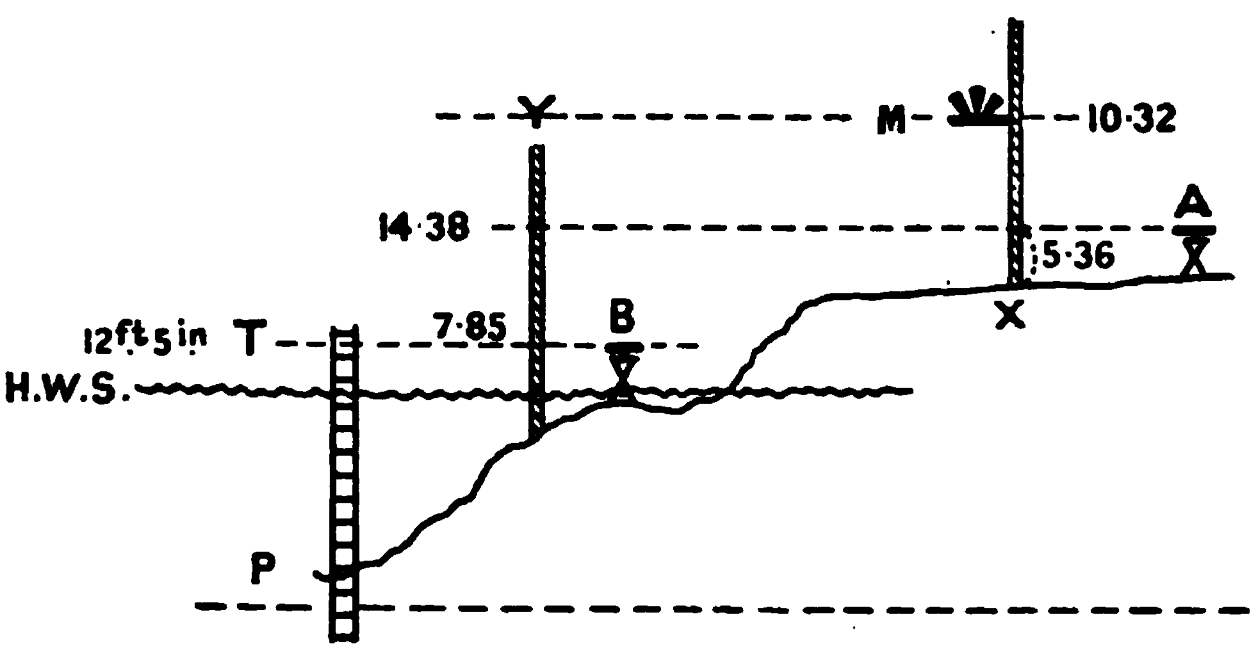


FIG. 106.

par. 502). If this mark is 'referred' to a reading on the tide-pole, then the mark of the tide-pole, corresponding to the bench mark, is deduced with reference to high- or low-water springs.

Let M, fig. 106, be the bench mark, said to be 12 feet 6 inches above H.W.S. Place a levelling staff at X, close enough to M to read off its height; suppose this reading is 10·32 feet. Place a theodolite at A in such a position that it will see the levelling

staff when it is fleeted on toward the tide-pole. Let the reading at X from the theodolite be 5·36 feet.

Shift the levelling staff to Y, where its base will be below the level of the top of the tide-pole, and the reading on it from A is 14·38 feet.

Now since the bench mark is $10·32 - 5·36$ feet above the level of $A = 4·96$ feet; and the reading on Y of the level of A is 14·38 feet; then the bench mark is 19·34 feet above the level of the base of Y.

If the theodolite be moved to B, and the reading on Y staff from B is 7·85 feet, then the difference of level between B theodolite and the bench mark is $19·34 - 7·85 = 11·49$ feet, or, in other words, the bench mark is 11·99 feet above the level of B theodolite.

Let the reading of B theodolite on the tide pole be 12·5 feet, then the bench mark is 11·49 feet above the 12·5-foot mark on the pole.

But since the bench mark is 12·6 feet above high-water springs, then H.W.S. must be $12·6 - 11·49$ feet below the 12·5-foot mark $= 11·4$-foot mark nearly on the pole.

CHAPTER XIX.

STEEL CHAIN.

334. The steel chain is another instrument for linear measurements; that of the hydrographic surveyor and engineer is 100 feet long, the land surveyor's is 66 feet. Each link is 12 inches from crown to crown, and it is marked with a brass tag at every 10 feet. As the 10-foot pole is to a sextant, so is the chain to a theodolite; and as a flexible length can be used for several purposes. Its first and primary use is to measure a length in hundreds and in fractions of feet; and in that capacity it is used for the measurement of a base for whatever other purposes may follow.

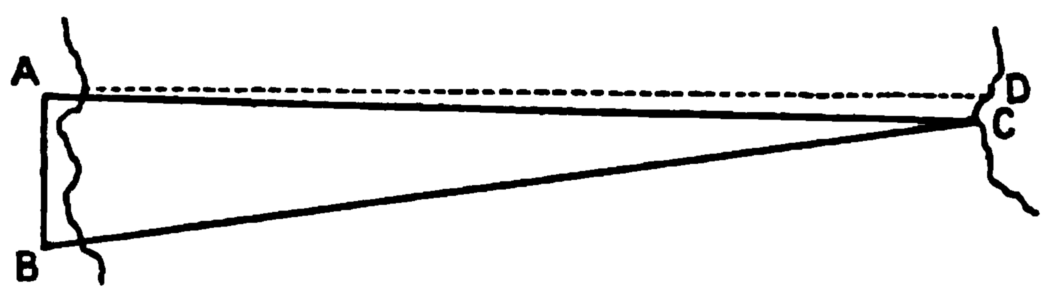


FIG. 107.

335. Obtaining a Distance from a Chained Length.—If, for instance, the horizontal length of A to B in triangle A B C is 100 feet, and all three angles of the triangle are observed, then the length of A C or B C can be calculated; or, supposing A B to be on one side of a river = 115 feet, and both angles, C A B and C B A, are measured on the same side of the river, to a point C on the opposite shore, then the distance A C or B C can be calculated without going to C.—With a sextant and a 10-foot pole, this order would be reversed, the observer being at C, while the pole is at A B.—$AC = AB . \sin B . \operatorname{cosec} C$; where $C = 180 - (A + B)$, though this is not quite so accurate as if all three angles of the triangle are observed; for there is no check that either or both the angles observed *i.e.* C A B and C B A are correct, without also observing the third, and hence, the sum of all three equalling 180°. A sextant in this particular case will not serve, because the dis-

tance A B is too short, and would introduce error due to parallax (see p. 3).

336. Making a Right Angle with a 'Chain.'—But neither A C nor B C would give the width of the river; the width would be A D, which is at right angles to B A. A D may, however, be obtained by other means, provided an observer is able to obtain a footing on the D side of the river.

Now the proportions of the three sides of a right-angled triangle can be 3, 4, 5, in any units, 5 being the hypotenuse; 6, 8, 10, or 60, 80, 100, will be in the same proportion.

Erect a staff at B, and with the chain measure from B to A, parallel with the bank. At a distance of 80 feet, put a staff up at A. From B, fig 108, measure inland 60 feet, and sweep a small arc of a circle with that distance; then, with one end of the chain at A, stretch out the whole 100 feet and sweep a small arc with this: where the arc from A cuts the arc from B, will give the point X. Then A B X is a right-angled triangle, B X being perpendicular to A B.

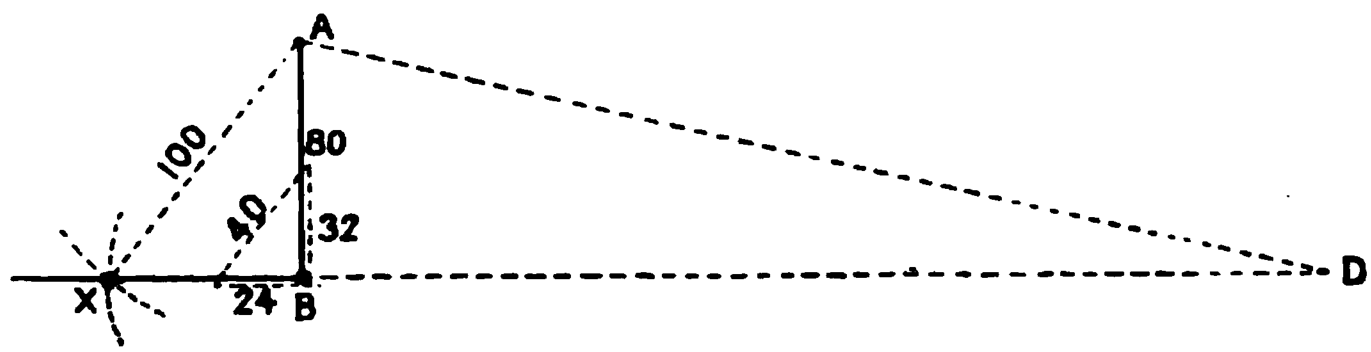


FIG. 108.

Erect another staff at X.

The observer at D, must now place himself so that the staff at B and the one at X are in line; from that position he measures the angles subtended by A B; if sufficiently far to neglect parallax, a sextant can be used. Then B D (the width of the river) = A B cot angle at D. Here there may be objections, owing to the difficulty of sweeping small arcs with the chain, or there may not be room: in that case the whole length of the chain may be used as the total length of the three sides of a right-angled triangle. Take that indicated in fig. 108.

If A B is 32 feet; then B X will be 24 feet and A X 40 feet, the proportion still remaining 3, 4, 5. If a peg is placed inside the triangle, against the end of the chain at B, another inside A where B A = 32 feet, then, still holding the end link of the chain at B, place the end of 96 feet link against it, and pull the 'bight' of the chain out till both A X and B X are stretched taut; place a peg at X; then will A B = 32 feet, A X 40 feet, and B X 24 feet, and A B X is a right angle. Then, as before, the observer D places himself with B and X in line, and measures *with a sextant* or a theodolite the angle subtended by A B.

337. To Chain a Length.—And this distance B D may serve for the beginning of the triangulation up the river. Where a level straight length can be found at either high or low water or any intermediate state of the tide, then there is no explanation necessary in the measurement of a length, beyond, that one assistant keeps the line of direction straight with the aid of the telescope on a theodolite; that the chain be stretched out taut without being strained; that the iron pins be used at the end of each length measured; and placed *inside* the end link, while the next length is taken from *outside* the pin—thus: see fig. 109.

FIG. 109.

337a. Difference between Horizontal Length and Chained Length.—When the ground is not level, the difference between the length measured and the *horizontal* distance is about 1 per cent. less, for a rise of 8°; the elevation or depression can be either guessed or absolutely measured. (See *Levelling Staff*, p. 155; and *Theodolite*, p. 127, par. 276.)

338. Measuring a Base by Chained Portions.—If there is not a sufficiently long straight run, though otherwise adaptable for chaining a length for a base, then it can be measured in portions; heretofore, when the ground was inclined, it was a deflection in the chain vertically, now it is a deflection horizontally; and in both cases it is required to find the horizontal and *nearest distance* between two points.

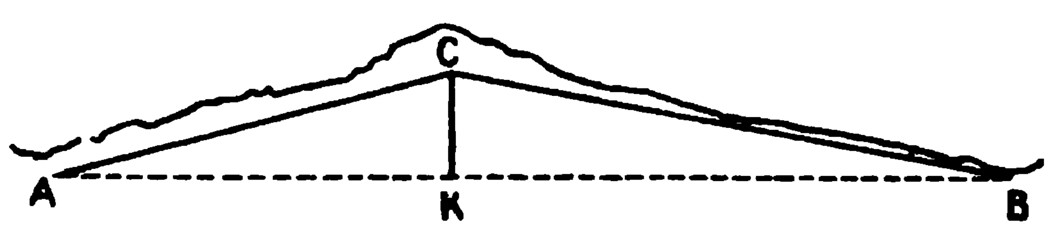


FIG. 110.

Suppose, for instance, across a small bay, A round C to B (fig. 110) is a fairly level beach, required the distance from A to B. Measure from A to C, and from C to B. Observe angles B A C, A C B, and A B C with a theodolite; then A B = A C . sin C cosec B; this gives one value for A B *through* angle B. Also A B = B C . sin C cosec A; this gives another value for A B *through* angle A. The mean value of A B is as correct as you can get it.

Supposing the coast from A to C and from C to B to be unsuitable for measuring a length, then an assistant at K (fig. 110), who is in exact transit with A and B, can, by means of the chain, obtain the direction of C K nearly perpendicular to A B (see par. 336), and also measure C K; and A K = C K . cot . C A K;

B K = C K cot C B K. *Neither C A K nor C B K* should, if possible, be less than 5°, on account of errors of observation; and see *Parallax*, p. 3, if the angles are taken with a sextant. It is still necessary to measure angle A C B as a check. If C K is very short, then a small error in placing C K perpendicular to A K will make but a very small error in angle at A or at B (see par. 325). K C may be placed right-angled to A B by theodolite.

In fig. 111 distances are measured from A to C, then to D, on to E, and finally to B. A B is the distance required.

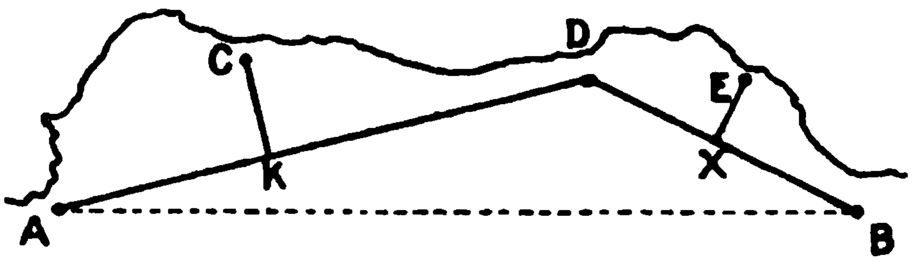


FIG. 111.

In triangle A C D, given A C and C D and angles C D A and C A D, find A D, through both angles A and D.

$$AD = AC \sin C \operatorname{cosec} D$$
$$AD = CD \sin C \operatorname{cosec} A$$

In triangle B E D, similarly, through angles D and B find B D. In triangle A B D, through angles D A B and D B A, find A B. Necessary staves are, of course, erected at the points A, C, D, E, and B; or, as before explained, the length A D and D B can be obtained by perpendiculars from K and X, and in triangle A D B, A B is calculated. There are innumerable such combinations; when in either figure the angles at A and B are less than 5°, the measurements should be made in shorter bits, with perpendiculars.

339. Errors in Measuring.—Under the most favourable conditions, without more elaborate apparatus, it is not possible to measure any base without errors, and the more lengths and more angles there are, the greater the probable total error. It is probable that the error of each calculated side will be about ½ foot p.c. in the first triangle, and will possibly increase by about the same ratio, in arithmetical progression, in all the sides of the succeeding triangles, providing they are fairly conditioned. If time and patience admit, the whole operation may be repeated, working back from B to A; and, under any circumstances, each piece must be measured forward and back: the mean will probably give an enhanced result.

A 'well-stretched' lead line can be used as a *very* rough substitute for a chain; this does not mean a line stretched well.

CHAPTER XX.

DISTANCE BY M.H. ANGLE.

340. Distance obtained by the Angles Subtended by Ship's Mast.—For the hydrographic surveyor, there is still another length of reference—the height of a ship's mast-head above the water-line, or a mark on the ship's side near water-line; this can be directly measured with a lead line or a piece of string.

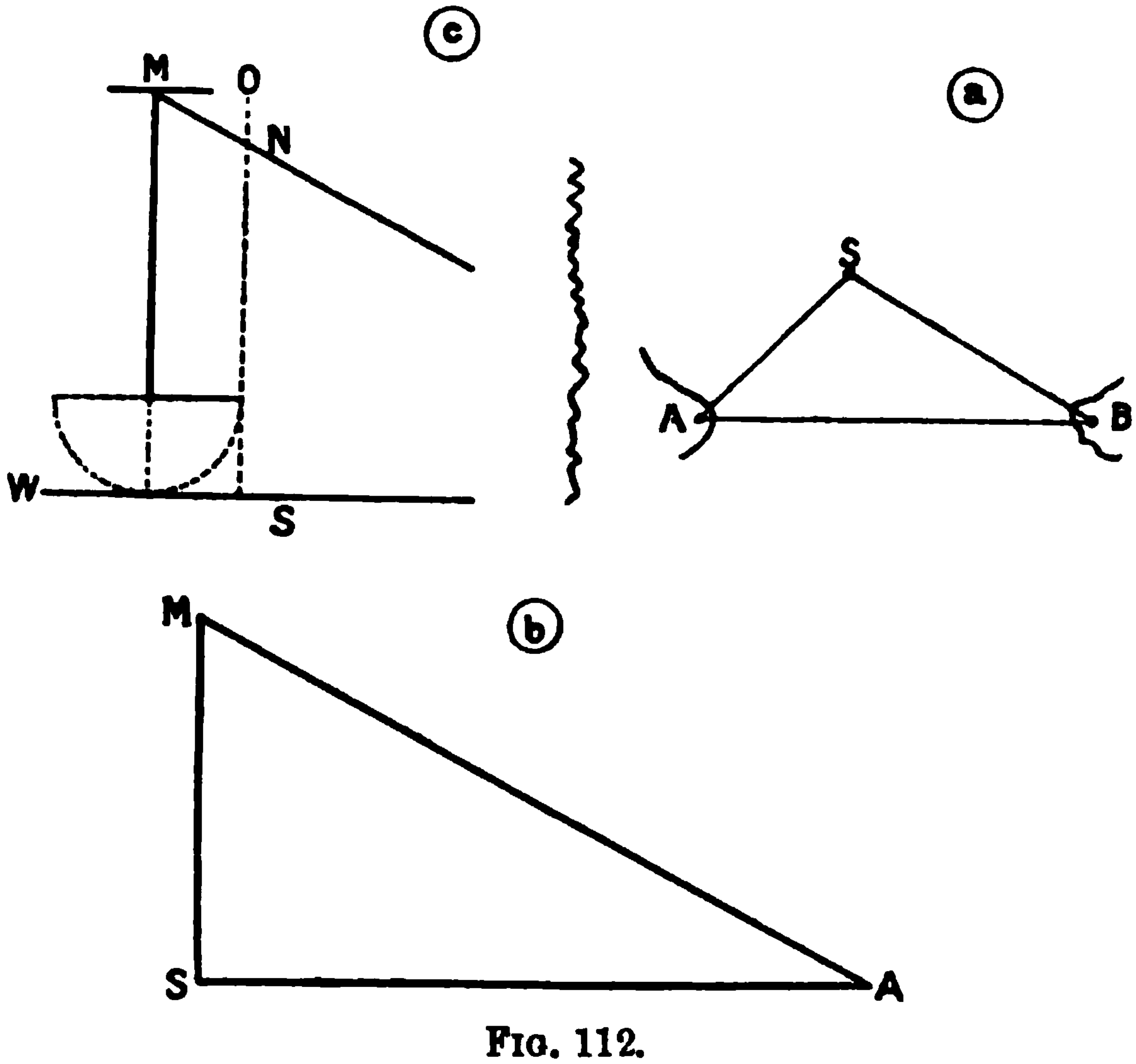


FIG. 112.

In fig. 112 (*a*) let S be the ship and A B two positions at the entrance of a river. If from A and B simultaneous angles with a sextant are taken of the ship's mast-head to the mark near the water-line at S, then S A, fig. 112 (*b*), = S M cot M A S. S M is the height of M above S.

In fig. (*a*) S A is now known, and S B can be calculated in the same manner; it is necessary that the angles S A B and S B A shall be taken, also A S B, as a check. Then calculate A B through

A = S B . sin S cosec B ; and A B through B = S A . sin S cosec A : mean the values of A B = distance from A to B.

By fig. (*c*) it will be seen that, owing to half the 'beam' of the ship, the vertical line from S goes through position N ; and in the small right-angled triangle M O N, O M is half the width of the ship, and O M N = angle of elevation ; therefore O N = O M tan elevation. Suppose O M = 30 feet, and the mean of the readings for elevation observed both 'on' and 'off' the arc + parallax to be 2° 14′ 30″, then O N = 30 . ·039 = 1·17 feet, and therefore N S = height of mast-head − 1·2 feet. Hence in triangle M S A, M S must be modified to this new height. Practically it is 1 foot less for every 2° of elevation, and if M S = 120 feet, then N S = 119, and A S = 119 . cot 2° 14′ 30″ = 2975 feet ; if 120 feet be used instead, A S will equal 3000 feet : at 3° elevation the error in distance would be 86 feet in 3000.

341. Obtaining a Base by Ship's M.H. in Transit between two Observers.—The observation is much simplified, and the calculation is reduced, and probably more correct because no other angles are involved, if the observers place themselves in exact transit with the ship, and the ship is nearly midway between them. See fig. 113 ; here, as before, simultaneous angles of elevation are taken from A and B, of S (the ship's mast-head) ; and A S = height of mast-head cot elevation from A, B S = height of mast-head cot elevation from B. The same or a similar correction should be made as before in dealing with the 'presentation' of the ship to each observer in consideration of the distance M to O, fig. 112 ; in that figure M O is given as the width of the ship, whereas M O is the distance from the base of the mast to the position at the water-line to which M is reflected ; if the ship is bows on to one, and stern on to the other, supposing M to be in the centre of the ship, M O will equal half the length of the ship. If the ship is 300 feet long, then O M = 150. In the case given, O N = 150 tan 2° 14′ 30″ = 150 × ·04 = 6 feet. Therefore M S in fig. 112 (*b*) = 144, and, continuing the same case, A S = 114 (120 − 6) . cot 2° 14′ 30″ = 2850 feet ; whereas, if 120 feet be used, A S = 3000 feet. To find the total length A S or B S, 150 feet will be added to both : if this is not allowed for both from A and B, each will be 150 feet in error ; hence A B would be 300 feet too long, or 100 yards out in a mile.

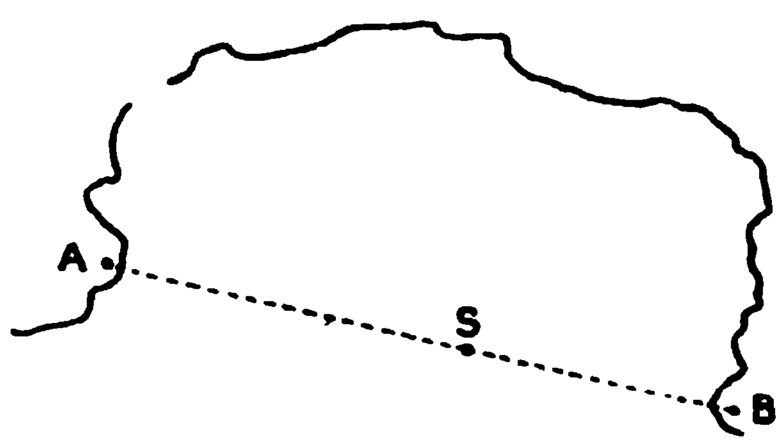


FIG. 113.

342. Errors of Observation in M.H. Angles and in

Distance.—No account is here taken of error of observation. If the angles are measured 'on' and 'off' the arc, with a sextant stand, the probable total error is $\pm 2'$. This includes instrumental error. In the above case such an error would produce about ± 50 feet in the side A S (*i.e.* in 3000 feet). The smaller the angle of elevation the greater the corresponding error in distance for a small error of observation. If the elevation is $1°$, an error of $2'$ for the height 120 feet = 200 feet. Knowing this, this form of measuring a distance for a base will then have its time and place (chaining a direct distance of 2000 feet over uneven ground would probably be within 2 feet of the correct horizontal distance, but when calculated through two or more triangles from a short measured length it will be about 20 to 30 feet in error).

If a cone (base up) or a black flag is hoisted to the mast-head, it is made plainer to an observer; and a streak of whitewash on the ship's side would mark the water-line.

The angles at the two places must be taken simultaneously; and, unless the ship is, for certain, stationary, only one set of observations can possibly be made.

343. The D.P.F. or 'Direct Position' Finder.—It is only necessary to mention the principle of this instrument, since its use for giving the position of a boat over a mine in a harbour is confined to the Torpedo School in H.M. Navy.

If a theodolite is set up on a spot whose height is known exactly (30 feet, for instance), and the angle of depression is taken to a spot (suppose it to be on the opposite side of a river or a boat in mid channel), then since distance = height cot elevation or depression, and if b is 30 feet and depression equals $3°$, then distance = 572 feet; therefore, placing the position of the observer at a suitable point on the paper, he can lay off this distance, and in a line of direction as is done on a plane-table or with a ruler and a piece of paper. The instrument combines the two motions, both in direction and distance. This process can be carried out for any number of points or objects in any direction.

There are bound to be errors of observation; but so long as the distance does not exceed half a mile, it will serve the purpose the D.P.F. is intended for, and the position of the boat will be within 20 feet of where the observer places her, near enough to be blown up by a mine there; or the river may be within 20 feet of its true width.

If this process is carried out for every detail of a coast-line surrounding the observer, he can obviously sketch it in straight away; for the instrument is adjustable for the height, and places the position without any calculation, on a scale of either $\frac{1}{2000}$ or $\frac{1}{5000}$; this works out to about 30 inches and 15 inches to a mile respectively. But the results with it, and the limits of its reach, do not justify its use beyond the purpose it is intended for.

PART II.

PROJECTIONS AND SCALES OF CHARTS AND PLANS.

CHAPTER I.

PROJECTION.

344. **Projections.**—The nearest distance between any two places is the arc of a great circle joining them.

If the arc joining them is transferred to paper as a straight line, and the distance measured along it, then we represent a great circle by a straight line.

345. **Gnomonic Projection.**—The projection that best satisfies this condition for hydrographic purposes is the gnomonic.

It is the projection of an area of the earth's surface, on to a plane touching a sphere at a tangent at the centre of the area, the parts being projected by imaginary lines drawn through them from the centre of the earth, and intercepted on the plane.

In fig. 114 a flat surface is laid touching the earth at one spot only, marked S. Lines drawn from the centre of the earth through the meridian A A^1 will be intercepted, so as to produce the straight line B B^1 on the plane. C C^1 will produce D D^1; E E^1 is F F^1, and so on—like, in fact, a shadow of the part A to A^1 thrown on to an intercepting plane, by a light at the centre of the earth. On this projection *all* great circles are straight lines; and the arcs of the great circles joining any three places are, within certain limits, considered to cut each other, as the straight lines of a plane triangle.

346. **Spherical Excess.**—When, however, large areas are included, the arcs joining three places must be accepted for what they are, viz. spherical arcs; and, for plotting purposes, will have to be reduced to a plane triangle. The difference between the

spherical triangle and the plane triangle of the same length of sides, is known as spherical excess.

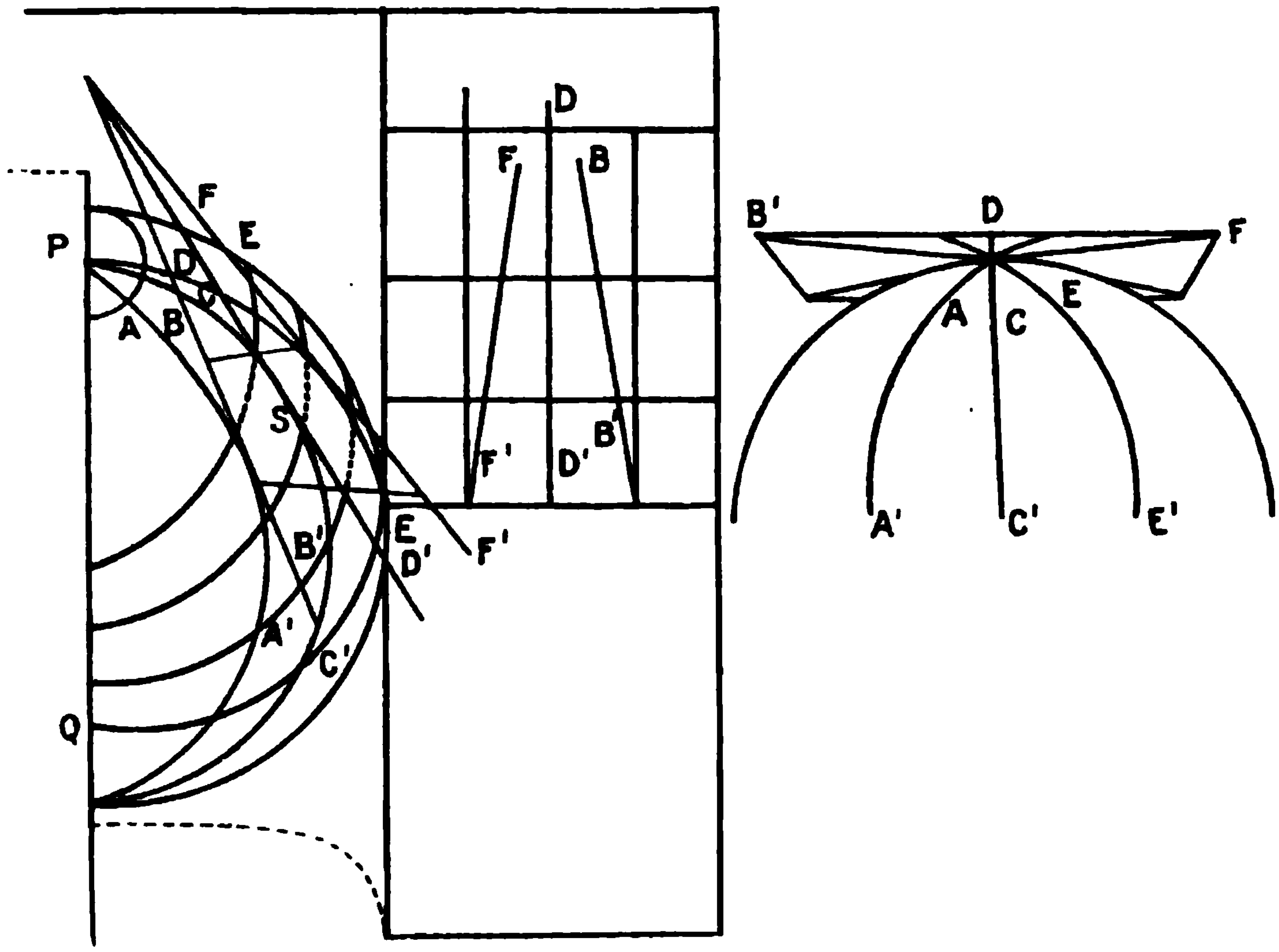


Fig. 114.

Let A B C in fig. 115 be a spherical triangle, the length of each side being 60′.

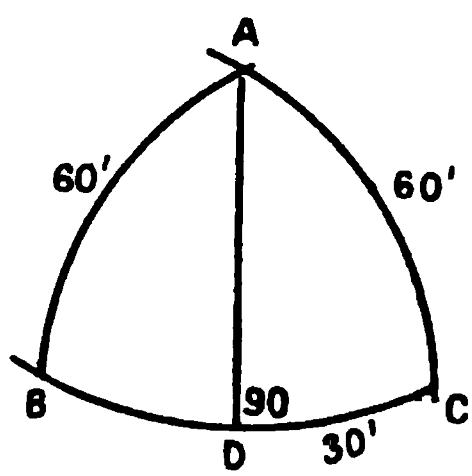


Fig. 115.

From A draw A D perpendicular to B C, bisecting B C in D; then D C = 30′, and cosec C = tangent 30′ cot 60′.

$$\tan 30' = 7{\cdot}940{,}858$$
$$\cot 60' = 11{\cdot}758{,}079$$

$$\log \operatorname{cosec} \; 9{\cdot}698{,}937$$
$$= 60^\circ\ 0'\ 9''.$$

∴ in a spherical equilateral triangle the sides of which are 60′

(miles), each angle = 60° 0′ 9″, whereas in a plane equilateral triangle, each angle = 60°; the difference being nearly $\frac{1}{10}$th mile in 60 miles.

Beyond a knowledge of the fact, it has no place in elementary surveying.

Now, it will be noticed that on the gnomonic projection the meridians are straight lines; but, like the meridians on the earth, they still converge to a point and, consequently, are not parallel. The position of this point of convergency varies with the latitude of the work under consideration. At the Equator the point is at infinity, and for an area round the Pole the point of convergency is the Pole. Fig. 116 represents bird's-eye views of the projection

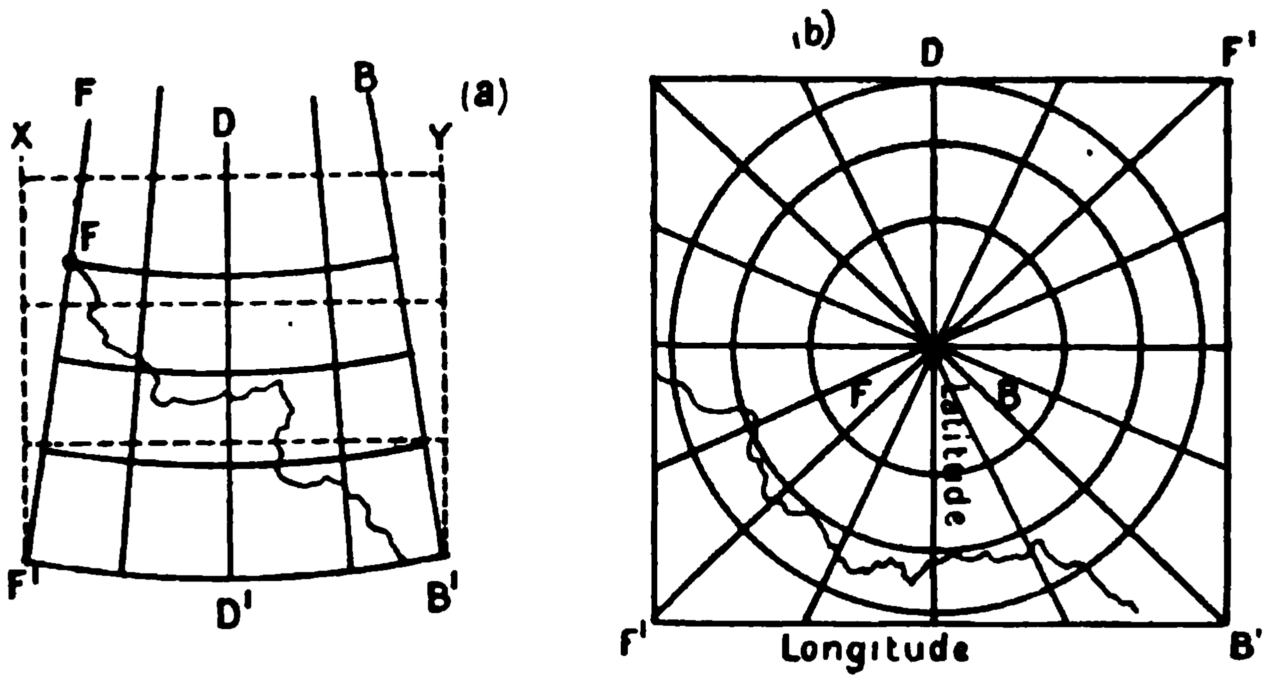


FIG. 116.

—(*a*) in middle latitudes, (*b*) at the Pole. It is, in fact, a portion of the surface of the earth, included between two meridians and two parallels of latitude, flattened out.

347. Distortion in Gnomonic Projections.—But in the flattening-out process a certain amount of distortion to the extreme parts takes place, amounting to about 1 foot in 60 miles. Parallels of latitude, which are arcs of small circles, appear on the projection as curve lines, their curve varying with the latitude. Above latitude 45° the curve is an ellipse, at 45° it is a parabola, and at less than 45° a hyperbola.

The straight line joining any two places is a great circle; this is, in fact, the *raison d'être* of the projection. But a course is a curve, because the meridians are not parallel.

348. Scale on a Gnomonic Projection.—Its scale of distance is a matter of convenience—4 inches to a mile, for instance. This is a scale of latitude and distance, and the meridian is graduated accordingly, or a suitable scale is drawn (see *Scales*, Chapter II. p. 176). The scale of longitude is set off along a parallel, according to its ratio to the scale of latitude (see table,

Inman's Tables, for example). In latitude 50° the degree of longitude is to the degree of latitude as 37·835 is to 59·962 = ·63; and the scale of longitude then = ·63 of the scale of latitude.

349. Mercator Projection.—A Mercator projection is formed by the expansion and consequent further distortion of the area under consideration, so as to make the sides parallel, and is likened to a projection on a cylinder (see fig. 114) encircling the earth round the equator. To this end F B is made equal to F^1B^1 or X Y (fig. 116); so that the parts will remain proportional to each other, then F^1X and B^1Y must be increased in the same proportion as B F has been to X Y (see fig. 116). For investigation of this see any work on navigation.

In this projection the meridians will be great circles, but all other great circles will be, correctly speaking, shown on it as curves, as exemplified in great circle sailing; but for short distances there is no appreciable difference between the curve and the straight line.

350. Reason for Mercator Projections.—A course, on the other hand, cutting all the meridians, which are parallel, at the same angle, will be a straight line; and this is its advantage for navigation purposes, and is the reason it is adopted, though for the original construction of large areas it is unsuitable, for the reason given above, viz. that the straight line on it is not the nearest distance between one point and another.

The original is then on a gnomonic projection; and it is transferred by construction to a Mercator projection.

351. To Construct a Mercator Projection.—To construct a Mercator chart, extending from the same latitudes and longitudes as the extremes of the gnomonic chart—viz., latitude 50° to 51° N. and longitude 1° to 2° W.

The scale of it is to be 1 inch to 5 miles of longitude (see fig. 117). Draw a horizontal line A B; from A draw A C perpendicular to A B.*

Now from A on the side A B, lay off a scale of miles or minutes of longitude, each mile being $\frac{1}{5}$ inch. For the practical use a Mercator chart serves, these lengths are accurately enough laid off by the edge of the protractor.

NOTE.—The most useful protractors have a scale on the edge of $\frac{1}{4}$ inch, which are again divided into tenths. Multiply any length desired by 4, and use those divisions: thus 2 inches (one-fifth) = 8 divisions; the next mark will be 16, or one and 6

* *To draw a line perpendicular to another from a point at one end of it.*—On a pair of dividers take a length of roughly half the length of the perpendicular desired; measure this off from a point A in a direction of roughly 45° with line A B. Let S be the point. From this point sweep a small arc across line A B, cutting it at T. From T through S draw a long line, and along it from S measure the same length S T; from this point to A a line will be perpendicular to A B.

divisions beyond; 3 will be 2·4, and so on. This method is far more accurate than measuring from mile to mile or from $\frac{1}{8}$ inch to the next $\frac{1}{8}$ inch, and so on; because in that case the errors of

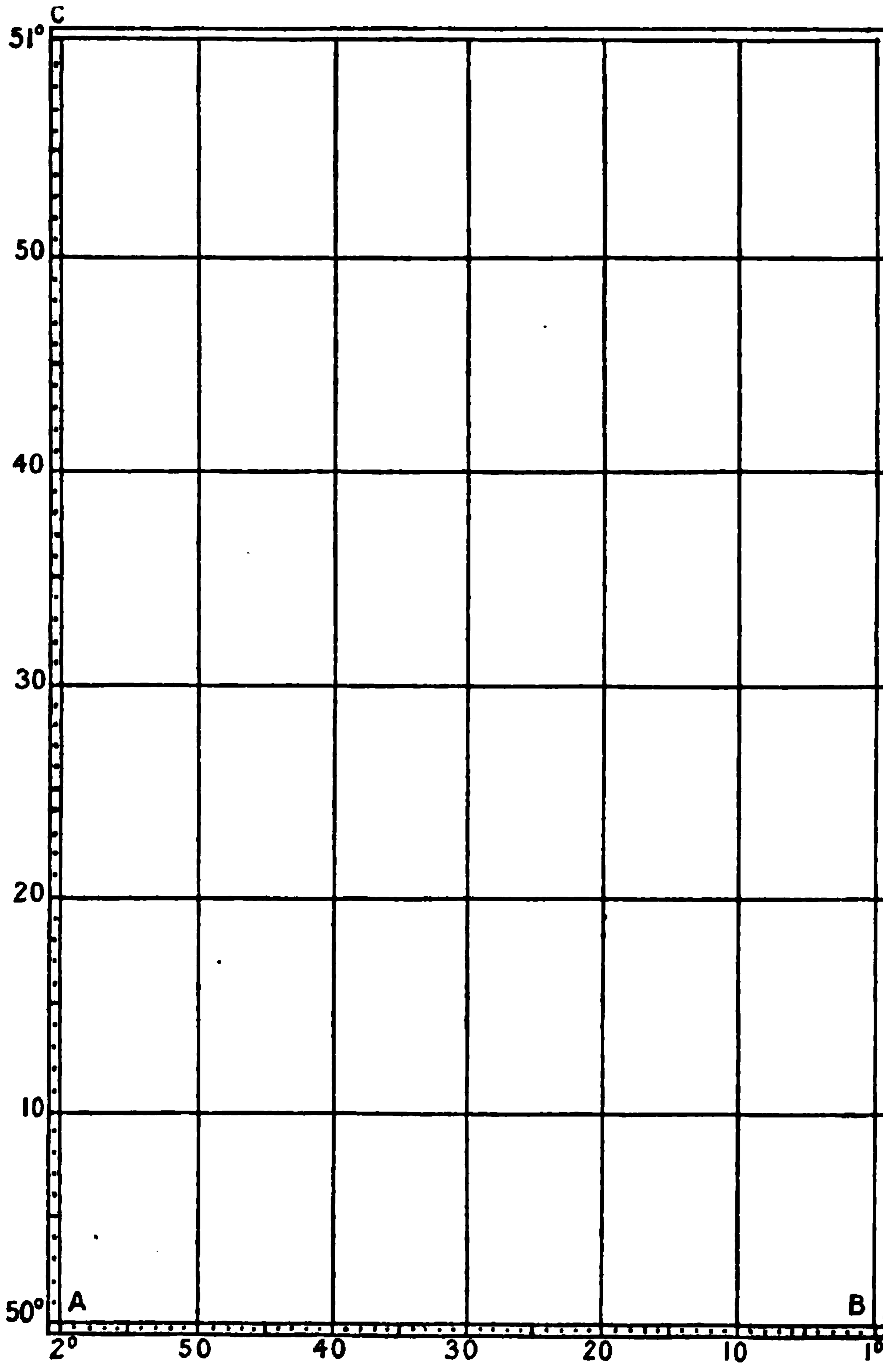


FIG. 117.

measurement accumulate; and, conversely, to divide a length represented by 2 inches or 8 quarter inches, that is 10 miles of our work, by 10, then each mile will be ·8 divisions.

The scale of latitude is now set off on line A C. To mark off

the scale of latitude: In a Mercator chart the scale of latitude = scale longitude × sec latitude; and since our scale of longitude is 1 inch to 5 miles and the latitude is 50°, the scale for 5 miles of latitude at 50°

$$= 1 \text{ sec } 50°$$
$$= 1·555 = 6·22 \text{ by the } \tfrac{1}{4}\text{-inch scale.}$$

In latitude 50° 30′ = 1·572 = 629

In latitude 51° = 1·589 = 6·35 by the same scale.

352. From this it will be noticed that while the scale of longitude on a Mercator's chart is unaltered, the scale of latitude changes in the ratio of the sec latitude; and therefore, that equal distances *on paper* represent different lengths in different latitudes: For example, an island 5 miles long in latitude 50° is 1·555 inches; in latitude 60° an island of the same dimensions would be 2 inches long; and to indicate the two islands of the same size, each will be measured by the scale in its latitude.

Distortion Mercator Chart.—Without referring to the different scales, each part of a Mercator chart is out of proportion to the other.

Then in our Mercator chart, the first 5 miles from latitude 50° may be set off 1·555 inches or 6·22 quarter inches, measuring by edge scale.

The next 5 miles should increase proportionately according to the sec: at 50° 30′, for instance, it will be 1·572; practically, this is midway between 1·555 and 1·589, and therefore is increasing ·017 inches for 60 miles; or, since each measurement is made in 5 miles, each will increase one-twelfth of ·017 or ·0014, *i.e.* ·006 nearly of the ¼-inch scale.

This is so small, that on the scale given it can be neglected, and no appreciable error is introduced by accepting the mean of 1·555 and 1·589; or, in other words, scale of 5 miles of latitude

$$= 1 \times \text{sec. } 50° \ 30' \text{ the mid latitude}$$
$$= 1·572 \text{ inches.}$$

Continuing on fig. 117, from A measure along A C portions of 1·572 inches; by the edge scale 6·29, then 12·58, then 18·86, without moving the protractor. Each mark will be a distance of 5 miles from the other.

When, however, the scale is much larger, such as 1 inch to a mile between the same latitudes, then the scale at every 10 miles of latitude would have to be found. (See par. 372.)

353. When Sides not Graduated.—If the scale is 3 inches to a mile, for instance, then from 50° to 50° 3′ the difference in scale would be ·005 inch, and for a 6-inch scale ·01 inch. The difference between these scales for such a short distance as 3 miles is so small, that charts included within a distance of 5 miles of

latitude, on a normal scale, in ordinary working latitudes—*i.e.* not higher than about 60°—do not need a graduated side for a scale, but have a separate scale. (See *Scales*, p. 176.)

354. Plans, Projection of.—Such charts are known as plans; and, the distance being limited to within those given, they are constructed on a plane projection.

Referring to fig. 118 (*a*), it will be noticed that the meridians of a gnomonic chart converge, and the higher the latitude the greater is the convergency. In fig. 118 (*b*) is shown the maximum convergency, the points of convergency being the Pole.

355. True and Mercatorial Bearings.—In fig. 118 (*a*), X F′B′Y is the corresponding Mercator projection to the gnomonic included within F F′B′B. Join F B′; this is a great circle joining

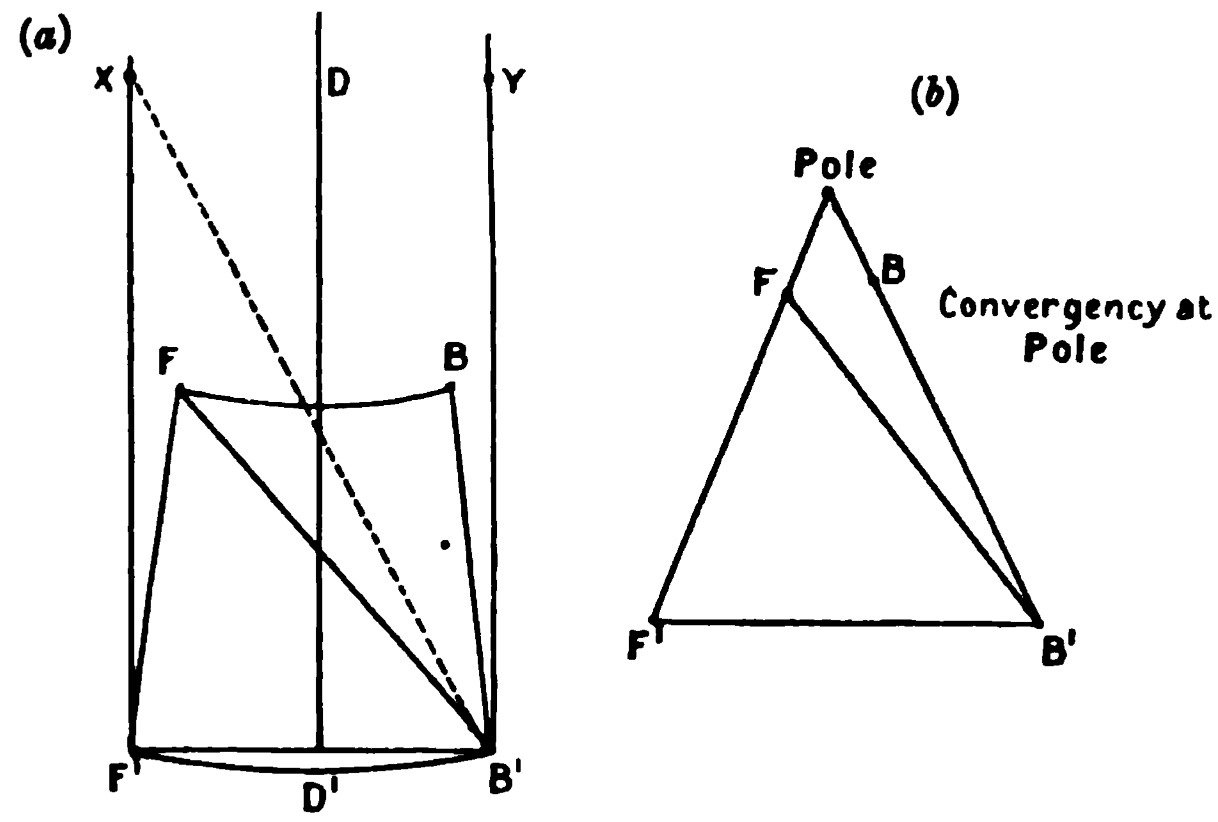


FIG. 118.

F and B′; consequently F′F B′ is the true bearing of B′ from F; and F B′B is the true bearing of F from B′.

Since F F′ is not parallel to B B′, the angle F′F B′ is not equal to F B′B: the difference between them will vary as the latitude. See, for example, the difference between the angle in the left figure 118 and the right-hand figure.

Now in fig. 118 (*a*), by the scheme of the Mercator projection F is placed at X. Join X B′; then F′X B′ is now equal to X B′Y; each of these is the Mercatorial bearing, and each differs from the true bearing by ½ the convergency; *i.e.* Mercatorial bearing = T.B ± ½ convergency.

356. Formula for Convergency.—The formula for convergency, when the distance or the convergency is not very large, is, convergency = *d*. longitude × sin mid latitude. This does not, strictly speaking, give the convergency at either place, but at the meridian of mid latitude; but is near enough in ordinary

working latitudes. A and B are two places on meridians S M and S N; then A S B is the convergency for mid latitude *ab* (fig. 119).

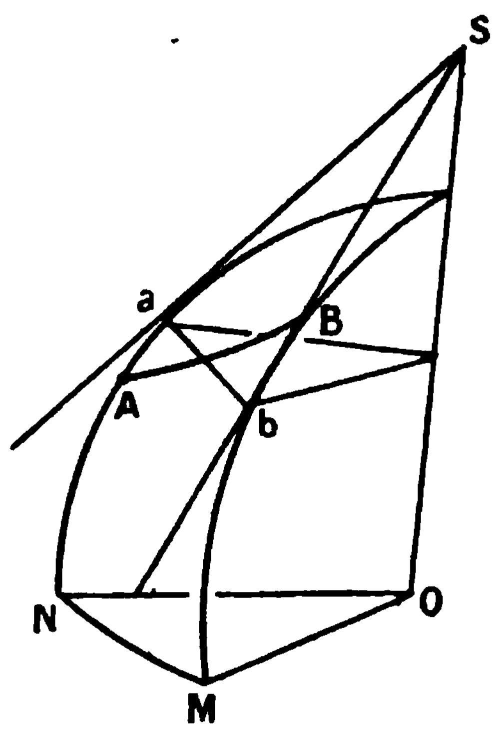


FIG. 119.

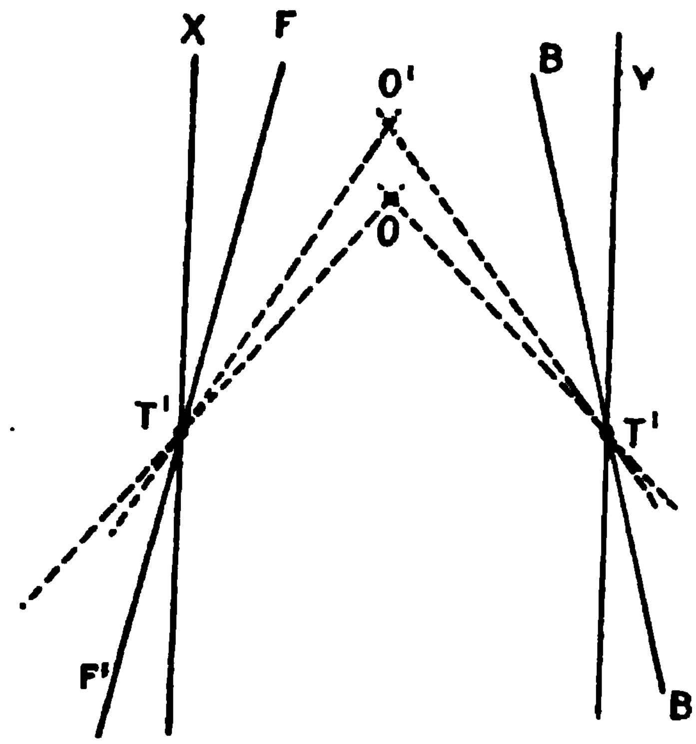


FIG. 120.

357. When Convergency Neglected. Extreme Cases of.—The correction in ordinary latitudes, and with normal distances, is inappreciable and disregarded for *navigating purposes*; but taking two extreme cases:—

Suppose two imaginary peaks of Teneriffe, 100 miles apart, one on each bow, visible 80 miles to the southward (see fig. 120). Let a true bearing be taken of each; if this bearing is laid off from the Mercatorial meridian X and Y, they would fix a position O′.

But if the convergency is applied to the bearings, or if they are laid off from their proper true meridians, F F′ and B B′, then O is the true position. The convergency correction is about 20′ in such a low latitude as Teneriffe is in, but such an error at a distance of 80 miles would make a material difference to the position for surveying purposes, though for navigation good enough.

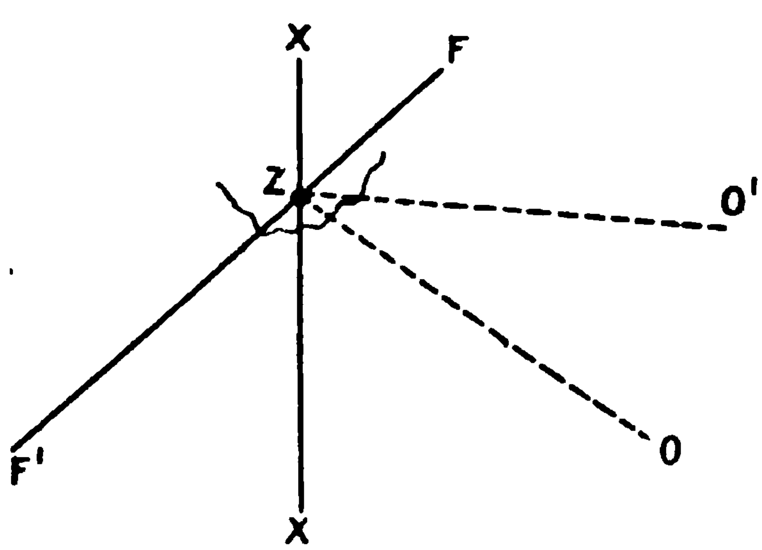


FIG. 121.

358.—The other extreme case is in high latitudes. Let Z be a point of land in, suppose, lat. 75° N., X the Mercatorial meridian, F F′ the true meridian through Z. A true bearing is taken of Z, N. 80° W. This bearing set off from the true meridian line F′F through Z gives line Z O, but from the Mercatorial meridian gives Z O′.

In this case, the error in laying off the bearing from X amounts to nearly 2°; and supposing the distance to be 30 miles, if a course 2° in error is laid back for 30 miles from Z, the probability is, if the ship steers on this course she will land on the shore somewhere. In such latitudes it will be advisable to navigate on a gnomonic projection, and lay the course off in a curve.

These examples are merely given to show the effect of convergency on a position; and if one has to account for minutes in the true bearing, as, for instance, when fixing a position accurately to within a handful of seconds, convergency will have to be included.

359. Example in Applying the Correction for Convergency to Find the Latitude and Longitude of the Observer's Position.

At O, fig. 122, the true bearing of a summit S is found to be S. 34° 30′ E. The latitude of S is 51° 28′ 56″ N., and longitude 6° 20′ 17″ W.

At O the angle between S and O′ is 87° 52′.

At O′ the angle between S and O is 73° 41′.

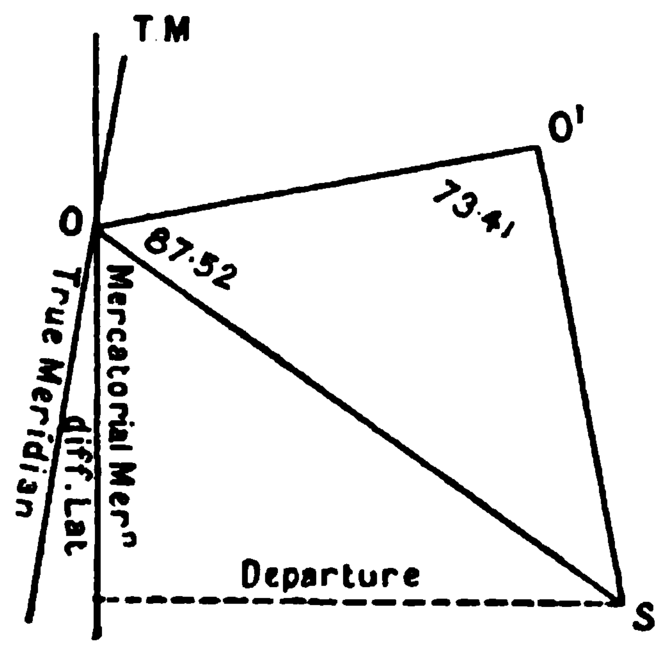


FIG. 122.

The distance of O to O′ is 24·2 inches of paper. The scale is 3·038 in. = 1 mile = 6076 feet.

Required the latitude and longitude of O.

1. Find the distance O S.

$$O\,S = O\,O'\,.\,\sin O'\,.\,\operatorname{cosec} S.$$

$$O\,O' = \frac{24{\cdot}2\,.\,3{\cdot}038}{6076} = 10{\cdot}12 \text{ miles.}$$

Angle $S = 180 - (O + O') = 180° - (87°\ 52' + 73°\ 41') = 18°\ 27'$.

log	10·12	1·005180
log sin	73° 41′	9·906204
log cosec	18° 27′	10·499658
log		1·411042 = 25·77 miles = distance O S.

2. Find the diff. lat. between S and O, so as to deduce the mid lat. required, for finding the convergency correction.

diff. lat. = O S . cos Mercatorial bearing.

Here we shall have to use the true for the Mercatorial bearing; and, after the correct Mercatorial bearing is found later on, then, if necessary, re-work the diff. lat.

log O S	1·411042		1·411042
log cos T.B.	9·915994	recalculated	9·916794
log	1·327036		1·327836
diff. lat. =	21·23 = 21′ 13″		21·27 = 21′ 16″

lat. S	51° 28′ 56″ N.
½ diff. lat.	0° 10′ 36″
approximate mid. lat.	51° 39′ 20″

3. To find the convergency, for the purpose of converting the T.B. into a Mercatorial bearing.

Conv. in minutes = dist . sin Merc. bearing . tan mid lat.

We must still use the true for the Mercatorial bearing, and recalculate the convergency after the true bearing is corrected to a Mercatorial bearing.

log dist. O S	1·411042		1·411042
log sin 34° 30′	9·753128	recalculated	9·751469
log tan mid lat.	0·101860		10·101860
log.	1·266030	log	1·264371
convergency	18·46 = 18′ 27″		18·38 = 18′ 22″.

The T.B. was	S. 34° 30′ E.		S. 34° 30′ E.
½ convergency	− 9′ 13″	recalculated	9′ 11″
Merc. bearing	S. 34° 20′ 47″ E.		S. 34° 20′ 49″ E.

and recalculating (1) gives diff. lat. 21′ 16″ (see above).

Since the lat. of S is	51° 28′ 56″ N.
and diff. lat. . .	0° 21′ 16″ N.
lat. of O . .	51° 50′ 12″ N.

4. To find the diff. long.

diff. long. = dep . sec mid lat.
dep. = dist . sin Merc. bearing.
diff. long. = dist . sin Merc. bearing, sec mid lat.

log dist.	1·411042
log sin bearing	9·751469
log sec mid lat.	10·207364
	1·369875 = 23·44 = 23′ 26″.

The long. of S is	6° 20′ 17″ W.
diff. long. to O .	0° 23′ 26″ W.
long. of O . .	6° 43′ 43″ W.

360. From which is deduced the fact that in lat. 51° N. and for a distance of nearly 26 miles, applying the correction for convergency, for the purpose of calculating the latitude and longitude of a position, makes a difference of 3″ in latitude, and about the same in longitude, if the uncorrected bearing is used in the calculation; the uncorrected bearing being the true bearing, and when corrected is the Mercatorial bearing. This correction is then only necessary when results to the nearest seconds of arc are desired; or when in extreme high latitudes. As is stated in par. 358, the convergency correction is a matter of degrees.

NOTE.—*To draw the margins of a plan.*

1. Mark off approximately the corners of the plan.

2. Lay a straight-edge across the opposite corners, and note roughly the angle it makes with the *true bearing line.*

3. Lay off the noted angle accurately with chords, from the point of intersection on the T.B. line, and ascertain the T.B. of this diagonal.

4. Take a point near the centre of this diagonal, and lay off in the direction of the meridian *twice* its true bearing. This will be the second diagonal.

5. With a radius equal to the distance of the corners, strike a circle through each end of the diagonals, pricking the intersections.

6. The lines joining these marks give the margin in the positions required, and correctly in the meridian.

CHAPTER II.

SCALES.

361. Length of a Mile: Geographic; Hydrographic.—The English geographical mile is the old standard measurement of 5000 feet, modified to 5280 when the standard measurements of England were overhauled.

It is the length of a minute of arc, measured on the Equator, and bears a ratio to the equatorial diameter.

The length of a minute of arc, measured on the meridian at any one latitude, is the British hydrographic mile. Owing to the oblate spheroidal shape of the earth, the polar and equatorial axes differ in length; consequently a minute of arc varies, from 6046 feet in length at the equator, to 6108 feet at the pole: 6080 feet is the accepted length of the nautical mile. J. D. Potter, the mapseller at the Minories, London, publishes a table of the length in feet of the hydrographic mile for each ten miles of latitude; and see Appendix VII.

362. Common Unit of Measurement for Land and Sea.—The common unit of measurement for difference of latitude and distances used for a land survey and a hydrographic survey, is therefore the foot. A land survey of 6 inches to a mile is 6 inches = 5280 feet; for a hydrographic survey of 6 inches to a mile 6 inches = 6056·5 feet in latitude 24° 30′.

363. 'Natural' Scale.—In the one case, 6 inches = 5280 × 12 inches, and the proportion to nature is $\frac{6}{5280 \times 12} = \frac{1}{10560}$.

In the other, 6 inches = 6056·5 × 12 inches, and the natural scale is $\frac{6}{6056 \cdot 5 \times 12} = \frac{1}{12113}$. The proportions of any given distance on paper will be O : H : : 12,113 : 10,560 when the scale to a mile is the same for both.

364. Transferring Positions from an Ordnance Survey to a Hydrographic Chart of the Same Scale.—Again, to adapt an Ordnance chart on a scale of 6 inches = 5280 feet, to

a hydrographic plan or chart, the relation of the length to 6056·5 feet (in latitude 24° 30′) must be deduced; and if 6 inches = 5280 feet, how many inches = 6056·5 feet?— : 6 : : 6056·5 : 5280 = 6·88 inches; this would be the corresponding scale for a hydrographic chart, from which any measurements may be made (at that latitude) to adapt the positions in one chart to their corresponding places in the other, and details desired transferred to the hydrographic chart; or if the scale of the hydrographic is drawn with reference to feet (see examples which follow) then the calculation is simplified.

Example.—On an Ordnance map (scale 6 inches = 1 mile) the distance between two objects is 8·92 inches.

Required the corresponding distance in inches of paper, between the two objects on a hydrographic survey, the scale of which is 4·72 = 1 mile = 6083·8 feet.

If the Ordnance Survey scale is 6 inches = 1 mile = 5280 feet, then 8·92 inches $= \frac{5280 \,.\, 8{\cdot}92}{6}$ feet.

The hydrographic survey is 1 mile = 6083·8 feet = 4·72 inches.

If 4·72 inches = 6083·8 feet, how many inches $= \frac{5280 \,.\, 8{\cdot}92}{6}$ feet?

$$x : 4{\cdot}72 : : \frac{5280 \,.\, 8{\cdot}92}{6} : 6083{\cdot}8$$

$$x = \left(\frac{5280 \,.\, 4{\cdot}72}{6083{\cdot}8 \,.\, 6}\right) . \, 8{\cdot}92 = ({\cdot}6833) \,.\, 8{\cdot}92 = 6{\cdot}09 \text{ inches.}$$

The first fraction will be a constant for that particular hydrographic chart on to which the Ordnance points are being transferred, and it equals the proportion of the natural scales of each chart.

For, the natural scale of the Ordnance Survey $= \frac{6}{5280 \,.\, 12} = \frac{1}{10560}$,

the natural scale of the hydrographic survey $= \frac{4{\cdot}72}{6083{\cdot}8 \,.\, 12} = \frac{1}{15447}$.

Therefore the proportion of the hydrographic to the Ordnance scale is $\frac{1}{15447} : \frac{1}{10560} = \frac{10550}{15447} = {\cdot}6833$ as before.

The hydrographic scale corresponding to a 6-inch Ordnance scale in lat. 51° 30′ $= \frac{6 \,.\, 6083{\cdot}8}{5280}$, or $\frac{6 \,.\, 69{\cdot}116}{59{\cdot}962}$ (the proportions of a nautical to a land mile, as given in Table 29, *Inman's Tables*) and = 6·916 inches.

But the corresponding scale of longitude is very nearly the same on both charts.

That of the Ordnance would be $6 \cdot \frac{3797 \cdot 4}{5280}$ * $= 4 \cdot 315$,

while that of the hydrographic $= 6 \cdot 916 \cdot \cos \text{lat.} = 4 \cdot 305$.

365. Scales, Irregular Length.—A gnomonic chart is constructed on any convenient scale, depending on many circumstances, and, for reasons explained further on, it will be only a coincidence if this scale is a whole number; it may be 4·65 or 6·023, and so on.

366. How Deduced in Longitude.—The margin between the parallels of latitude is subdivided in its whole length in equal divisions; and the scale of longitude is deduced from the proportion of the length of a mile in longitude to a mile of latitude, in that latitude (see par. 348), and divided into equal divisions also; the scales on an Ordnance chart are the same as this, the scale of latitude being for the measurement of distances, while the scale of longitude, proportioned as explained in par. 364, is for the measurement of longitude.

367. Measurement of Latitude, Departure, and Distance.—On a Mercator chart, for the reasons already stated, the latitude margins are graduated for each latitude, proportional to a given scale of longitude; and from this scale, latitude, distance, and departure are measured. Each mile is usually subdivided into tenths, each subdivision being 1 cable.

368. Measurement of Longitude.—From the scale of longitude, difference of longitude is measured; and this scale is subdivided into sixths, each subdivision being 10″ of arc.

369. Scales on a Plan.—In a plan, however, for reasons already given, one scale suffices for the measurement of difference of latitude, departure, or distance, and this scale is arbitrary, depending upon the size of the work, the present or future importance of the locality, time at one's disposal, instruments, etc.

Some plans are constructed on 6, 10, 18, 24, or 30, or even nearly 150 inches to a mile. Portsmouth Harbour, for instance, would be more important than Poole, or Harwich more than Burnham-on-Crouch. There might be months devoted over the one, and weeks over the other, and the larger the scale the more details it will hold.

The scale attached to a plan may be drawn as so many inches to a mile; if so, each inch is subdivided into ten parts, and each part is a cable. If the scale is such that so many inches equal so many feet, say 2 inches = 1000 feet, then the unit of subdivision will be feet (see examples); but the system is not interchangeable unless the number of feet is exactly known in that latitude. If 2 inches = 1000 feet, and 1 mile = 6056 feet, then the scale can be 12·112 inches = 1 mile.

* This quantity is the length of a mile of longitude in lat. 51° 30′, and roughly = the length of 1 mile of latitude cos lat.

370. Scale of Longitude on a Plan.—Besides the scale of latitude and distance, it is customary in large plans to place a scale of longitude; and, as before explained, scale longitude = scale latitude . cos latitude. For example:

$$\begin{aligned} \text{the scale latitude} &= 12{\cdot}112 \text{ inches} = 1 \text{ mile;} \\ \text{then scale longitude for 1 mile} &= 12{\cdot}112 \cos 51^\circ\ 30' \text{ (Greenwich)} \\ &= 12{\cdot}112\,.\,{\cdot}622 \\ &= \ 7{\cdot}54 \text{ inches.} \end{aligned}$$

This length subdivided into 6 parts, each unit being 10″, will represent the scale of longitude, from which the difference of longitude between two points can be measured.

If there is no scale for this purpose, then the departure between the points—that is, the horizontal distance east and west of each other—would have to be measured from the chart and on the distance scale, and converted into d. long.; d. long. = dep. sec latitude; but this may be strenuous; hence the separate scale of longitude.

371. Natural Scale on a Plan.—Besides the scale of latitude and scale of longitude, a plan will have a natural scale, as already described, par. 363; if for no other purpose, it conveys the relative scales of an Ordnance chart and of a hydrographic chart of the same neighbourhood; and from the natural scales of each an Ordnance Survey can be connected up with a hydrographic survey from a point or mark common to both charts, see par. 364.

The position of the Ordnance Survey marks may be obtained locally, or from headquarters.

372. *Examples of Scales.*—1. For a plan, draw the scale of latitude and distance of 2·5 inches = 1 sea mile.

2. Draw a scale of 1 inch = 1000 feet.

3. Draw a scale of 1 inch = 1000 feet, *i.e.* 6065 feet = 1 mile. The length of the scale in this case will be 6·065 inches = 1 mile subdivided into 10 cables.

4. Draw the scales of a Mercator chart, extending from lat. 65° to 65° 10′ N., and from long. 1° to 1° 20′ W.

Given the scale of longitude 1 inch = 5 miles, *i.e.* 12 in. = 1°. The rigid method is by meridional parts (see Table of Meridional Parts, *Inman's Tables*):—

Mer. parts for 65°	5178·81	mer. parts 65° 05′	5190·66
,, ,, 65° 05′	5190·66	,, ,, 65° 10′	5202·55
	11·87		11·89
	12		12
	60|142·44		60|142·68
distance in inches between 60° 00′ and 60° 05′	2·374	between 60° 05′ and 60° 10′.	2·378

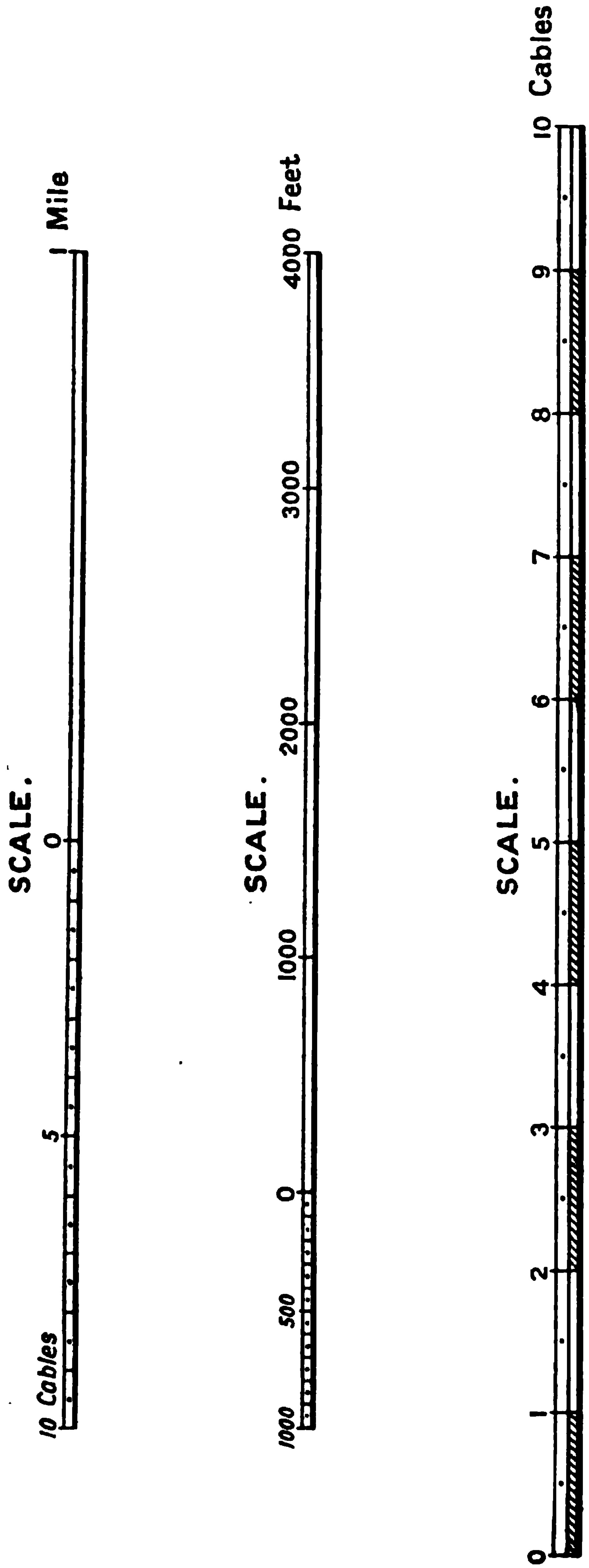

SCALE.
10 Cables
5
0
1 Mile
SCALE.
1000
500
0
1000
2000
3000
4000 Feet
SCALE.
0
1
2
3
4
5
6
7
8
9
10 Cables

FIG. 123.

By the rougher method,

where scale lat. for 5 miles = scale long. for 5 miles . sec mid lat.
= 1 . 2·373 inches.

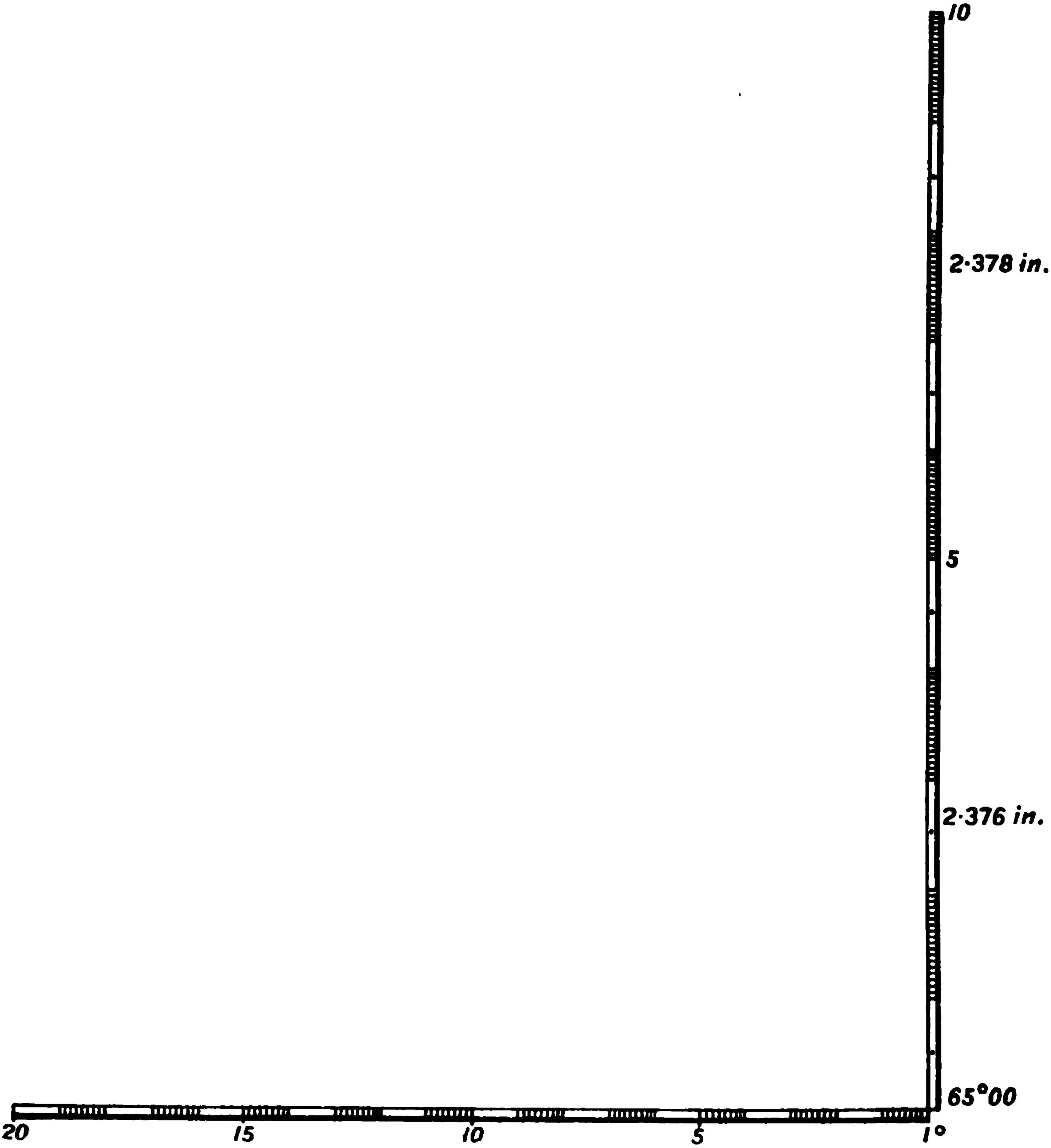


FIG. 124.

The difference between the results of the two methods is in the third place of decimals. When the scale is small, this rougher method is near enough; but when the scale exceeds ½ inch to a mile, the rigorous method should be adopted. For instance, in the above

case, had the scale been 1 inch to 1 mile, then the distance between 65° and 65° 5′ would have been 11·87 inches, and between 65° 05′ and 65° 10′, 11·89 inches; whereas by the rougher method the distance would be 11·86 inches—a loss of 0·3 inch per mile; and if a distance of 30 miles was measured by the latter scale the error would be 1 mile.

5. Draw a scale of longitude, 2·4 inches = 1 mile, and subdivide the scale into 5 seconds.

1 mile of longitude is 1 minute.

5″ is $\frac{1}{12}$ of 1′. Hence divide 2·4 inches into 12 parts: each part will be 5″.

6. Draw a scale of longitude, 8·4 inches = 1 mile of latitude in lat. 50°.

Since scale longitude = scale latitude × cos latitude

$$\begin{aligned} &= 8{\cdot}4 \times \cos 50^\circ \\ \log. 8{\cdot}4 &= {\cdot}924279 \\ \log. \cos 50^\circ &= \bar{1}{\cdot}808067 \\ \hline \log &= {\cdot}732346 = 5{\cdot}3995 = \text{length in inches of scale of } 1' \text{ of longitude,} \end{aligned}$$

or natural cos 50° = ·643′ multiplied by 8·4 = 5·40 inches.

7. Lay off with a protractor the scale of longitude, given 4·27 inches = scale of 1 mile of latitude in lat. 34° 32′.

From any point O in the horizontal line protract 34° 32′ (the amount of the latitude); along this line measure O S = 4·37 inches, the scale of 1 mile of latitude; from S draw a perpendicuiar to O L; where this intersects O L, at the point *o*, will be the length of the scale of 1 minute of longitude; subdivide into 5 or 10 seconds.

8. Required the natural scale; 2·63 inches = 1 mile. 1 mile in the latitude of the plan = 6071 feet.

The natural scale, being the proportion of 1 inch of the plan to the number of inches it represents on the earth's surface,

$$= \frac{2{\cdot}63 \text{ inches}}{(6071 \times 12) \text{ inches}} = \frac{1}{27700}.$$

9. Given the natural scale $\frac{1}{5000}$ in lat. 31°, where the number of feet in a mile = 6067 feet; required the scale of latitude and distance of the plan.

$$\text{Natural scale} = \frac{\text{scale of inches to 1 mile}}{\text{number of inches in 1 mile}};$$

$$\text{therefore } \frac{1}{5000} = \frac{x}{6067 \times 12},$$

$$x \text{ (scale of 1 mile)} = \frac{6067 \times 12}{5000} = 14{\cdot}56 \text{ inches.}$$

Fig. 125.

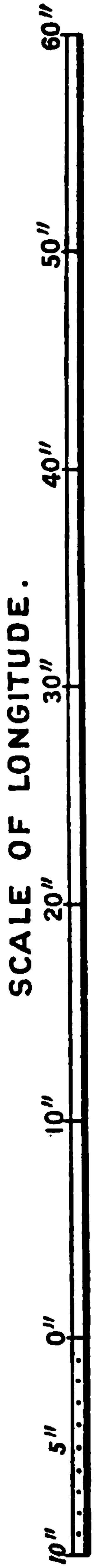


Fig. 126.

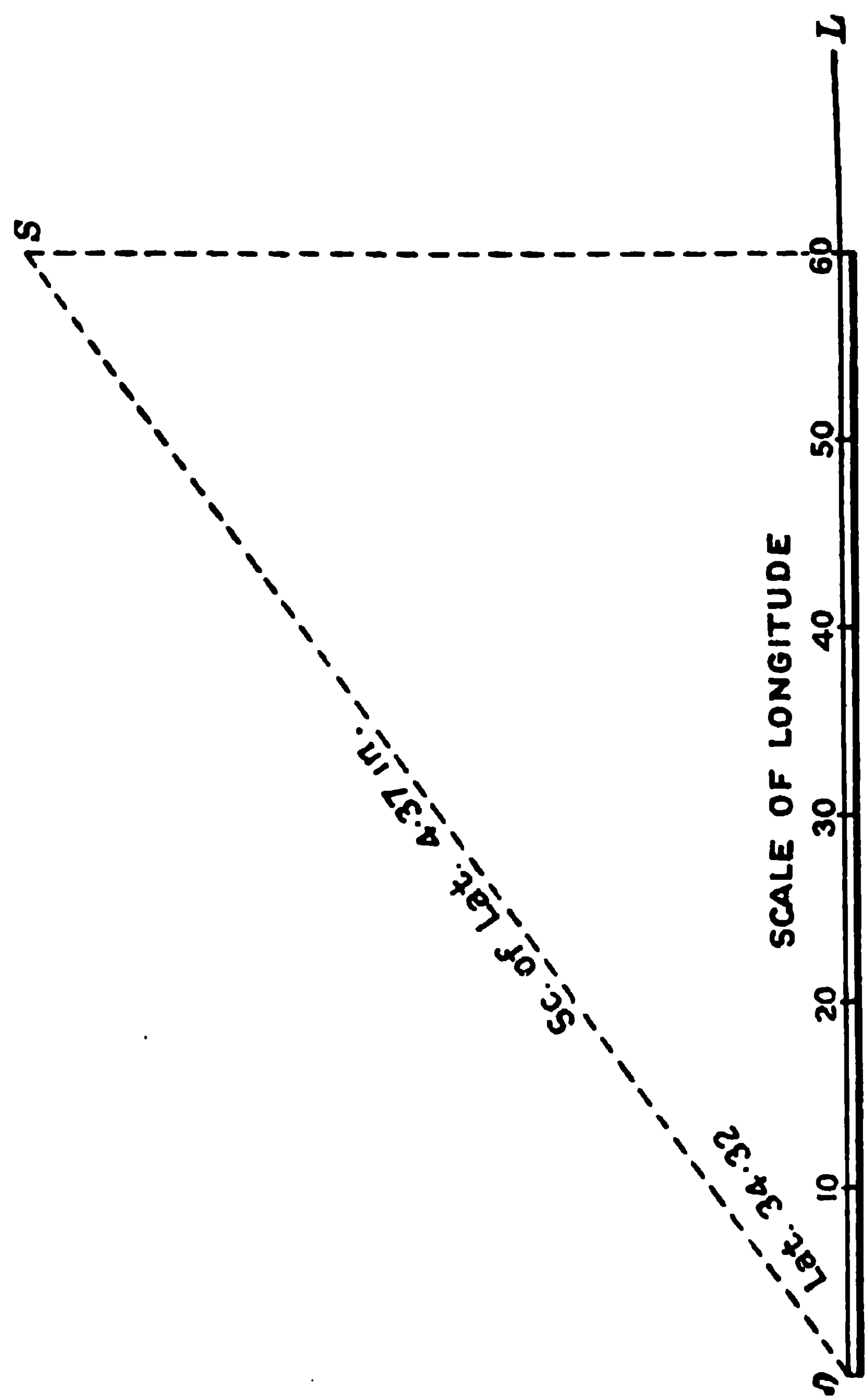


Fig. 127.

10. The calculated distance of B from A being 9120 feet, and the measured distance of the plan being 7·5 inches; find

(1) the scale of latitude and distance;
(2) the scale of longitude;
(3) the natural scale.

In lat. 48° 10′ N. 6080 feet = 1 nautical mile.

If 9120 is represented by 7·5 inches, how many inches represent 1 mile or 6080 feet?

$$9120 : 6080 :: 7{\cdot}5 : x.$$

(1) $\therefore x = 5$ inches = scale of latitude and distance.
(2) Scale of long. = scale of lat. × cos lat.
= 5·667 (nat. cos lat.) = 3·335 = 1 mile.

(3) Natural scale $= \dfrac{5}{6080 \times 12} = \dfrac{1}{14592}$.

11. A hydrographic plan is on a scale of latitude of 6·75 inches to a mile, and on this plan there is a spot marked O as the observation spot, in lat. 51° 28′ 56″ N., long. 1° 10′ 15″ W.

Given an Ordnance chart on a 6-inch scale whereon there is a triangulated mark T. The latitude of T on the Ordnance chart is 51° 30′ 15″ N. and long. 1° 12′ 47″·4 W.

It is required to place T on the hydrographic chart by its difference of latitude and difference of longitude from O.

Lat. O	51° 28′ 56″
lat. T	51° 30′ 14″
d. lat.	0° 1′ 18″ = 1·3 miles.

Since the hydrographic chart scale is 6·75 in. = 1 mile, then inches of d. lat. = 6·75 . 1·3 = 8·775 inches.

Long. of O	1° 10′ 15″ W.
long. of T	1° 12′ 47″·4
diff. long.	0° 2′ 32″·4 = 2·54 miles.

dep. = d. long. . cos mid lat.

log. d. long.	·404834
log. cos lat.	9·794229
log.	·199063 = 1·5815.

Multiply by 6·75. log. ·829304

1·829367 = 10·675 inches.

Then T on the hydrographic chart will be 8·775 inches to the north of O, and 10·675 inches to the west.

How to transfer the bearing and distance of any object from the Ordnance to the hydrographic chart is shown in par. 364.

CHAPTER III.

SYMBOLS.

373. High-Water Line.—*Topographical Symbols and Abbreviations used in Delineating the Details of a Plan* (see Plate I.).—Looking from seaward, the first 'hard' line or continuous line represents the high-water line of the coast at ordinary spring tides. This is the coast-line.

Inside of the coast-line, that is, on the shore side, topographical symbols are used to convey the nature of the ground at the coast; it may be a sandy H.W. line, at the back of which may be sand-hills; or shingle, such as the Chesil beach at Portland; or it may be a steep bank, such as in parts of the Isle of Wight, perhaps wooded to the water's edge; or it may be a steep cliff, like near Margate; or there may be nothing particular.

Each one of the foregoing is shown, in the order mentioned, in Plate I.

When the coast-line is either fringed or masked by mangroves a foot or so deep, or where it disappears behind an impenetrable and considerable depth of mangroves, sometimes hundreds of yards deep, then the hard line of the coast is replaced by an imitation of mangrove bushes.

The symbol is something like a cabbage head with a short stalk, many of them close together; but there is no line. If the H.W. line behind the mangroves is only surmised, it may be put in as a 'pecked' line.

374. View of the Land: Conspicuous Objects.—Beyond the coast-line, at the back of it, are huts or houses (the prominent single ones are marked), as also conspicuous large factory chimneys, churches, high monuments, pile lights, mills for wind or water, flag-staffs, conspicuous *rows* and detached masses of trees, and even lines of hedges; in fact, a multitude of details that should have a place on the chart.

Most of them are not usually required for navigation purposes,

though some may be; they are a sign-board to distinguish parts of the harbour by, but not for the exercise of draughtsmanship. Their principal function is as objects to 'fix' with; they are, in fact, the permanent marks left and probably used by the surveyor.

Without them no chart could be corrected, nor verified, nor 'amended,' unless re-triangulated; and it should be the principal duty of a harbour-master, not only to verify his chart constantly, but also to 'amend' the position of soundings and buoys, or pile lights, as well as the conspicuous details on the shore.

'Amending' includes 'adding to.'

It was the author's experience once to find himself unable to fix his position at certain parts of a chart; because most of the conspicuous marks had either been shifted, or demolished and erected elsewhere, and the positions or changes were not amended on the published chart.

Neither cultivated ground nor a forest of trees are objects to 'fix' with; but they serve as sign-boards from which a detail will be recognised, and they should be included in the details of the land.

375. Rise of the Land.—Beyond, again, there is the rise of the land; and a well-fixed summit is unmistakably a permanent and usually an indestructible mark.

To distinguish one summit from another, not only are their heights affixed, but there is a series of contour lines round them, showing the valley or the saddle behind two rises; and from the bird's-eye view of these hills given on the chart an experienced navigator should be able to form in his mind's eye their elevation views.

The surveyor reverses the operation; from an elevation he creates the 'plan' view.

This part of surveying is not elementary; a sketch, as shown at p. 49, should be obtained from three or four points of view; the summits plotted separately, then the direction of the main valleys and the tributary indentations sketched in, from both of which the contour lines are derived; the finishing touch is practice.

The contour lines need not, as in Ordnance charts, represent any definite height, for they are but sketched, while those of Ordnance maps are obtained by levelling; but relative contour lines must represent relative heights, that is, a hill 1000 feet high must show twice as many as one of 500 feet.

Contours wide apart show a gradual incline, while those close together indicate that the grade is steep.

Equidistant means equally, whether steep or gradual.

If there are no hills marked on a chart, the natural conclusion is that there are none, and the land is quite flat.

376. Low-Water Line.—On the sea side of the H.W. line

abbreviations are used to denote the appearance of the shore at low water, ordinary springs.

The ebbed tide will expose a sandy beach, or a shingle or gravel beach, or rocks which were under water will now show dry (dries 4 feet), or shelving rock or coral (the same abbreviation is used for both), or mud-flats (with the height above *low* water), ($\underline{6}$) or if there is nothing exposed and the coast is 'steep to' dots close up to the shore; each is here described as shown on Plate I. from left to right.

Only matter which is exposed at low water will have a symbol of supposed resemblance.

377. **Below-Low-Water Abbreviations.**—For anything under water at L.W.S., a number of graphic and verbal abbreviations are used.

The soundings are in sloping figures; whether feet or fathoms must be stated in the title of the chart. If in fathoms, they are inserted in $\frac{1}{4}$'s up to 6; between 6 and 7, $\frac{1}{2}$ fathom only: $6\frac{1}{4}$ would be 6, and $6\frac{3}{4}$ would be $6\frac{1}{2}$. After 7 all fractions are omitted: $7\frac{1}{4}$ and $7\frac{1}{2}$ are 7; $7\frac{3}{4}$ is 8.

The nature of the bottom, for which there are special verbal abbreviations (see *Inman's Tables*, Table 60), is inserted below and to the right of the sounding.

This is especially important where the anchorage is indicated; elsewhere it may turn out useful, either for a ship that wants to go on a soft bottom, or for one that wants to come off: a pumice bottom, for instance, would not serve to 'lay out' anchors on, from which to heave a ship off; and the information is serviceable.

In the open, soundings and the nature of the bottom combined do in some parts give a fairly accurate fix for navigational purposes, and it is for navigators that the chart is intended.

Seaward, contour lines of equal depths are inserted. The requirements vary on different plans, principally depending upon the scale: some contain the contour of 1, 2, 3, 4, and 5 fathoms, while those of a smaller scale include the 1, 3, and 5 fathoms only. (See Plate I.)

Contours of depth up to 5 fathoms may be looked upon in the light of danger lines, and therefore must be considered in connection with the draught of water of, and with the nature of, shipping which frequents the port; and also with the rise and fall of the tide. For instance, a sounding of 2 fathoms at L.W.S. will not be dangerous at H.W. springs to a ship drawing 20 feet, when the rise of the tide is 30 feet. Here, then, at this period of the tides, the 1-fathom line in the above case would be the warning line.

The height above L.W.S., or the amount a mud- or sand-bank dried, is of use to smaller craft; also by reason of the rise of tide, they will know when it is a danger, and when not.

In deep water the lines of contours are an aid to navigational fixing, for indicating the best line of direction to sound in when position finding by soundings is resorted to.

The verbal abbreviation *r* indicates a rocky bottom, and a sounding of $1\frac{1}{4}$ *r* means $1\frac{1}{4}$ fathoms over a rocky bottom or a rock; but when the depth is less than 6 feet, the numeral is omitted, and a cross + substituted; and if there is a cluster of rocks 6 feet or less below L.W.S., they will be shown severally by crosses enclosed by a dotted line thus .

A coral reef, for instance, may dry at L.W.S. for a distance; but beyond the L.W. spring line there may still be a continuation of the reef. This continuation would be shown with a number of crosses, until the depth above the sunken part of the reef exceeds 1 fathom (see Plate I.), when the actual depth is recorded, and 'crl.' (coral) is attached.

Another graphic abbreviation, ⁜, is used for rocks which are awash at L.W.S. or at the L.W.S. datum adopted.

But this must be an unusual coincidence; if it does without doubt exist, its existence is very useful for future reference, as the datum from which all the present soundings on the plan are shown.

There is no abbreviation for a rock which is awash at H.W. springs, or at the H.W. springs datum adopted; if, however, one such is found, the fact is stated alongside it—'awash at H.W.S.' As in the case of the rock awash at L.W.S., this will also serve as a datum mark for future reference, or for any future amendments to the plan.

378. Heights.—Heights are, as stated, always shown above H.W. springs; in the case of small islets or rocks where there is no room to put the figures on dry land, the heights are written on the water part of the chart, in upright figures, and enclosed in brackets to prevent a confusion with surrounding figures—thus (15).

379. The margins of a plan should be true north and south, though there may occasionally be an exception to this rule; and it is customary to draw the true and mag. meridian through the spot where the T.B. and variation are actually observed (see p. 175).

380. Anchorages and Clearing Lines for Dangers.—It is the business of the surveyor to suggest suitable anchorages for small or large shipping; also to lay down 'leading lines' if there are any, shown by two parallel lines, to the suggested anchorage; as well as to indicate 'clearing lines.' (See Plate I.)

The rate and direction of tidal streams is as important as everything else; they should be determined, and shown by arrows; and it is equally the duty of the harbour-master to amend these when necessary.

381. Scale and Title.—Without a scale, a plan is almost useless.

And, finally, the title should convey all such information as will assist a navigator. The name of the port; whether the soundings are feet or fathoms; the date of the survey, and who the surveyor was, both as regards his name, credentials, and implied capacity, are all absolutely necessary.

382. The position of the datum mark from which all the soundings are shown, is equally important for future reference, and should be included whenever practicable; also lat. and long. of observation spot, which also should be distinguished.

In submitting the finished plan, the reader should refer to *Hydrographic Surveying*, by the late Sir William Wharton, K.C.B. In the capacity of Hydrographer to the Admiralty, the highest possible authority, he states in Chapter XVII. what the requirements of the Hydrographer are.

The neophyte can do no better than to submit his work and await 'information required,' and this will be his best education and future guidance.

PART III.

PRACTICAL CONSTRUCTION.

INTRODUCTION.

383. **Symbols and Abbreviations.**—To save time in writing down, the following symbols and abbreviations are generally recognised :—

⊖	The sun's centre in altitude as applied to a theodolite angle.
⦶	The sun's centre in azimuth as applied to a theodolite angle.
$\underline{\bigcirc}$ $\overline{\bigcirc}$	The sun's lower and upper limb respectively, single altitudes.
$\underline{\underline{\bigcirc}}$ $\overline{\overline{\bigcirc}}$	The sun's lower and upper limb respectively, double altitudes, as observed in the artificial horizon (Art. Hor.).
$\bigcirc\vert$ $\vert\bigcirc$	The sun's right and left limb respectively as observed with a sextant.
	The sun's right limb as seen through the non-inverting telescope of a theodolite.
	The sun's left limb under the same conditions as the right.
R.T. or ⊣ or >	is the right tangent, or tangent of the right extreme of a point of land, or island, or object.
L.T. or ⊢ or <	is the left tangent, or tangent of the left extreme of a point of land, island, or object.
ϕ	In transit with : the objects in transit being stated on each side of the symbol : thus Arch ϕ Bee. It is of no moment which is the nearer.
△	A primary station of the triangulation.
⊙	A secondary station or 'fixed mark.'

◎ A station transferred from an Ordnance Survey.

⊕ The zero object or direction of the initial line of reference. This symbol is applicable either for sextant or for theodolite angles: in the first the reading of the zero is 0° 0′; while with a theodolite the reading of the zero follows the symbol. Thus at 0 ⊕ H – 100° 0′.

M.H. Mast-head.

W.L. Water-line.

W.W. Whitewash.

+ or ⊕ on a plan marks the observation spot.

X or ● following after a sounding means that the last sounding is repeated; thus $4\frac{3}{4}$ ● ● ● ● means that there is a series of $4\frac{3}{4}$ fathoms.

5/2, 20/5. When sounding in feet and inches, 5/2 means 5 feet 2 inches; 20/5, 20 feet 5 inches: or, if used for fathoms and feet, 5/2 would be 5 fathoms 2 feet.

Z.O.K. is zero O.K., or zero correct.

Time of day is usually written in Roman letters; and some enthusiasts, for brevity, use the signs of the zodiac for the days of the week, thus, in order, as shown: ☿ ♀ ♂ ♃ ♄ ♅ ○.

Δ	Triangle.	M.T.P.	Mean time of place.
Lat.	Latitude.	A.N.	Apparent noon.
Long.	Longitude.	Mer. Pass.	Meridian passage.
T.B.	True bearing.	Mag. Mer.	Magnetic meridian.
H.A.	Hour-angle.		

CHAPTER I.

SURVEY IN GENERAL.

384. All methods of surveying consist of the connecting together of a number of triangles.

385. To construct a triangle it is necessary to know the length of one side and the dimensions of two angles, or the length of the three sides. Thus, given the length of the line A B in triangle A B C (fig. 128, *a*) and the horizontal angles A B C and B A C, the triangle A B C can be constructed, the angle C being $= \{180 - (A + B)\}$ (see *Protractor*, p. 51). Thus the position of C may be fixed, and the length of A C and B C can be measured by *protractor*, the distances A to C, B to C, depending upon the same scale as that on which A B is drawn. Triangle B A D can be constructed on the same side A B, and also A B E, and any number of others. Or take triangle A B C (fig. 128, *b*), B A C being a right angle and B C A being an observed vertical angle, the length of A C can be *protracted* by laying off the angles A B C and B A C; or it can be calculated, A C = A B . tan A B C or A B . cot A C B, and the position C determined either way; and similarly any number of others that may follow.

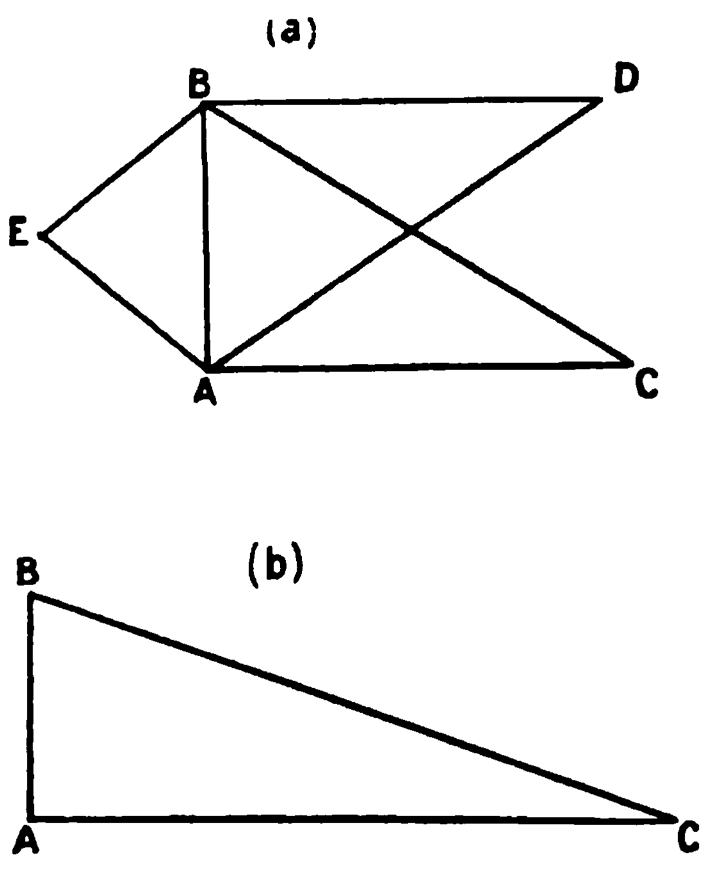


FIG. 128.

Referring to fig. 128 (*a*): If from each end of A B angles are taken to a number of objects such as D and E, the positions of E and D can be determined, relative to A B.

386. Accuracy of Angles and Sides.—Now, the value of a survey, or the correctness of it, since it rests on a *number* of triangles, will depend upon the accuracy with which C, D, etc., are fixed; and this, again, will depend upon the accuracy of the angles taken at B and A, and projected from these points.

387. Angles Correct, the Objects are Relatively Correct.—If the angles are correct, A, B, C, D, E are all quite correct relative to each other.

388. The Accuracy of the Lengths depends upon Correctness of Side Plotted from.—The accuracy of the scale will depend upon the correctness of the length A B. For example, if A B is *said* to be 1000 feet, and is drawn 10 inches in length, then A C and B C, A D, B D, A E and B E, etc., have a certain ratio to A B, depending upon the angles of each triangle; and their distance depends upon A B being 1000 feet, or a scale of 1 inch equals 100 feet.

But if A B is really 900 feet, then the correct scale is 1 inch equals 90 feet, and all the distances measured by the assumed scale are wrong.

389. Necessity to obtain Correct Length as well as Correct Angles.—Therefore the surveyor must attempt to obtain correct angles, as also a correct length for the common base.

390. The most Correct Instrument for Angles.—The most correct angles can be obtained with a theodolite, for reasons already stated.

A sextant on shore will give wonderfully good results; a sextant afloat not quite so good.

391. The most Correct Means of Obtaining Length.—Distances can be measured with a chain more correctly than by any other known means. (See *Chain*, p. 158).

For distances measured with a theodolite and a known length, see *Ten-foot Pole and Levelling Staff*, pp. 153–155. Distances obtained by sextant measurements, such as by M.H. angle or 10-foot pole (see pp. 152 and 162), are not so good; distances measured by the difference between two positions fixed by station pointers are much less accurate; and distances obtained by patent log, from one position to another, are still less so.

As will be shown hereafter in sequence, each represents grades of correctness; and there must be an occasion when by necessity it is one form, and the best under the circumstances; and another occasion when it is the other, or a happy combination of the best of either.

392. All Details depend upon the Triangulated Points.—Everything, *i.e.* all further 'fixing' and 'sounding,' depends upon the accuracy of the triangulated points which are used for fixing; and hence the total value of the work.

393. First Thing Necessary: Plotting Side.—The first thing required in any survey is the length of a side from which to plot a number of objects; in fact, a base common to a number of triangles.

This side is technically called the 'plotting side.'

394. The most Correct Side to Plot from.—One side of a triangle is more correct to plot from than another (see triangle A B C, fig. 129).

Let a line A B be drawn on which to construct triangle A B C. From A and B lay off successively the angles B A C and A B C.

As is represented in the figure, angle A C B is the 'receiving' angle, and it is very small. On p. 76 it has been demonstrated that when the receiving angle is small, an error of observation makes a large error in position, and in Part I. whole chapters are devoted to errors of observation; hence, a small error in either or both angles B A C, A B C, will place C considerably out of position, apart from any error in drawing. A B is the shortest side of the triangle, opposite to the smallest angle; therefore the short side is the worst to plot from.

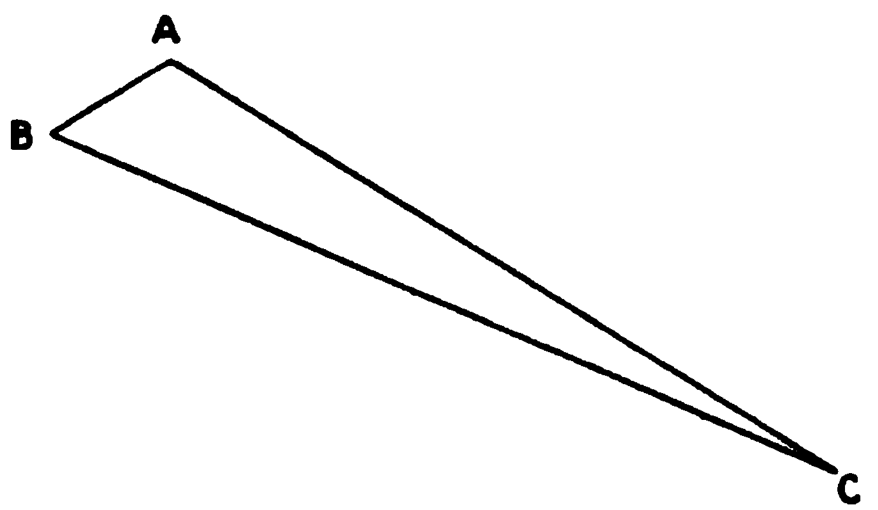


FIG. 129.

Now consider the converse position. Suppose B C to be the plotting side; then errors of observation in angles B and C will make but a small error in the position of A.

B C is the side opposite the greatest angle.

The nearer the receiving angle at A is to a right angle, the better it will be (see p. 211).

And, in every triangle, the largest angle is nearer 90° than any other angle of the triangle, and since the longest side is opposite the greatest angle, the plotting side of a triangle should be the longest side.

And since there will be a number of triangles constructed on the plotting side, then the plotting side should be the longest side of each triangle, or the longest possible in the whole area included, *consistent with other requirements.*

395. The most Suitable Side to Plot from.—And since we require to take the angles to the third object from each end of that side, then the positions at the end of it must be visible from each other; this may not be always practicable.

Then, that side the ends of which are visible from each other

and from the third point of the triangle of which it is the base, and which most nearly fulfils the condition that the receiving angle to the other points is most nearly 90°, is the plotting side that should be selected.

396. Extending the 'Measured Base' to the 'Plotting Side.'—But this longest possible side is probably a long diameter of the plan, and its direction may be across the water; if so, some means must be devised for obtaining its length.

If a chain, or a line of known length, or a 10-foot pole, or a levelling staff, be used in connection with a theodolite, the distance E W (fig. 130, *b*) can be measured: this piece is known as the base or 'measured base.'

It is desired to use C W as a plotting side. Then, the three angles of the triangle having been observed, and E W measured,

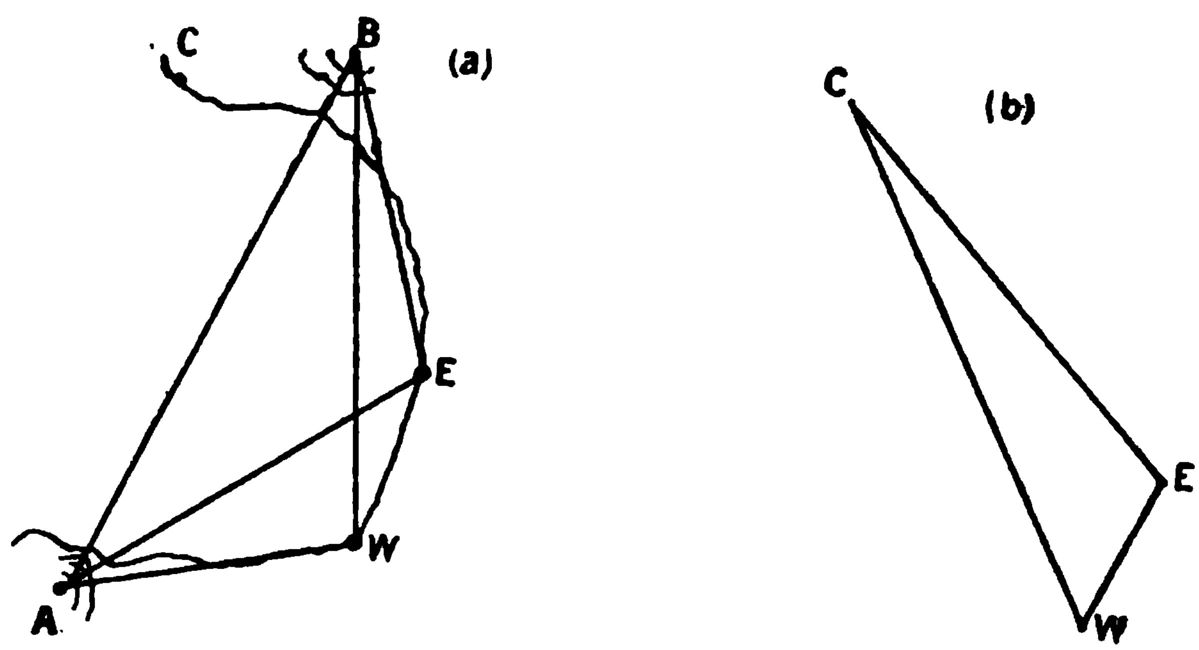


FIG. 130.

$CW = EW . \sin E . \operatorname{cosec} C$; and the length of the plotting side is calculated (see fig. 130 (*b*), and refer to example, p. 218). C W might be found by using the ship (see *Example*, p. 206, par. 416).

Fig. 130 (*a*) shows where the base E W was extended to the plotting side A B, by first finding B W in Δ B E W: then in Δ B A W find A B; or in Δ B E W first find B E, and in Δ B E A find A B; or, and more correctly, calculate A B by both ways, and mean the results (see *Examples*, pp. 222 and 320).

397. Project Angles from the Ends of the 'Plotting Side.'—From the ends of this side, angles are taken to all those objects that are to be included in the 'triangulation.' But before the plotting side can be put on paper, we must know what the true bearing of one end of it is from the other; merely and only because we want it to lay on its right slue according to the sides of the paper.

398. Placing the Plotting Side on the Paper.—Also we must know what will be its position on the paper, or the lines

projected from its two ends if badly placed on the paper may intersect beyond the margin of the paper.

Then, having a true bearing—or, at first, an approximate T.B. —and the length of the line in feet, the position on the paper can be decided.

The direction and position of the plotting side is drawn, reaching from one end of the paper to the other (see *Protractor*, p. 52, par. 138). The scale must be decided on, but no rule can be given for this, as it will depend upon the size of the harbour and the length of time that it is intended to devote to the work; remembering that the larger the scale the more detail it will hold. The length of the plotting side in inches of paper is dependent on the scale adopted.

399. Length of Plotting Side in Inches of Paper.—Suppose the plotting side is found to be 8000 feet, and the scale adopted is 6 inches = 1 mile = 6060 feet, then 8000 feet = $\frac{8000}{6060} \cdot 6$ (inches of paper) = 7·92 inches.

Prick on the line drawn the position of one end of the plotting side; measure 7·92 along it; and prick the other position.

Whether a theodolite or sextant is used, the three angles of each triangle, of the first part of the triangulation, must together equal 180°; if they do not, then the triangle cannot be plotted (see p. 228). When the necessary adjustment is made, each angle is projected from the end of the plotting side. This will give two lines through every object which forms the third point of the triangle; and at the third point of the triangle, the angle, whether observed or calculated from the other two, = 180° − the sum of the other two.

400. First Lot of Angles Projected.—Let A B in fig. 131 be the plotting side; the observed angles at A and B are projected, producing intersection of two lines at respectively E, C, and D. The plotting must be carried out rigorously, as explained on p. 55, par. 147, and the lines be drawn the full length of the paper. But, since any two lines projected from the ends of a line must meet somewhere, there is nothing to indicate that all the positions may not be in error, due to a mistake in the plotting; it cannot be in the angles, because the sum of the three angles of each triangle makes 180°.

But since they have all been plotted from the common side, then each, if *plotted* correctly, should be relatively correct to the other.

401. 'Checking' Lines.—So that, if an angle is projected from one, C, for instance, to D, *C B being the line of reference because it is the longer line of the two drawn through C*, then the line so projected should go exactly through D; and also, if a similar line is projected from C, with C B as a line of reference, with the angle B C E, then the line should go exactly through E.

If there is no cocked hat (see p. 60) at either D or E, then E, C, D are not only relatively correct to A B, but also to each other.

The check angles are suggested from C. But is this the best place to project angles from? For (see fig. 131) the line from C to D is obviously a bad one, because it makes such a small angle with the line A D. (See *Explanation of a 'Fix' by Two Lines and the Best Position of the Third Line*, p. 211.)

There is the same objection to taking the check lines from D, for D C is almost in line with A C.

But from E is evidently the best; and all the check lines of reference should have been plotted from E (also see fig. 144, p. 211).

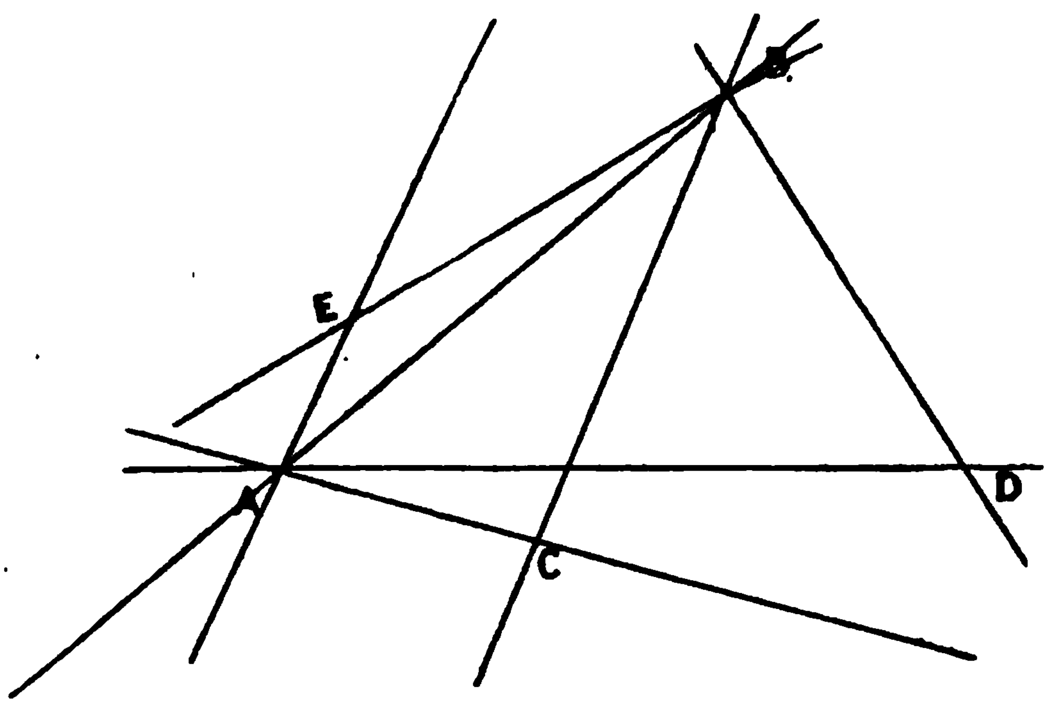


FIG. 131.

402. Line of Reference for 'Check Lines.'—Now from E, which shall be the line of reference from which the angles are to be laid off? It must be either E B or E A.

It must not be forgotten we require as long a line as possible for an initial line for chording the angles. (See *Protractor*, p. 53, par. 141.)

And since E B is supposed longer than E A, that is the proper side to plot from, and angles B E D, B E C should be projected from it.

403. Selection of Zero ⊕.—Also, an observer at E must or should, for the reason given above, take B for a zero (see *Zero*, p. 122, par. 265). He has the choice of either A or B, and B it must be. Therefore B is the zero from E, both to observe from and also to plot from.

The following elementary plans are examples of, first, plotting angles; followed by an illustration of the difference between a measured base and a plotting side; and then by a measured base which is the plotting side; and, lastly, a case is given of an erroneous triangulation.

TRIANGULATION BY SEXTANT ANGLES.

404. *Example* 1. (See fig. 137.)

Surveying harbour Date etc.

The T.B. of Flag from Cairn is N. 67° 20′ E. and Var. 10° 06′ W.

The calculated distance from Cairn to Flag is 7225 feet, obtained from a series of measurements from C to F (see p. 160, par. 338).

The scale adopted is 5 inches = 1 mile = 6084 feet.

Cairn, Flag, N.W. (merely a name), White, are the adopted 'main' stations (△).

Cairn to Flag is not the best plotting side, on account of the small 'receiving angle' from C and F. N.W. to Flag is better; its length must therefore be calculated, and the T.B. must also be deduced.

The main angles taken at the following positions (1), (2), (3), (4), must be chorded (chord $a = 2r \cdot \sin \frac{1}{2}a$):—

(1).
At Cairn △.

N.W.	78°	00′	Flag
	51	12	White
	12	48	Islet (25)

(2).
At Flag △.

Cairn	63°	40′	N.W.
Islet (25)	13	04	N.W.
N.W.	51	46	White
Rock (4)	27	40	N.W.

(3).
At White △.

Cairn	22°	00′	Rock (4)
„	33	32	Islet (25)
„	62	54	N.W.
Flag	110	40	N.W.

(4).
At N.W. △.

White 65° 54′ Flag 38° 20′ Cairn

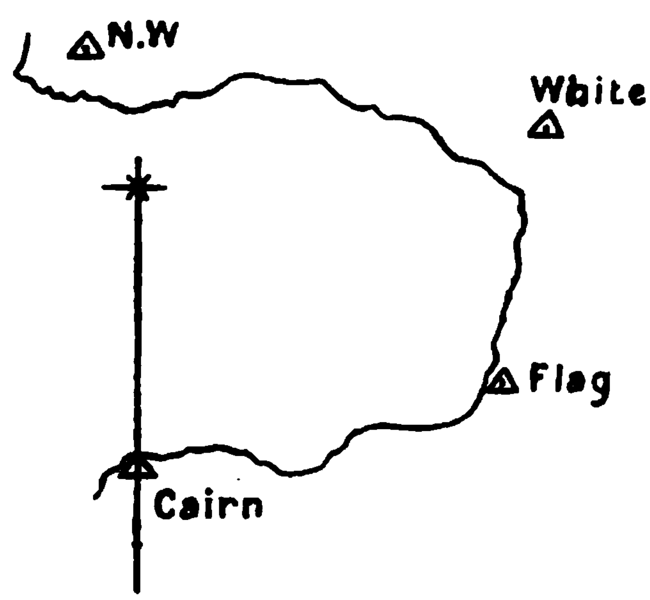


FIG. 132.

And fix the following positions:—

(5) At Rock (4), N.W. 120° 00′ Flag.

(6) At conical buoy, Cairn 122° N.W. ϕ Rock (4) 90° 00′ Flag.

(7) „ „ N.W. 24° 40′ Rock (4) ϕ Islet 78° 00′ Flag.

(5) and (6) *are fixed by calculated angles.*

(8) At Rock dries 4 ft., N.W. 90° 00′ White.
" " Islet 85° 30′ "
(9) Rock awash at L.W., Islet 10° 00′ N.W.
" " Cairn 110° 00′ N.W.
(10) Rock with 2 feet over it at L.W., N.W. 123° 00′ White 57° 00′ Flag.
(11) Rock awash at H.W., White 39° 00′ Flag 51° 00′ Islet ϕ Cairn.

} Station pointer fixes.

(12) Ship at anchor, Islet 23° 15′ N.W.
" " Cairn 90° 00′ N.W.
" " Flag 90° 0′ Cairn.
" " White 90° 0′ Flag.

} Tracing-paper fix.

(13) N.W. 2° Islet 86° 30′ Flag.
(14) Cairn 61° 30′ Islet 62° 30′ N.W.
(15) " 41° 30′ " 21° 0′ N.W.
At White, Fix (15) 17° 50′ Flag.

This angle at White is a 'check' to Fix (15).

(16) Flag 80° 30′ Islet 43° 30′ N.W.
(17) " 61° 00′ " 83° 30′ N.W.
White 14° 30′ Fix (15).
Flag 14° 00′ Fix (14).
Fix (13) 16° 20′ Cairn.

These last three angles are 'checks' to Fixes (13), (14), (15).

405. To Start Plotting.—Starting with a clean sheet of drawing-paper about 12 inches square, we want to place the plan on the paper so that the edges of the paper will be nearly true N. and S.

406. Draw a True Meridian.—The simplest way to do this is to draw a true meridian roughly parallel with the edges.

Where shall it be drawn?

Now since, for the reasons already stated, the side Flag to N.W. is to be the plotting side, this will be the first line drawn on the paper; and one end of it will for choice, and also for accuracy's sake, be on the true meridian. So, then, the true meridian will either be drawn through Flag or through N.W.

Let it be through Flag.

Flag is apparently nearly the easternmost station (see sketch, fig. 132); but, in order to allow for anything that may turn up to be to the right of it, draw the true meridian about 3 inches from the right edge. The symbol for the T.M. is $\underset{|}{\ast}$, and it had better be so labelled to prevent further confusion. On this meridian we want to place the position of Flag. Well, it appears to be about $\frac{1}{4}$ of the way up the plan from the southernmost station. Sup-

posing Cairn will eventually be 3 inches from the bottom, put Flag about 5 inches from bottom.

Prick through, on the true meridian, the position of Flag, about 5 inches from the bottom, and label it '△ Flag.'

407. Lay off Direction of Plotting Side.—The next thing is to find the *direction* of N.W. from Flag.

From Flag, Cairn bears S. 67° 20′ W.; and since the angle at Flag, between Cairn and N.W., is 63° 40′ (see under column 2), then N.W. must bear from Flag S. 67° 20′ W + 63° 40′ to the West = S. 131° 0′ W., that is, N. 49° W.

Hence N.W. bears from Flag N. 49° W.

To project 49° from the true meridian by chords:

Adopt a circle of 5-inch radius (see *Protractor*, p. 57, par. 152).

Measure off 5 inches from the scale on the protractor.

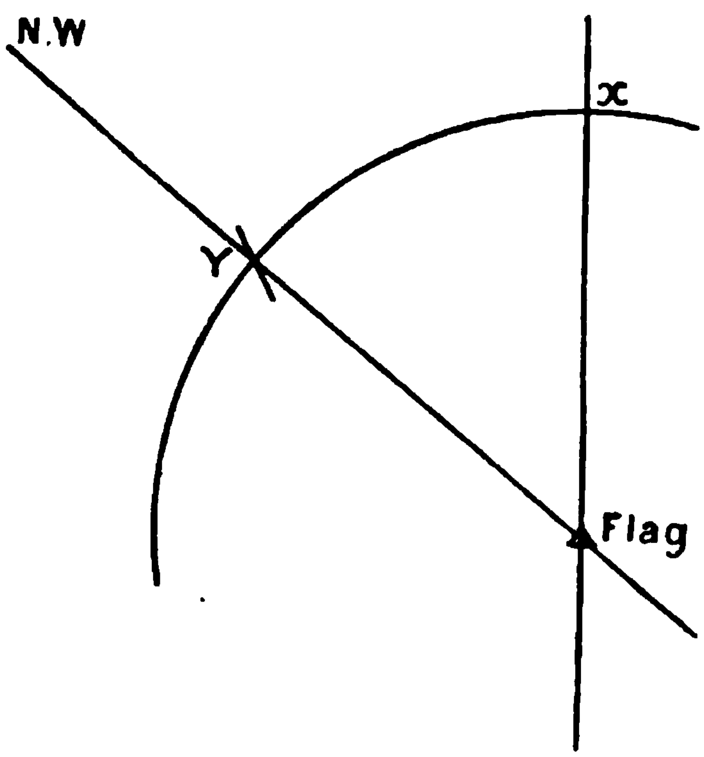


Fig. 133.

With one leg of the dividers at Flag, *scratch* a circle so that it cuts the true meridian at x (see fig. 133), and leave a mark all round the paper on each side of it.

Now the length of the chord

$$\text{of } 49° = 2 \text{ rad.} \sin \frac{49°}{2}$$
$$= 10 \times \cdot 4147$$
$$= 4 \cdot 147 \text{ inches.}$$

Measure off this length on the protractor from the 'diagonal' scale. With one leg of the dividers at x, scratch across the first circle as shown in fig. 133. Suppose it to cut the other circle at Y, prick a hole at Y. Then from F, F Y is the line of direction for N. 49° 0′ W.

Place the straight-edge on the paper with the bevel edge uppermost, and join F with Y. Feel with the point of the pencil whether, when placed at the edge of the straight-edge, it goes through the centre of the holes pricked at F and Y.

Hold the pencil at such an angle that the little finger rests on the straight-edge; and, *keeping it at that angle*, draw the line from one end of the straight-edge to the other. A straight-edge for this purpose should not be less than 18 inches long: a 5-inch protractor is practically useless for such a purpose.

Then F Y is the direction of N.W. This fact had better be noted at the very end of the line, lightly, in pencil.

408. Length in Inches of Paper of Plotting Side.—The distance along the line F Y must now be calculated.

In triangle N C F (fig. 134)—

N = 38° 28′ (see col. 4).
C = 78 00 " " 1
F = 63 40 " " 2

180 00

and C F = 7225: to find N F.

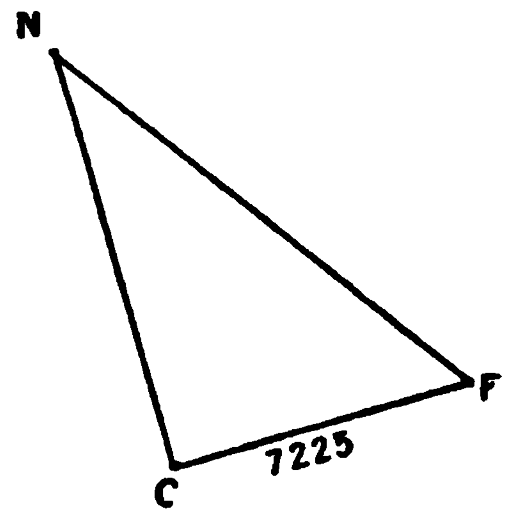


FIG. 134.

N F = C F sin C . cosec N = 7225 . sin 78° 00′ cosec 38° 20′ = 11,344 feet.

As 5 inches = 6084 feet, how many inches = 11,344 feet $= \frac{11344}{6084} . 5 = 9{\cdot}364$ inches? Measure, along the line F Y, 9·364 inches; prick a hole on the line; that will be the position of N.W.

409. Projecting the Main Angles.—Make a triangle round this hole and label it '△ N.W.' All the 'main' angles taken at F will now be projected from this initial line by chords. The first angle mentioned at F is Cairn 63° 40′ N.W.

The circle is already on the paper; there only remains to measure the chord of 63° 40′ (see *Protractor and Chording Angles above 60°*, p. 55, par. 149) to the left of Y.

This will give a line of direction for Cairn.

The same process is carried out for White on the right of Y.

The paper now shows three lines radiating from F, each being labelled.

If the angles are laid off in the same way at N.W. from the line N F, using the same length of radius, there will then be three lines radiating from N.W.: the initial line N F, one to Cairn, and one to White.

Where the two Cairn lines intersect, is its plotted position, and where the White lines meet is White's position.

If the angles are projected without error of any sort, then Cairn and White are relatively quite correct to N F.

But we are never certain that the angles are correctly laid off until the check line is drawn.

410. Projecting Check Lines from the Best Lines of References.—In this particular case it does not matter much which is taken—that is, whether the check line is projected at C or at W, though that from the Cairn is slightly the better.

And if from Cairn, which angle shall be used?

It must be either N C W (62° 54′) from line N C, or W C F (47° 46′, *i.e.* 110° 40′ − 62° 54′) from line C F.

From N C is undoubtedly the better line, because the arc of the circle swept for chording can fall within C N and beyond C W; but C F being nearer than C N, the line C F *produced*

would have to be the zero line, and this is obviously not so correct as C N.

So then angle N C W is laid off from line C N, with a chord of any suitable length of radius so long as it falls short of N and if possible beyond W.

At the beginning of this explanation, the length of the radius was suggested as 5, merely because it was more convenient, inasmuch as $2r =$ twice $5 = 10$; as a matter of fact, since F to N was 9 inches we could or should, for greater accuracy, have used an 8 or $8\frac{1}{2}$ radius.

In the final angle N C W, we still have the option of any radius shorter than the distance C N.

If the line C W goes through the plotted position of W, then not only are C and W correct relative to N and F, but they are also relatively correct as regards each other; hence their position is established.

Put △ round them, and mark them.

These four points are the primary triangulation in this particular case.

In practice there will be many more, but all are plotted in exactly the same way as shown.

411. Plotting Secondary Stations Depending upon Accuracy of Primary Points.—From these four the secondary stations are derived. But all the three angles of the triangle of which they are the apex are not necessarily observed: it will be satisfactory enough to plot them from the angles at the base; *and if the bases of each triangle* (*they are the sides of the main triangulation*) *of which any secondary mark is the common apex are relatively correct* to each other, then that point can be fixed without a cocked hat (see *Cocked Hat*, p. 60). If there is an error, then the points *from which* a secondary station is plotted are in error, or the lines are projected in error; therefore, all possible accuracy should be employed in fixing the first and main points.

To fix Islet there is an angle from C, from F, and from W: one from N also would not assist.

These angles are projected either with the edge of the protractor, or the 'chord scale' on the protractor, or with the chord as before.

It very much depends on their distance from it, and also the receiving angle at it and the accuracy desired. If near, the edge of the protractor might do, but if the receiving angles are bad, then chording must be resorted to.

412. Using the Chord Scale on the Protractor.—In this case we temporise and use the chord scale on the protractor. Let the radius have any value.

Taking the length of the chord of 60° as a radius, scratch this

across the line F N, and for a short distance to the left of F N; because the angle is 13° 04′ to the left of it.

Then measure off the scale on the protractor, the length of the chord for 13° 04′, interpolating as near as possible for the odd 4′; with this distance, sweep across the previous scratched arc, from its point of intersection on the line F N.

Where the scratches cross, is the point of direction for the angle 13° 04′ drawn from F. Repeat the same thing from C; there the angle is 12° 48′ from the N line from C.

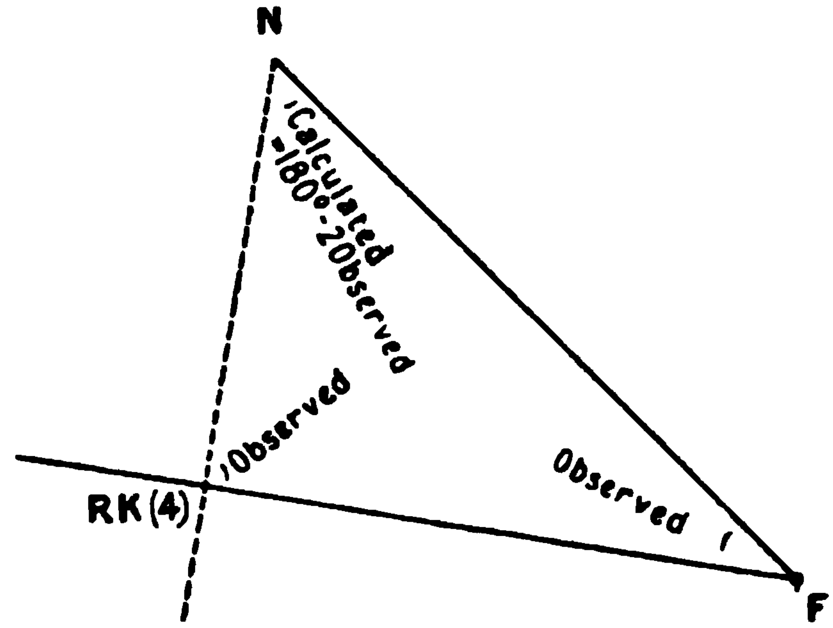


Fig. 135.

The next should be from W; but, though the angle is given from C 33° 32′, it is slightly more accurate to lay it off from N, the angle from there being (62° 54′ − 33° 32′) = 29° 22′; this line of reference being the longer, and making a smaller angle with the line intended to be projected: this applies to all the secondary stations plotted.

413. 'Fixing' an Object by the Calculated Angle to it.— The case of the rock (4 feet) is slightly different. It was only 'shot up' from F and from W (see fig. 137), so there is still the doubt, for the want of a third shot, as to whether the angles were taken correctly, or if they have been plotted correctly. So an angle was taken at Rock (4). See fix (5) 'at Rock (4) N W 120° 00′ Flag.' In fig. 135 N R F = 120° 00′; and since it was observed from F, angle N F R = 27° 40′; and if N R F = 120° 00′, then R N F must = 180° − (120° 00′ + 27° 40′) = 32° 20′. Here, then, is the check angle, *derived* from a *calculated angle*, and this is known as the process of fixing by a 'calculated angle'; and see example, fix (6), where two calculated angles are employed.

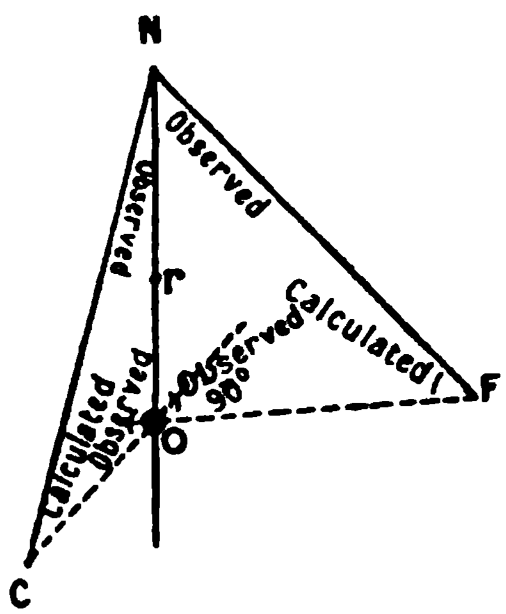


Fig. 136.

Fix (6). This is again a case of fixing by 'calculated' angle, but this time from two calculated angles, one of them being the 'check.'

In fig. 136, O is the observer at the conical buoy.

And N ϕ *r* = 90° 00′ F, therefore N O F = 90° 00′. But F N *r* or F N O is 32° 20′ from the previous fix; and if

$$\begin{array}{lll} & FNO = 32^\circ\ 20' & \text{observed} \\ & FON = 90\ \ 00 & \text{observed} \\ \text{then} & NFO = 57\ \ 40 & (\text{'calculated'}) \\ \hline & \quad 180\ \ 00 & \end{array}$$

and the angle 57° 40′ is projected at F from line F N.

Again, in fig. 136, if O N F = 32° 20′ and F N C = 38° 20′ (see angles at N.W.), then C N O = 6° 00′. And since N O C = 122° 00′ (observed), then N C O, the angle at C between N and O, = 180° − (6° 00′ + 122° 00′) = 52° 00′. This angle gives the check. O being fixed from a line at N.W., O being ⏀ Rock (4), a 'calculated' angle at F, N F O, being 57° 40′ F; and a 'calculated' angle from C, N 52° 00′ O.

414. Usual Method of Fixing Coast-Line Marks, or

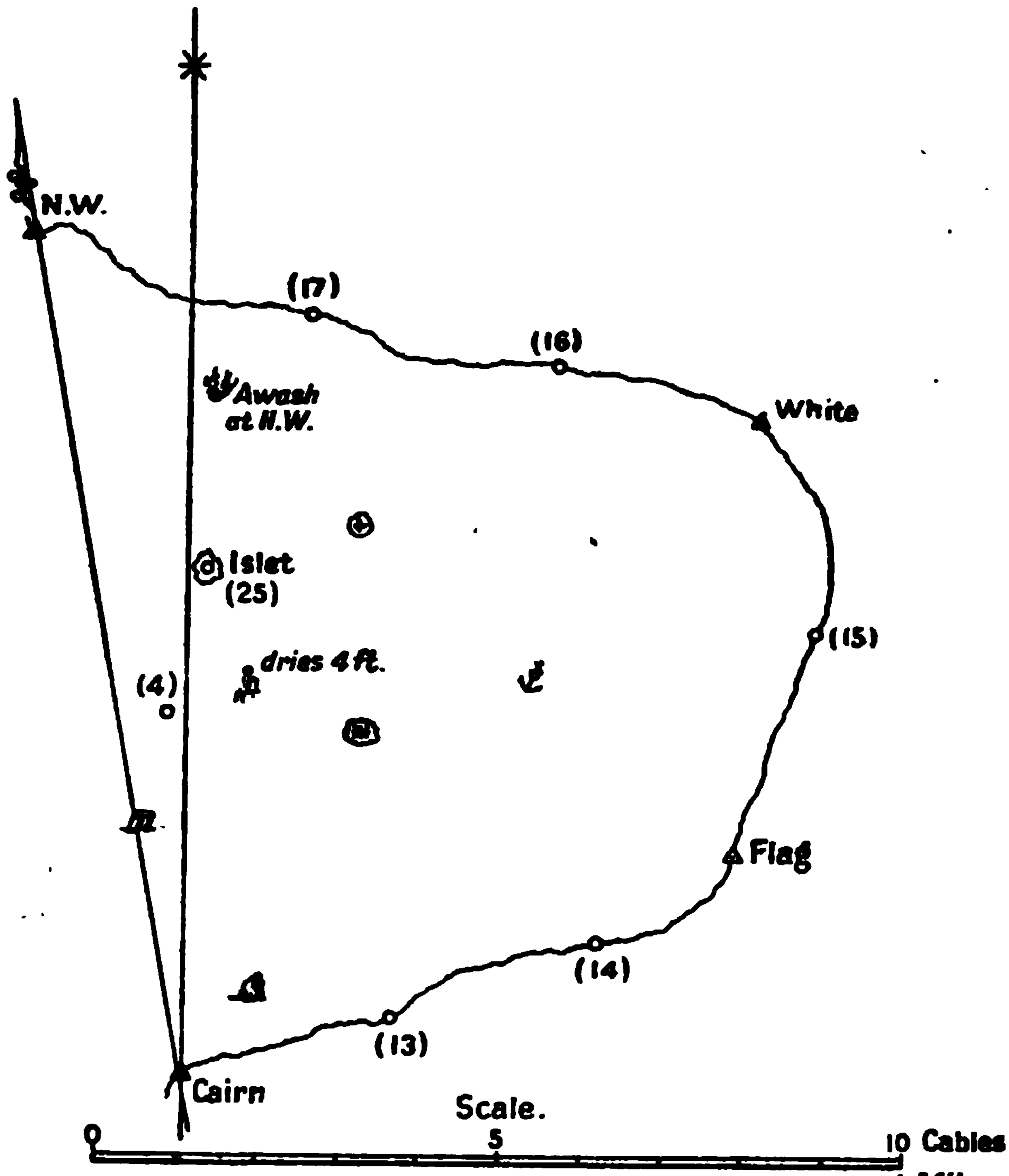


Fig. 137.

Sounding Marks.—This is the usual method of fixing sounding marks and coast-line marks.

There are then first the main △, the three angles of which

are observed, if practicable, and the positions plotted with all possible accuracy; then the secondary stations, ⊙, placed on prominent points of land, or where the best view is obtained to fix other marks, usually 'shot up' from main △ and plotted to the nearest minute; then the coast-line and sounding marks 'fixed,' in the manner shown above by 'calculated' angles (see p. 224).

Then follow 'fixes' as for soundings, and these are illustrated by fixes 7 to 11, using a station pointer.

Fix 12 is a fix at anchor, when the angles are projected on tracing-paper, and the lines so projected made to cover each point mentioned.

The coast-line 'fixes' are station pointer fixes.

415. *Example 2.—Of a small triangulation using a base measured on shore* (see fig. 138).

In the survey of a harbour, a base of 2107 feet has been

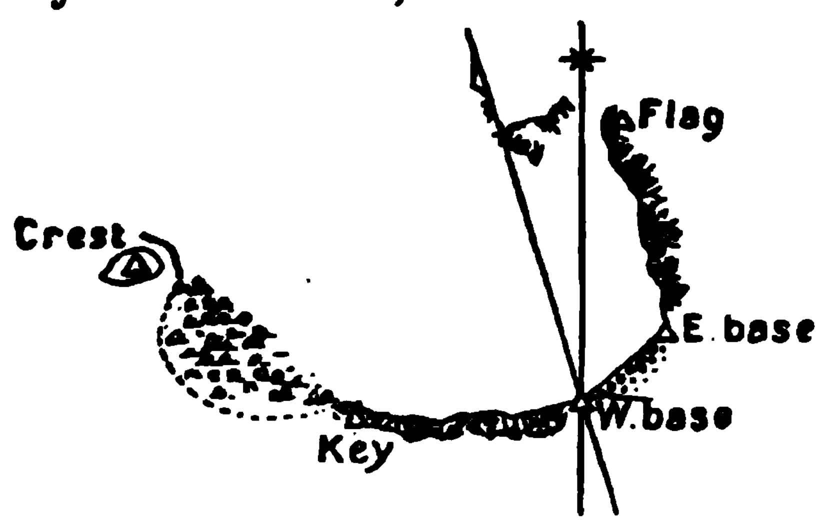


FIG. 138.

measured on a sandy beach (fig. 138). In triangle C E W the following angles were observed:—

Crest . . .	11° 50′
E. base . . .	62 10
W. base . . .	106 00

In triangle C E W, E to W being 2107 feet, calculate the length of W. base to Crest.

This will be the plotting side. The scale adopted is 3·8 inches = 1 mile = 6058 feet, from which is deduced the length in *inches* of the plotting side. The true bearing of Crest from W. base is N. 73° 06′ W., and the variation found to be 17° 55′ W.

These data will give the direction of the true and magnetic meridians, and they are projected through W. base.

Angles at W. base △.	At Crest △.	At Key △.
Crest 66° 32′ Flag	W. base 16° 10′ Key	Flag 52° 48′ W. base
Key 82 18 „	Flag 51 40 „	Rock 61 32 „
Rock 16 36 „	Rock ϕ Flag	

The position of West base △ and Crest △ having been laid

down on the paper, the above angles are projected from them, and the remaining △ fixed.

Crest △ is the summit of a small hillock; the coast from thence to Key △ is fringed with mangroves; from Key to W. base it is cliffy; and beyond the sandy beach from W. to E. base to Flag the coast is steep: the abbreviations are inserted as in figure.

It will be observed that E. base △ takes no part in the triangulation, though, if fixed by angles from Crest and West and the measured distance 2107 feet, it may be useful as a secondary station, or as a mark.

416. *Example 3.—Of a small triangulation using a base by mast-head angle* (fig. 139) for the purpose of illustrating when a measured base will also be the plotting side.

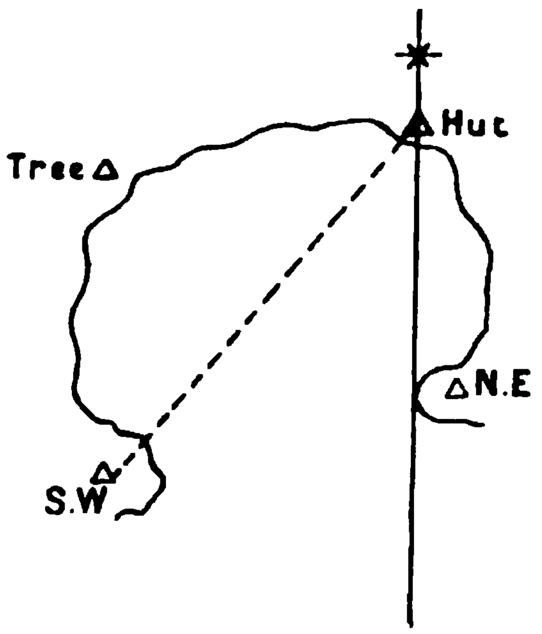


FIG. 139.

From Hut △, S.W. △ is ϕ ship, and bore S. 41° 31′ W. (true).

At Hut △ and S.W. △, angles are taken simultaneously of the ship's mast-head—140 feet—to a water-line mark which is ϕ the mast.

At Hut △, readings 3° 19′ 10″ on,
3° 15′ 20″ off.

At S.W. △, readings 2° 15′ 40″ on,
2° 15′ 00″ off.

Ship's head was south (true); ship's beam, 45 feet.

Half the beam, $\frac{45}{2}$ feet, resolved into the direction of observation, *i.e.* 41° with the direction of the ship's head, $= \frac{45}{2} . \text{cosec } 41°$ $= 34{\cdot}6$ feet.

This 34·6 feet is the horizontal distance from the centre of the mast to the mark on the ship's side used for observation.

Referring to par. 340, and fig. 112, *c*:

$$\text{O N} = \text{O M} . \tan \text{elev. (here O M} = 34{\cdot}6 \text{ feet)};$$

$$\begin{aligned}\therefore \text{ perpendicular N S from S.W. } \triangle &= 140 - (\text{O M} . \tan 3°\ 17'\ 15'') \\ &= 140 - (34{\cdot}6 . {\cdot}057) \\ &= 140 - 2 \text{ (nearly)} = 138 \text{ feet};\end{aligned}$$

$$\begin{aligned}\text{also perpendicular N S from Hut } \triangle &= 140 - (34{\cdot}6 . \tan 2°\ 15'\ 20'') \\ &= 140 - (34{\cdot}6 . {\cdot}04) \\ &= 140 - (1{\cdot}4 = 138{\cdot}6 \text{ feet}).\end{aligned}$$

$$\begin{aligned}\text{Then dist. of S.W. } \triangle \text{ from ship's side} &= 138 . \cot \text{elev. } (3°\ 17'\ 15'') \\ &= 2402{\cdot}5 \text{ feet};\end{aligned}$$

$$\begin{aligned}\text{and dist. of Hut } \triangle \text{ from ship's side} &= 138{\cdot}6 . \cot \text{elev. } (2°\ 15'\ 20'') \\ &= 3518{\cdot}9 \text{ feet.}\end{aligned}$$

Parallax is neglected in both sets of angles, and since the distance of the mast to the mark on the ship's side, to where the elevations were observed, is 34·6 feet in both cases, then the total distances in feet between S.W. △ and Hut △

$$\begin{aligned} &= 2402{\cdot}5 + 3518{\cdot}9 + 34{\cdot}6 + 34{\cdot}6 \\ &= 5990{\cdot}6 \text{ feet.} \end{aligned}$$

The scale adopted is 4 inches = 1 mile, which in the latitude of the place = 6084 feet.

If 6084 = 4 inches, how many inches will represent 5990 feet? = 3·942.

Consider the sides of the paper to be roughly true N. and S., and draw a line, in the direction N. 41° E. roughly, from one end of the paper to the other. Assume Hut △ to be a point on this line near about the centre of the paper; 3·942 inches from Hut △, measured along the line drawn, will be the position of S.W. △. *Hut △ to S.W. △ is the plotting side.*

An angle of 41° 31′, projected by chords at Hut △ to the left of the plotting side, gives the direction of the true meridian through Hut △.

Angles taken at Hut △.	Angles at S.W. △.	Angles at Tree.
S.W. △ 44° Tree	Hut 15° N.E.	N.E. 68° 38′ S.W.
N.E. △ 84° „	Tree 55° „	

At Hut △, Tree will be 44° to the right of S.W.; if this angle is laid off (by chords), the line projected will be the direction of the Tree.

Since Tree is 44° to the right of S.W., and N.E. is 84° to the left of Tree, evidently N.E. must be 40° to the left of S.W., and this angle laid off by chords will give the direction of N.E.

The same process is carried on at S.W. △, the lines projecting giving the direction of N.E. and Tree.

The intersection of the Tree line from Hut and from S.W. will give the position of tree, and the intersection of the N.E. lines gives the position of N.E.

417. *An example here follows of a survey which was carried out by an amateur; the data on which it was constructed were so incomplete, that the work was useless.*

It was desired to survey, and to sound out, the anchorage (see fig. 140). There was a pier *a b* off the point near the village, which suggested itself as a suitable place along which to measure a base. So a base was measured between *a* and *b*.

C, D, E were well-defined natural objects: E a church steeple, D a conspicuous tree at the end of the point, and C the white-washed corner of a building.

There was probably the usual difficulty of securing the

necessary boat and 'hands'; and—to save time and trouble!—it was deemed sufficient to take only the angles at each end of the pier, to C, D, and E. These, according to all the rules of plane trigonometry, were sufficient to plot C, D, and E (see fig. 141), and they were plotted accordingly from the length *a b*.

Undoubtedly the lines from *a* and *b* intersected somewhere; and C, D, and E were considered 'fixed.' With these fixed marks the

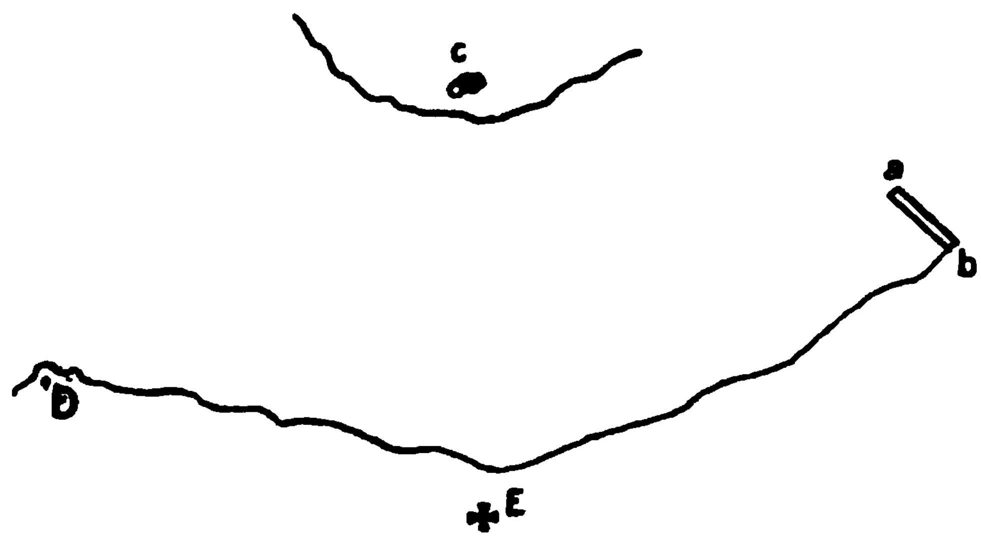


FIG. 140.

harbour was sounded out: but the work was useless. Putting aside errors of plotting into the ridiculously small receiving angles at C, D, and E, and also, for a moment, the small error of observation; it is noticeable that there was nothing to indicate that either of the angles at *a* and at *b* was correct; for, after all, most of us at some time or another read off an angle perhaps 10° wrong.

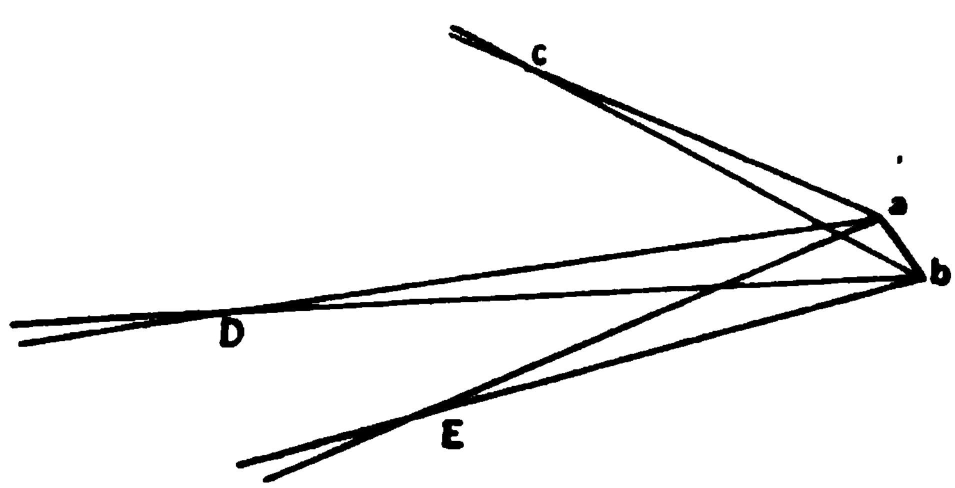


FIG. 141.

The only possible check to the angles taken at *a* and *b*, is the third angle of each triangle. Had the surveyor gone to D, observed the angle *a* D *b*, and then found that the three angles of the Δ *a* D *b* did not make 180° or anything near it, he must have known there was an error somewhere; even if they nearly approached 180°, they still required correcting so that their sum shall = 180° (see *Correcting Triangles*, p. 228). The same remark applies to the triangle *a* C *b*; but, since E is a church steeple,

the third angle of that triangle could not be taken, but other means are found to check this position—see further on.

Without the third angle, there still remain the errors of observation to be dealt with, that is, the amount of the error applied to each angle in the 'cooking' of the complete triangles—a matter of perhaps 5′.

Obviously, if a line is projected from *a* or *b*, and there is a small error in either the *a* or the *b* angle of say ± 5′, C and D will be plotted a considerable distance in error from their true position.

There is, besides this, the error in plotting, and the indeterminate position of the cut of the lines.

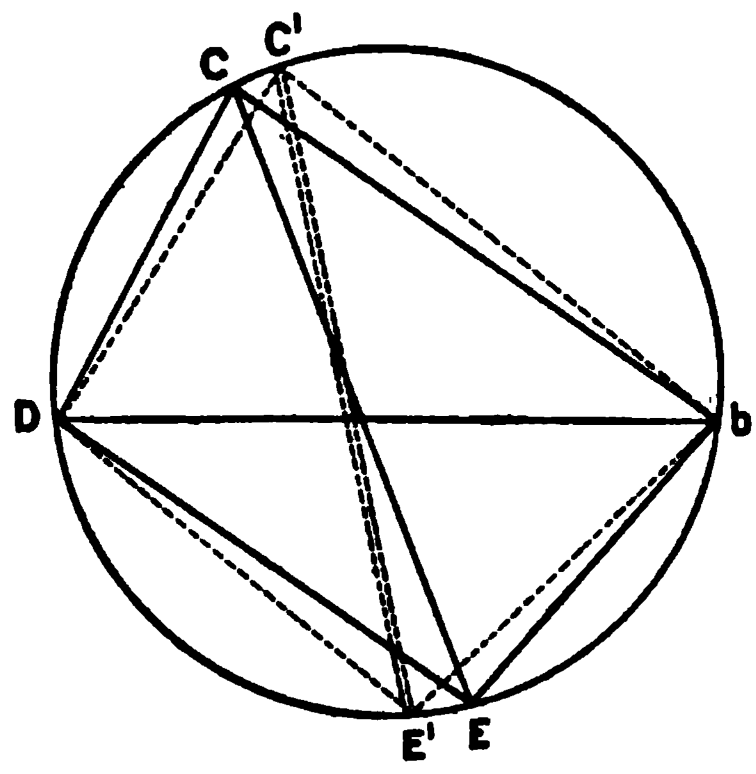


FIG. 142.

Instead of the line *a b*, D to *b* should have been the plotting side: it was quite right to measure *a b* as a base if that was the greatest length obtainable. Then, having observed *all three* angles of the triangle D *a b*, and coaxed them to make 180°, the length of D *b* could have been calculated by plane trigonometry (also see *Example*, p. 205, par. 415). C could then have been plotted from side D *b* with angles C *b* D and C D *b*.

If both the angles from D and *b* are plotted with the same amount of error but of opposite signs, by which a position C is determined (see fig. 142), and the angle at C′ between D and *b*, remains the same as that at C, then C′ and C may possibly be anywhere along the circumference of a circle going through C, C′, D, and *b* (see fig. 142). *So that the third angle of the triangle* D *b* C, *if plotted from* C′, *through* D *or b*, *would not check the accuracy of plotting for either the angles* D *or b*.

In the accompanying figure the hard line shows the correct lines, while the dotted lines are those projected.

By the same argument it may be possible to plot E in error, giving a position E′ from the line D *b*, so that by coincidence its angle, that is D E *b* = D E′ *b*, is unaffected. But it would be a very extraordinary coincidence, if the third check line, projected from C to E, should be also laid off in error; shown as C′ to E′ in figure, making C E *b* equal to C′ E′ *b*. If, therefore, the angle at E ('calculated' in Δ C *b* E) is *also* projected through to C, such errors of plotting as described would be discovered; but it would be better still if the angles at E are observed.

Trusting to luck then, since E is inaccessible, we can do no better. It is fixed from D and b; and the third line, projected from C to E, should go through the previously plotted position of E.

In this case, the angles should be taken from a and b, the ends of the base $a\,b$; *and from* C *and* D, all to each other. Though some of the angles will not be projected, for the reason given above, they should, as a rule, be taken.

In the survey of a plan, the general elementary principles deduced are:—

418. 1. For a plotting side, select two points, one at or near each extremity of the plan. They must be visible from each other, and, of course, be accessible.

How to obtain its length is explained in Chapter I., *Examples* 1, 2, 3, pp. 198, 205, 206.

For its true bearing see p. 107, Chapter XV.

2. Select another *accessible* point, near the outside limits of the plan, and visible from the first two; this, when plotted from them, will have, if possible, a receiving angle between 90° and 30°.

3. On either side of the plotting side, select a fourth point, also accessible if possible, visible from the other three, and yielding a good receiving angle when plotted from the ends of the plotting side.

419. Positions of Main △.—Let A B (fig. 143) be the plotting side; trisect it in x and y; with centres x and y and distance x B, describe two circles; they intersect at C and D. With the same radius, and with centres A and B, describe the small arcs cutting the first circles in E, F, G, H. Then probably all these points will fulfil the best conditions as regards main △; and, generally, the nearer they are to these points, the better are the best conditions, as regards receiving angles, fulfilled. Three △'s must always be visible from each other.

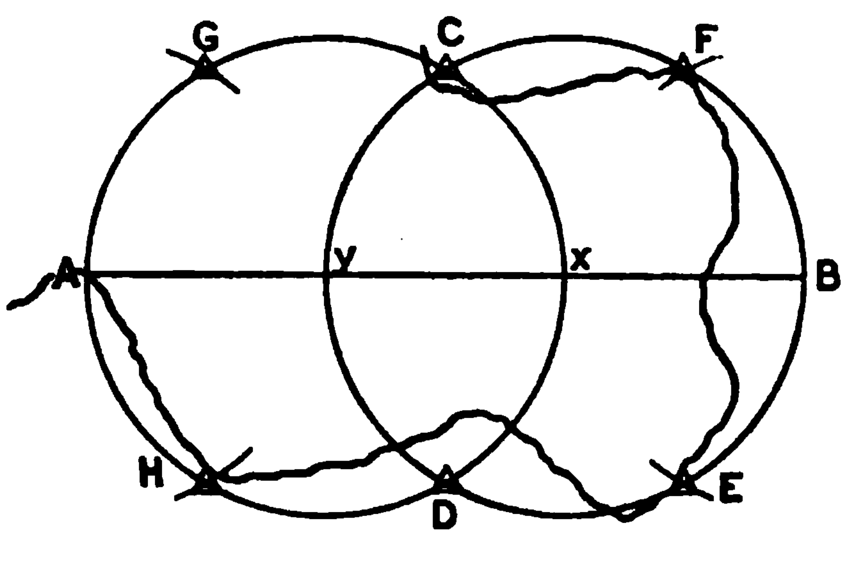


Fig. 143.

These first points constitute the primary or main stations, and since all other stations and marks depend upon them, they are bound to be most accurately fixed.

The more main stations there are, the larger will be the number of angles, and the greater the liability to errors of observation and plotting; the fewer there are the better. As the area of an equilateral triangle is greater than that of any other

triangle, the sum of the sides being equal, consequently the more often this shape is approached the better.

420. Direction of 'Check' Lines.—As regards receiving angles and 'cuts': *The distances of the points from which the lines are projected being equal*, two lines at right angles, shown by hard lines in fig. 144 (*a*), while the *check* (*dotted*) lines make an angle of 45° with either, are the best possible for an error of observation. Otherwise, if the receiving angle is between 30° and 60° (fig. *c*), then the third line should cut either of the other two at 90°.

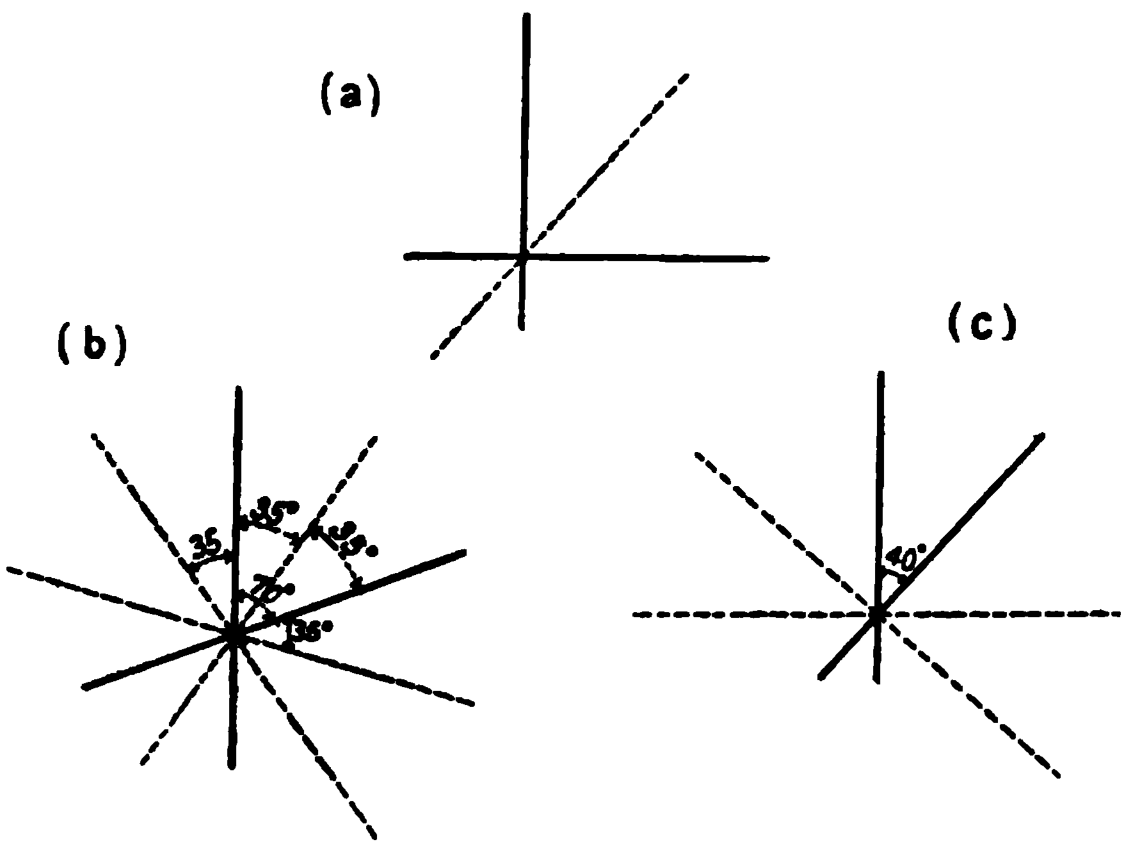


FIG. 144.

If two lines cut each other at an angle between 60° and 90° (fig. *b*), an angle of half the value, from either line, gives the best check.

There is always, however, the consideration of the length of the line to be taken into account (see *Errors of Position for Errors of Observation*, p. 76). A receiving angle of 30° from a distance *d* gives the same results as one of 90° at a distance 2*d*.

421. Distribution of Triangulation into 'Primary,' 'Secondary,' and 'Coast-line.'—The general idea that lies uppermost, is a system of triangles, embracing the outside limits. From the points obtained, secondary triangles, within the first, are plotted to prominent parts of the coast; and within these again, a number of sounding marks, serving the twofold purpose of fixing the coast between the secondary marks, and as marks for station pointer fixing when the soundings and details of the coast are filled in.

422. Plotting 'Inwards' and 'Outwards.'—The outer or larger triangles are taken first; because plotting inwards, as it is called, or plotting the inner marks from the outer, relieves the

errors of the fixed position; whereas working the other way would increase the errors already existing in the points first plotted.

423. When the instruction is given to select two points, etc., the *position for* two points is meant; because it is rarely, if ever, that nature, or the modern builder, provides two marks well adapted for taking angles from, and also serving as a good mark; though a Druid's monolith whitewashed, or a built-up cairn with standing-room for a theodolite or a man with a sextant on the top, would be the perfection of a natural mark for a surveyor.

Marks.—But, generally, marks have to be erected for main stations. The best and simplest form of mark is the tripod, or multipod, constructed like an Indian's wigwam, with any number of branches or poles or legs. After all the angles have been taken, then the tripod is erected over the spot, big enough, and leaving room inside without demolishing it, for a return visit if it should be found necessary. With a dark background, if surrounded by some whitewashed canvas, it will show up splendidly. If on the skyline, and the inside space is filled in with brushwood, it can be seen from a distance of miles around.

424. Marking Secondary △.—A similar mark may be erected for secondary stations, but all depends upon the general tone of the survey, the distances apart, the scale, the accuracy aimed at, and the particular part the secondary station plays. Generally, a secondary station should have a good all-round view, more especially *into* bights and bays, where probably coast-line marks will be required, and who in their turn will requisition the secondary station to fix by.

425. Coast-line Marks.—For the remainder, when no natural marks exist, something must be added to nature, such as a heap of stones, a whitewashed rock, etc.

Flags have the happy knack of blowing out either directly from, or towards, the very person who must see them to enable him to fix his position; or they persistently wind themselves round the staff; under either condition they are exasperating.

When, however, a staff with a flag on it can be of any use, it is at the end of a base while being measured. The same plan is convenient, though aggravating, for re-marking a surveyed harbour, in order to distinguish the particular mark while the operator is fixing additional soundings or otherwise amending the chart; its use on that occasion would only extend over a few hours. Colours, black and white perpendicular, black nearest the staff, will be found best; as then, when blowing out, the white flickers and catches the eye.

Every mark for *coast-line* purposes should be erected at H.W.S.; the additional ones for sounding, are, for convenience, usually erected at a similar height.

Natural objects as marks will, of course, be either in the desired

place, or will be additional and subsidiary to coast marks. For example, a conspicuous bathing machine, pulled up to the H.W. mark, will, in that situation, be a splendid mark. A coastguard flag-staff, when at high-water mark, is also quite what one wants; so will a wreck be, that has the mast standing that is nearest the beach; a conspicuous tree on the beach, decorated with white-washed canvas, is easily 'picked up'; but the canvas may be stolen.

A bush on the beach, decorated with a piece of white canvas, serves well; and in mangroves, a staff or stick tied to a branch, so that its attached pendant reaches over the water, not only distinguishes it well, but the observer can get under it and take his angles. A whitewashed stone or heap of stones, is indestructible by storm or tempest, and may therefore be trusted to 'stand up.'

Further inland, for fixing purposes, many natural and distinctive-looking objects serve as marks.

The centre of a haystack, the door of a barn, a farm-house, a chimney, church steeples, factory chimneys, clock towers, an isolated red or whitewashed house, windmills, etc., are all inaccessible for taking angles from, but may be fixed by taking direct angles to them from three or more fixed marks.

But in the case of the H.W. line marks, they are usually fixed by a combination of one distant shot, and two or more angles calculated from secondary stations; hence the necessity for the secondary $\triangle$ being visible (see par. 424).

426. The Person Erecting a Coast Mark must take Suitable Angles himself, to Fix it.—But as each mark is erected, it must be known how exactly *the erector* is going to fix it; and he cannot depend on his *vis-à-vis*, perhaps on the opposite shore, to wait and watch for him, and to shoot him up, because he himself is occupied with his own affairs.

427. Distance Apart of Marks.—Generally, the coast should have marks, natural or artificial, including secondary stations $1\frac{1}{2}$ inches of paper apart; thus for a plan of 6 inches to the mile they would be $\frac{1}{4}$ mile apart, that is $1\frac{1}{2}$ inches measured on the chart; and their position must be such, that they will serve both as a secondary station, and for fixing purposes from a boat, from the shore, or in fact from any part of the survey.

428. Angles taken at Marks.—It must be borne in mind that at none of the natural marks is the observer at the centre of the mark when he fixes it, and that in such cases he fixes himself at a false station.

Now when taking sextant angles at any time, it is from a false station (as explained in Chapter I., p. 3), though the error in doing so, when the index glass is over the spot, is very small. But when the observer stands a number of feet from the mark, he is deliberately making a false station. In a sextant survey, or in

fixing sounding marks, it is almost accurate enough for all practical purposes, if he measures the small distance he is off the centre, or as near as he can get to it, and from the measurement and direction makes the prick mark as near the centre as he can judge: such an example would happen in the case of a tree being the object fixed. But in laying off his angles to project other lines, he must use the false station.

429. False or Satellite △.—In the secondary stations which are erected, he first stands over the right spot, and then erects the mark over where he stood; but if that station has to be visited again, it might be inconvenient to have to move a substantially

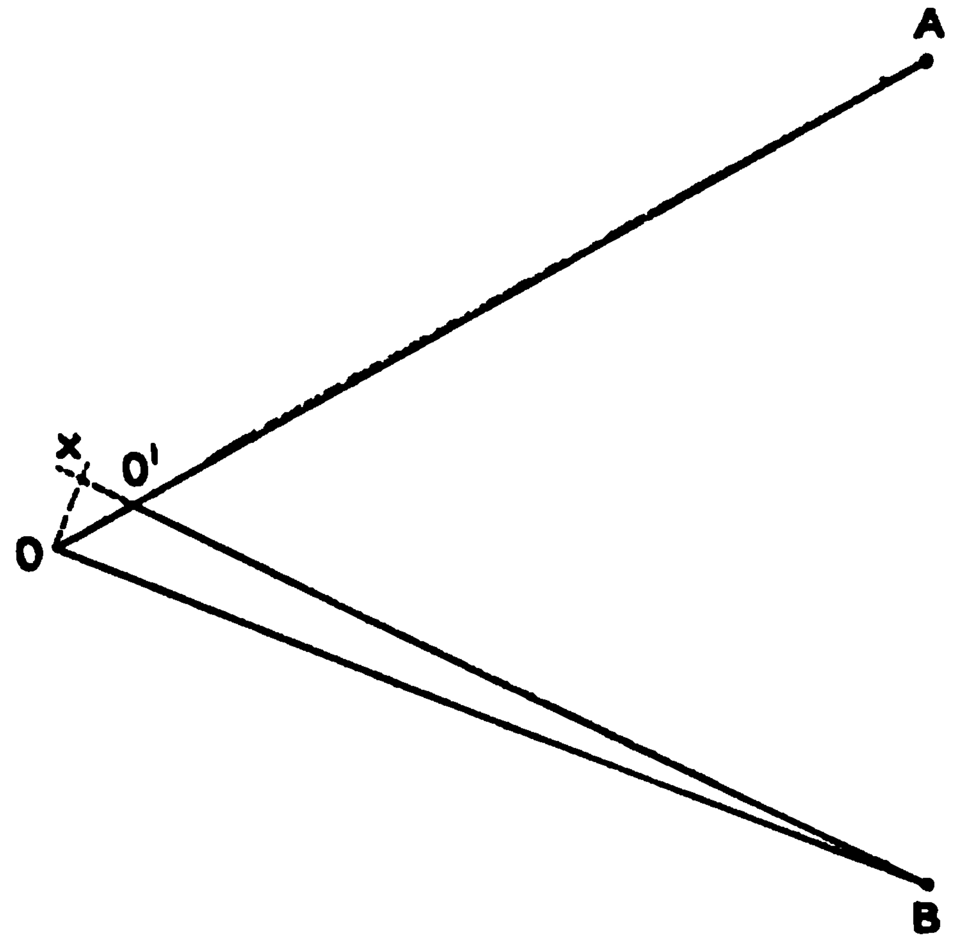


Fig. 145.

made mark; in such a case, greater accuracy is required, since other angles have to be projected from the *true* station.

In this case he observes at a false station, and corrects his angles to their true value, for the centre of the true station.

Take the simplest case. In fig. 145, O is the true station, and O′ the false; O A is the zero line. In this case the false station is made directly in that line. A O′B is the observed angle; suppose it to be 60°, while A O B is the angle required.

A O′ B = A O B + O B O′; therefore O B O′ is the correction required. O O′ is by supposition a short distance as compared to O B or O′ B; and O x, drawn perpendicular to B O′ x, represents the maximum perpendicular distance subtended by an angle O B x; Ox = O O′ . sin O O′ X = O O′ sin A O′ B, the observed angle. From the traverse table (*Inman's Tables*), 5 feet in the upper column and an angle of 60° gives 2·5 and 4·3.

These values are those of the sides opposite the angles of a right-

angled triangle, the greater side being opposite the greater angle. Therefore the side opposite the greater angle $=4 \cdot 3$.

The next question is, What angle does 4 feet (near enough) subtend at a distance equal to O′ B or O B; O B x being small $\frac{Ox}{OB}=$ tan or sin O Bx. This may be calculated from the logarithm tables. But, by referring to the table in the Appendix, the angle can be taken out by inspection, and is the amount of the correction required, in this case, $-$ to the observed *reading* A O′ B; if O′ is on the other side of O, then the correction would be $+$. Above 180° the sign is reversed.

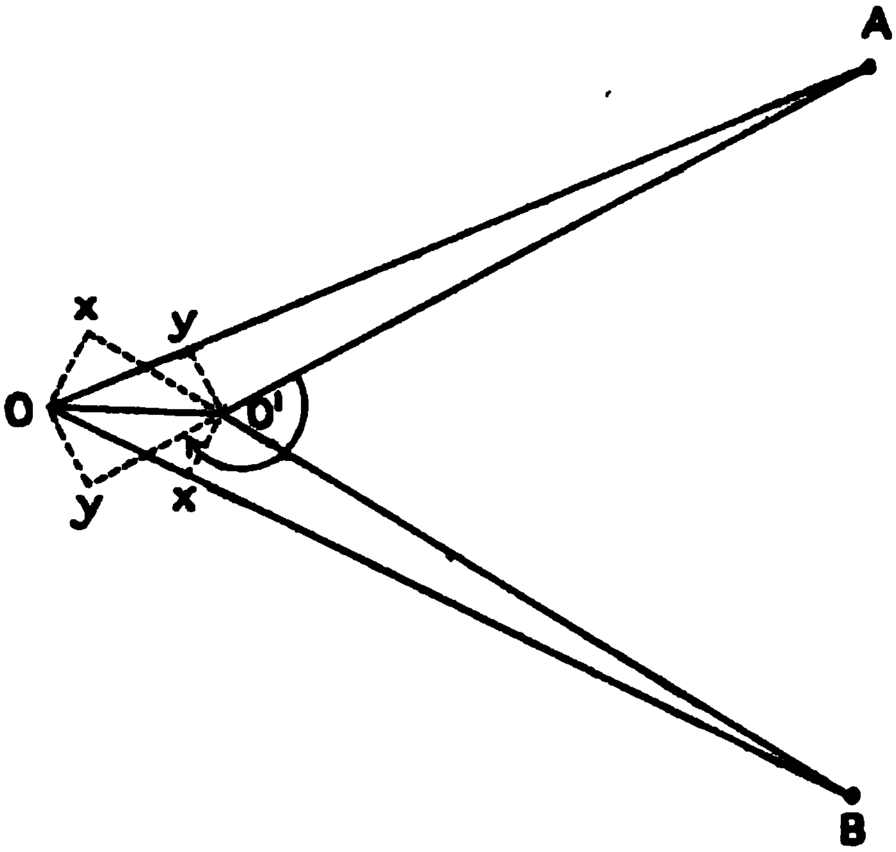


FIG. 146.

B is only a single angle taken from O′; but if there are any more, each would have to be corrected.

Secondly, when the position O′ is not on the line O A, but at O′, as shown in fig. 146, then the correction to A O′ B $=$ O A O′ $+$ O B O′, or the angle at A subtended by O y + angle at B subtended by the perpendicular distance O x.

From A the perpendicular distance is O Y, and it is O X from B. In this case O X $=$ O O^1 sin O O^1 X. O O^1 is measured, and angle A O^1 O observed. Suppose the ⊕ is A; sin O O^1 X $=$ sin 180° $-$ A O^1 O (measured to the right) $-$ A O^1 B $=$ sin 180° $-$ (reading to O $\sim$ reading to B), *i.e.* the sin 180° $-$ difference between the readings to B and O.

If the reading to O is $>$ that to B, the correction is $-$; for obviously reading A O^1 B is $>$ A O B.

Take out the value of O X from traverse table, and proceed as before; find the angle subtended by O X from Appendix, and do the same for O Y; subtract each from the reading observed

A O^1 B. It is always safest to draw the figure before applying the signs + or − ; and it suggests itself, that to save calculation

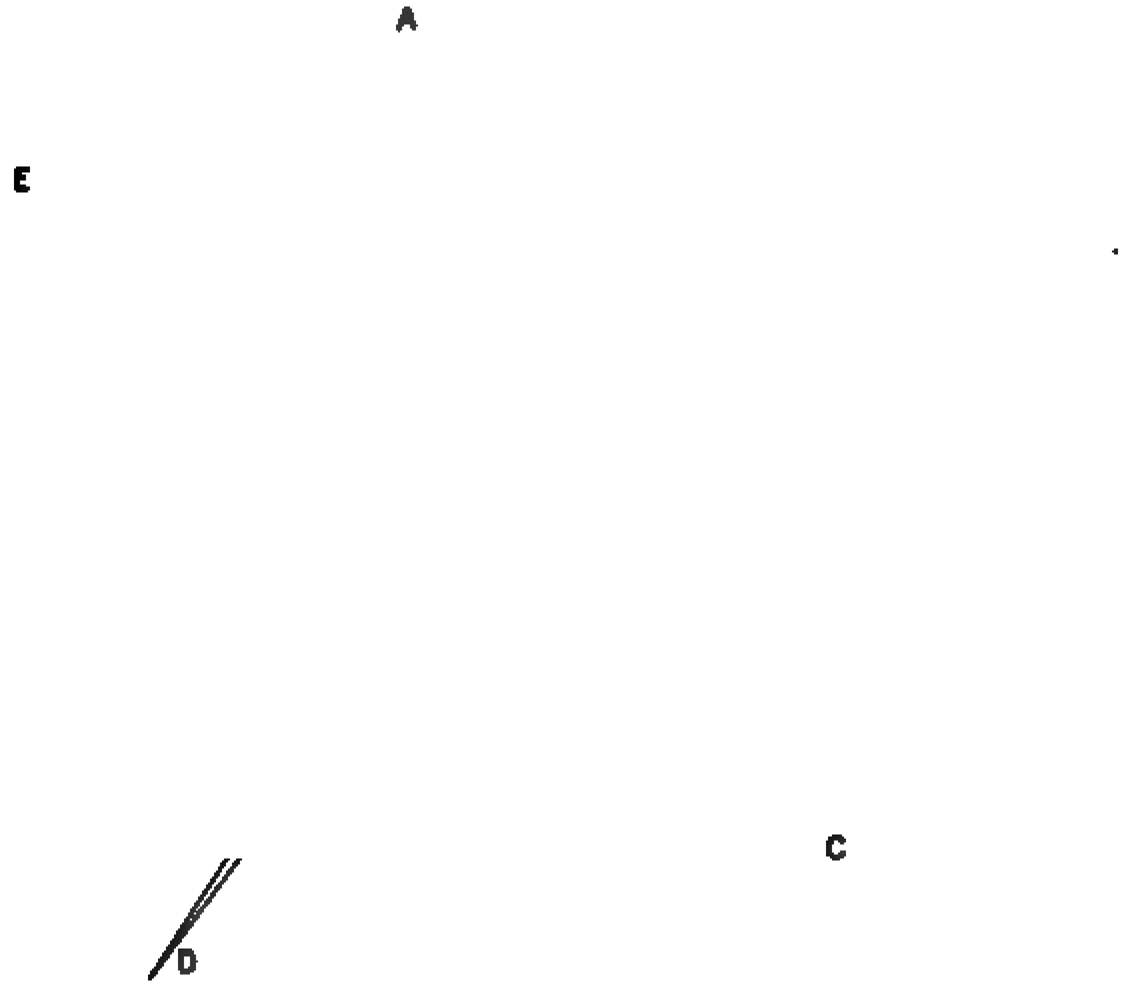


FIG. 147.

the false △ should, if possible, be made on the line O A or O A produced, A being the ⊕.

Example.—At false O^1—

⊕ A	360° 00′	distance	3	miles
B	31 10	"	1·4	"
C	120 53	"	2·3	"
D	220 12	"	2·1	"
E	322 06	"	2·9	"

Distance to mark, 10 feet : angle to mark, 240°.

Required the readings at O △.

1. In △ Y O O^1—

$$Y\,O\,O^1 = (360° - 240°) = 120°,$$

and $O\,O^1 = 10$ feet ;

$$O^1\,Y = O\,O^1 \,.\, \sin Y\,O\,O^1$$
$$= 8{\cdot}7 \text{ feet (traverse table or natural sine table).}$$

∴ angle subtended at A by 8·7 feet dist. 3 miles = 1′ 45″ (see table, Appendix X.).

2. In Δ X O O^1—

X O O^1 = (240° − 31°) = 209°;

O O^1 = 10 feet;

O^1 X = O O^1 . sin X O O^1

= 4·8 feet (traverse table or natural sine table).

∴ angle at B subtended by 4·8 feet at 1·4 miles = 2′.

3. In Δ X^1 O O^1, O^1 X^1 = O O^1 . sin O^1 O X(240° − 120°)^1 = 120°

= 8·7 feet.

∴ angle at C subtended by 8·7 feet at 2·3 miles = $2\frac{1}{2}$′.

4. In Δ O O^1 X^2, O X^2 = O O^1 . sin O O^1 X^2(240° − 220°) = 20°

= 3·4 feet.

∴ angle at D subtended by 3·4 feet at 2·1 miles = 1′.

5. In Δ O O^1 X^3, O X^3 = O O^1 sin . (322° − 240°) = 82°

= 9·9 feet.

∴ angle at E subtended by 9·9 feet at 2·9 miles = 2′ nearly.

To correct the reading A O^1 B : A O B = A O^1 B − A + B

31° 10′ − $1\frac{3}{4}$′ + 2′ = 31° 10′

" " A O^1 C : A O C = A O^1 C − A − C

120° 53′ − $1\frac{3}{4}$′ − $2\frac{1}{2}$′ = 120° 49′

" " A O^1 D : A O D = A O^1 D − A − D

220° 12′ − $1\frac{3}{4}$′ − 1′ = 220° 09′

" " A O^1 E : A O E = A O^1 E − A + E

322° 06′ − $1\frac{3}{4}$′ + 2′ = 322° 06′

From which is deduced:—Take the difference between the observed *reading* and the reading to the true △; with this angle, and the distance off the true △ find the corresponding perpendicular length from the traverse table or from natural sine table, where perp. = dist. sin angle; and with this length find the angle it subtends at the distance of each object observed, from the table in the Appendix. This will be the correction to be applied.

The safest method of applying the right signs is to draw a figure. When O^1 lies between the objects, such as between A and C, or A and D, as in the above figure, the proper sign is obviously − in the first, and + in the second, to the angle A O^1 D, but − to the 'reading' to D. In the other case it is not so evident, but it follows the law: ± A and ∓ B, or ∓ E.

CHAPTER II.

SEXTANT ANGLE SURVEY.

430. The following is an example of the work of one man for one day, with a boat and crew at his disposal: it is a practical illustration, introducing all that has been explained in Chapter I.

Plan of Sucker's Cove.

A Sextant Angle Survey.
Single-handed; time occupied 2 days, ☿ 3.8.08.
A rough shetch is made of the cove (fig. 148).

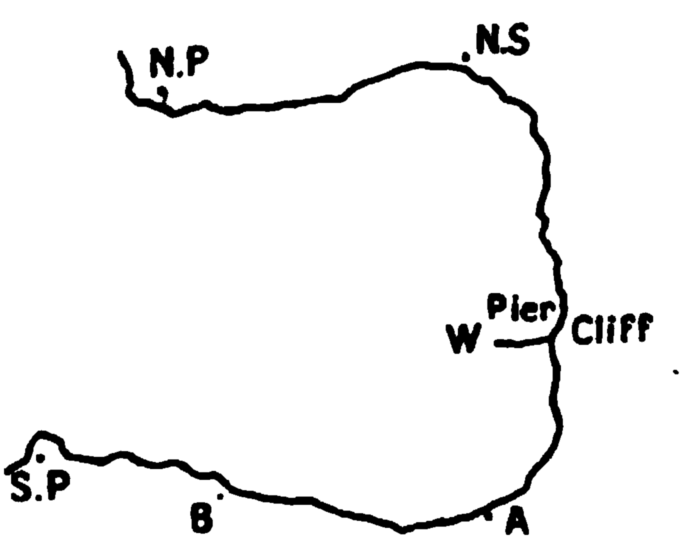


Fig. 148.

It is decided to use N. Sand to B for a plotting side.

The chief of the survey lands at the pier, and decides to measure a base on it, to enable him to find the length of B N.

The boat's crew are sent round the coast to mark it. N. Point is on a small rise, where they cut a number of sticks and make a multipod, filling in the space with brushwood. At S. Point they tie a piece of canvas round a tree and whitewash it; at B, in the mangroves, they tie a stick on to one of the mangroves to extend the branch outwards, and tie a piece of white calico to it.

At A they collect a lot of stones and make a small cairn, and

whitewash it. Calling for the chief operator at the pier, they all proceed to the North Sand, and there make a tripod out of spare oars, and fasten a piece of whitewashed canvas on the front; here the background is trees.

With a landing compass, a magnetic bearing of B is taken from N. Sand, reading S. 24° 30′ W.; the variation is accepted from the published chart as 10° E.

The scale is to be 6 inches = 1 mile = 6075 feet in that latitude.

Angles taken at W end of Pier—the west end of the base.

N.S. 119° 50′ Cliff; B 91° 30′ N.S.

Cliff 148° 36′ B. This angle had to be taken in two bits.

At Cliff (the other end of the base)—

W 55° 35′ N.S.; B 30° 13′ W.

The base from W to Cliff measured 270 feet.

At N. Sand (1).				At B (2).					
Peak	26° 05′	B 23° 10′	S. Pt.	S. Pt.	82° 50′	N.S.	9° 10′	←Village
A	45 30	47 30	N. Pt.	N. Pt.	42 40		22 50	→Vill. tree
W. Pier	60 00			Mill	17 30		28 30	W. Pier
Cliff	64 29						29 59	Cliff
→Vill. tree	77 15						59 10	A
←Village	112 30						A 32 20	Peak

Elevation of Peak to shore horizon, 2° 53′; height of eye, 5 feet.

From the above, the following complete triangles are picked out (for their adjustment and correction for parallax, see p. 227):

(1) N C W.			(2) N B W.			(3) C B W.		
N.S.	4°	30′	N.S.	60°	00′	Cliff	30°	01′
Cliff	55	40	B	28	30	B	1	29
W	119	50	W	91	30	W	148	30
	180	00		180	00		180	00

Angles taken at A (3).

N. Pt.	36°	20′	N.S.	12°	10′	←Village
B	75	20		16	00	W
Mill	10	20		20	20	→Village Φ Cliff

431. **Making a False △.**—From A, S. Point was not visible, and a false station had to be made 20 feet in the direction of N.S.; S. Point 62° 10′ N.S. All the above angles are taken with an ordinary sextant: that is to say, not a first-class observing sextant nor a sounding sextant, but the instrument that is used ordinarily for sight-taking at sea.

After adjusting it carefully, it is preferable to remove all the index error, or to reduce it to a minimum.

The short telescope was used throughout; and, in the case of the overhanging bough from the mangrove tree, the observer carefully placed himself as near as he could under the mark.

After fixing a number of points along the coast, the H.W. line was sketched in between them. Each place was marked by a daub of whitewash on a few stones heaped up.

Fix (1)	N.S.	103° 50′	B	88° 20′	N. Pt.	to N. Pt. rocky coast.
(2)	N.S.	115 53	B	95 30	S. Pt.	to Fix (1) shingly beach.
			S. Pt.	22 30	Fix (1)	to N. Sand shingle.
(3)	A	47 10	B	45 18	N. Pt.	to N. Sand sand.
			S. Pt.	44 30	Fix (2)	to Cliff sand.
				25 15	Fix (1)	
(4)	B	79 40	N.S. ⏀ W. Pier			to Cliff is cliffy, about 50 feet high. to A is cliffy.
(5) a mangrove tree.	B	93 55	N.S.	75 20	A	mangrove to A and B.

At B .	N. Pt.	11 30	Fix. 1
	„	33 50	„ 2
	„	59 05	„ 3
	„	82 55	„ 4
	„	107 10	„ 5

Fix (6)	N. Pt.	39 50	N.S.	to S. Pt. low rocky coast.
	„	69 20	Cliff	to B low rocky coast.
	„	93 50	A	
	S. Pt.	48 10	N. Pt.	

Fixes 1, 2, and 3 must be fixed by 'calculated' angles.

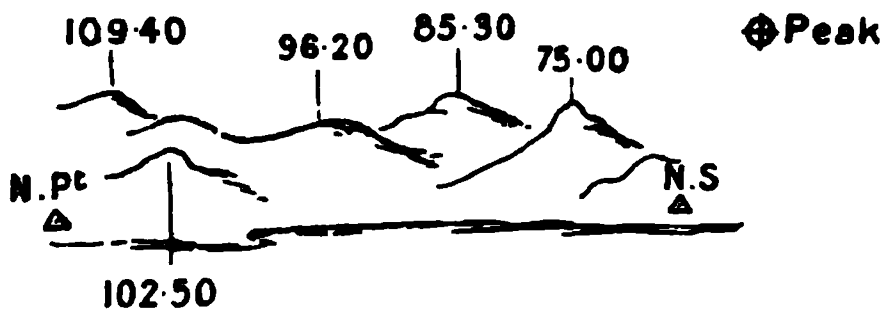
Fixes 4 and 5 must be fixed by station pointer.

Fix 6 must be fixed by tracing-paper.

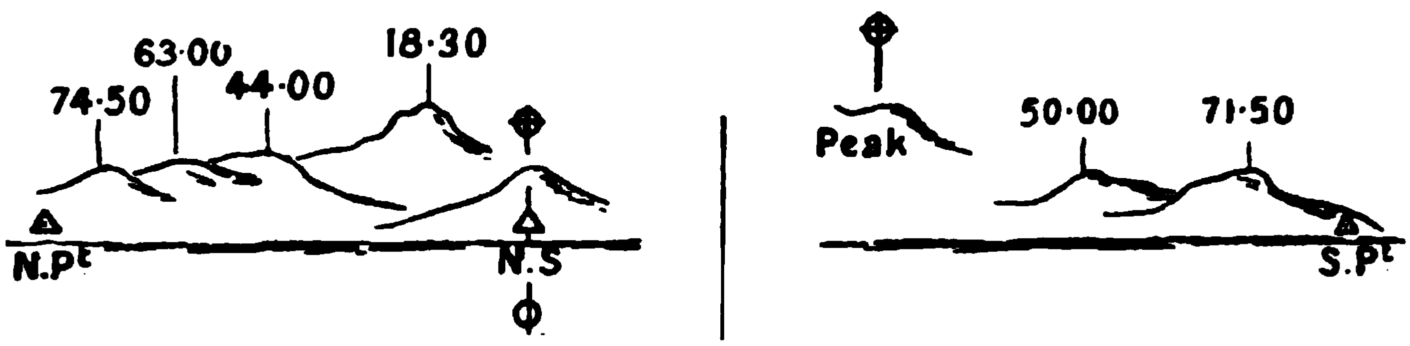
Later the Cove was sounded out; the tide-pole being nailed to the pier.

432. Sketches for Topography.—The following sketches were made, to fix the summits:—

A. At South Point.



B. At Ship. N.S. 62° 30′ Cliff 117° 30′ B.



C. At Boat. N. Pt. 65° 00′ S. Pt. Cliff 61° 50′.

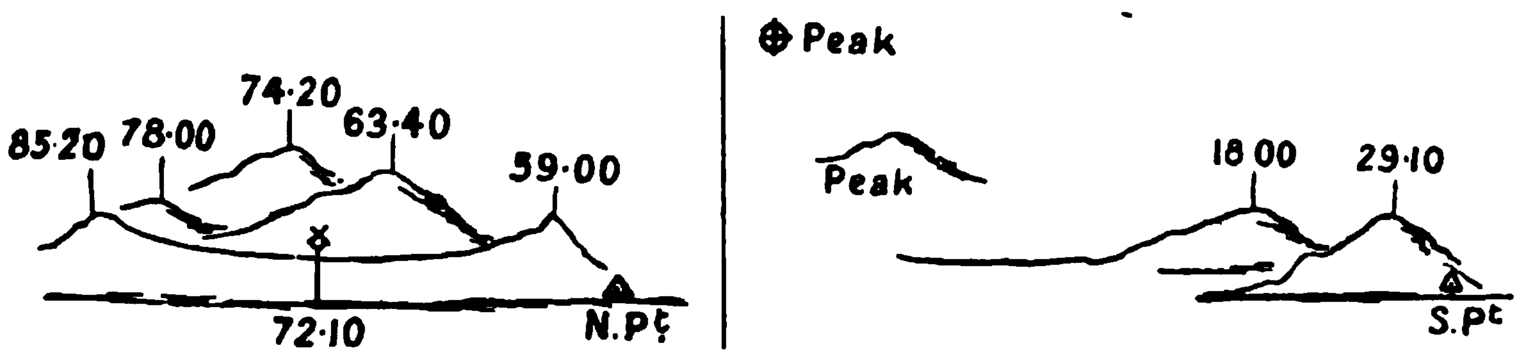


Fig. 149.

Method of plotting the above example (the harbour is about 2 miles across):—

Since B to N. Sand is the plotting side chosen, we must find its proper place on the paper. The true meridian may be drawn through either, but N.S. is the proper one.

It is about $\frac{1}{4}$ of the width of the plan from the right edge.

Then, leaving room for any details to the right of it, draw the true meridian the whole length of the paper, roughly parallel to, and about 3 inches from, the right edge.

Label it with ✱.

On this line, the position allotted to N.S. will be pricked (see sketch, p. 218): allowing a space above it for more details, and for a similar space south of B, take N.S. about $\frac{1}{4}$ of the way down the paper. This gives the position of one end of the line.

Now B bears from N.S. S. 24° 30′ W. (Mag.) = S. 34° 30′ W. (true), the variation being 10° E.

From N.S., with a radius of say 5 inches, scratch a circle, as far as the paper admits; on this circle, and measured from its intersection with the true meridian, chord the bearing S. 34° 30′ E. Join N.S. with the point of direction so found, drawing the line from one end of the paper to the other—B is somewhere on this line.

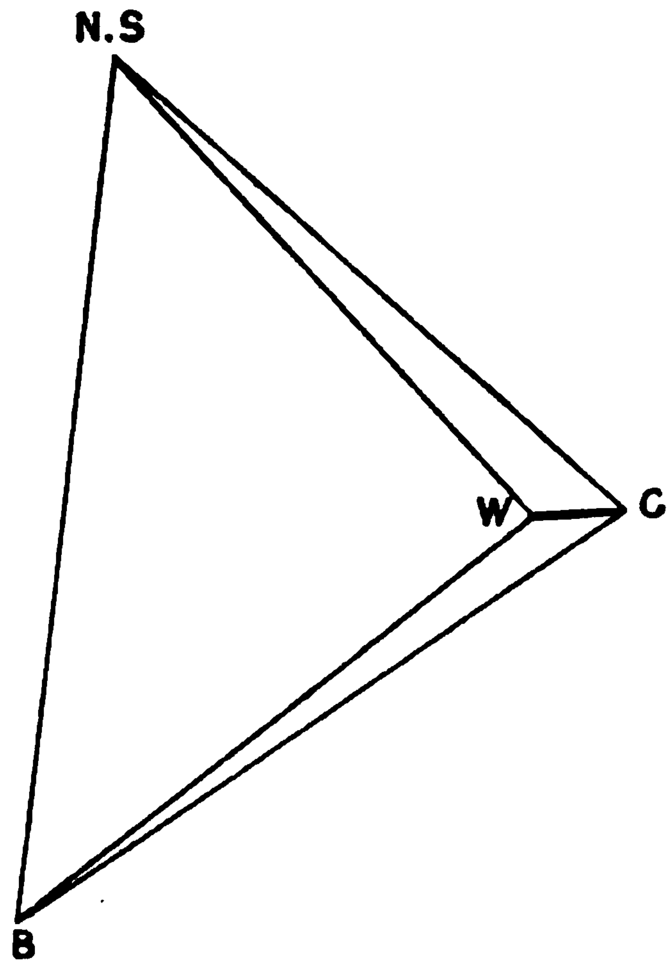


FIG. 150.

As to the length of N.S. to B:

In Δ N C W (fig. 150),

$$\left.\begin{aligned} WC &= 270 \text{ feet.} \\ N &= 4^\circ\ 30' \\ C &= 55^\circ\ 40' \\ W &= 119^\circ\ 30' \end{aligned}\right\}\text{ See pp. 219, 227.}$$

Then

$$\begin{aligned} NW &= WC \,.\, \sin C \,.\, \operatorname{cosec} N \\ &= 270 . \sin 55^\circ 40' . \operatorname{cosec} 4^\circ 30' \\ &= 2841{\cdot}7 \text{ feet.} \end{aligned}$$

In Δ N W B,

N W is found = 2841·7 feet, and since

$$\left.\begin{aligned} N &= 60^\circ\ 00' \\ W &= 91^\circ\ 30' \\ B &= 28^\circ\ 30' \end{aligned}\right\}\text{ See p. 219.}$$

Then N B = N W . sin W . cosec B.

$$= 2841{\cdot}7 \sin 91^\circ\ 30' \operatorname{cosec} 28^\circ\ 30' = 5953{\cdot}4 \text{ feet.}$$

N B calculating 'through Δ N W C' = 5953·4 feet.

If all the angles taken were quite correct, which they are not, then the side N B, calculated through W B obtained from Δ B W C, should give the same result as before. Then calculating through W B:—

In Δ B W C,

$$\left.\begin{aligned} WC &= 270 \text{ feet} \\ B &= 1^\circ\ 29' \\ W &= 148^\circ\ 30' \\ C &= 30^\circ\ 01' \end{aligned}\right\}\text{ See pp. 219, 227.}$$

And

$$\begin{aligned} WB &= WC \sin C \operatorname{cosec} B \\ &= 270 \sin 30^\circ\ 01' \operatorname{cosec} 1^\circ\ 29' \\ &= 5217{\cdot}8 \text{ feet.} \end{aligned}$$

In Δ B N W,

$$\left.\begin{aligned} WB &= 5474{\cdot}2 \text{ feet} \\ N &= 60^\circ\ 00' \\ W &= 91^\circ\ 30' \\ B &= 28^\circ\ 20' \end{aligned}\right\}\text{ See p. 219.}$$

Find B N.

$$\begin{aligned} BN &= BW . \sin W . \operatorname{cosec} N \\ &= 5217 \cdot 8 \sin 91° \ 30' \operatorname{cosec} 60° \ 00' \\ &= 6009 \text{ feet.} \end{aligned}$$

Now there are two values for B N: 'through' N W it = 5953·4 feet, and 'through' B W it = 6009 feet. The mean of them will be accepted, = 5981·2 feet (see note, p. 227).

The scale adopted is 6 inches = 6075 feet, therefore 5981·2 feet $= \frac{5981 \cdot 2 \times 6}{6075} = 5 \cdot 907$ inches.

Measure this length along the line N B, from N; and prick the position of B.

From N, and from B, project, by chords, all the main angles; but not such an angle as the ⟵ or ⟶ of the village; these can be projected by the edge of the protractor, if the protractor is of a reasonable size. This will give two lines through all the positions.

Then, finally, project the angles from A, also by chords—and making use of the longest zero line, that is the longest line that has already been plotted through A.

There is a complication about the angle to South Point, on account of the false station; and the angle taken at false A must be reduced to the true station.

433. Calculating Angle from False △ without Special Tables.—In fig. 151, A is the true station; A′, the false station. The angle observed is N A′ S, and N A S is required.

$$N A'S = N A S + A S A'.$$

$$\therefore \quad N A S = N A'S - A S A'.$$

(A S A′ is the correction.)

Draw $A'x$ perpendicular to S A.

Then, in the right-angled △ A $A'x$, angle x being the right angle, x A′ = A A′ sin x A A′ = A A′ sin N A S; and since the error is small, N A′ S is practically = N A S.

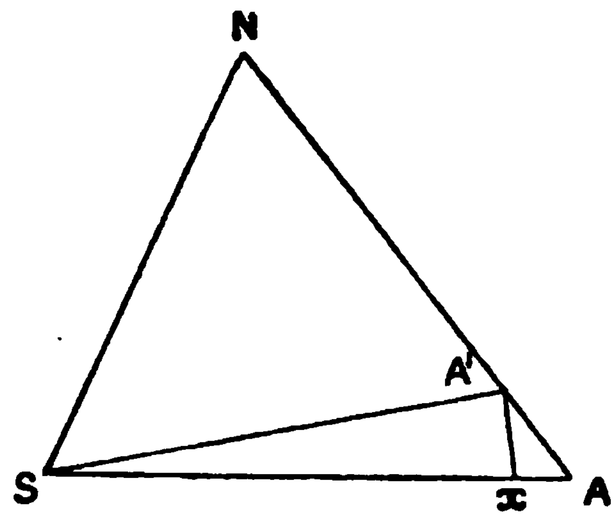


FIG. 151.

From the traverse table, or from the natural sine table, with distance A A′ and angle N A′ S, find $A'x$. In this case A A′ is 20 feet and N A′ S 62°. $A'x = 20 . \sin 62° = 20 . \cdot 88$. $\therefore A'x =$ 17·7 feet.

The angle of correction is A S A′, and the value of it is the angle subtended by the perpendicular $A'x$ (17·7 feet) at a distance A S or A S′. To calculate it without special tables:—

$\frac{A'x}{A'S} = \sin S$. The traverse table does not deal with anything

less than 1°, and is not suitable for this, the amount being too small.

Since A′S = 1 mile (approximately), then $\frac{A'x}{A'S} = \frac{18}{6000}$ (nearly) = ·003. Find the angle whose sine is ·003 (see nat. sine table, *Inman's Tables*); and A S A = 10′.

In this case N A S = N A′S − S, and the measured angle is too large; therefore the angle at A will be 62° 10′ − 10′ = 62° 00′. Project this angle from A; then all the main stations are plotted and labelled in ink.

Coast Fixing.—The observer goes to fix (1), makes his mark, and fixes by two angles, from N.P. to B and from B to N.S.

Now this is a very doubtful station pointer 'fix'; so doubtful that he does not plot it with station pointers. He moves on to (2), and there is the same difficulty there with regard to fixing

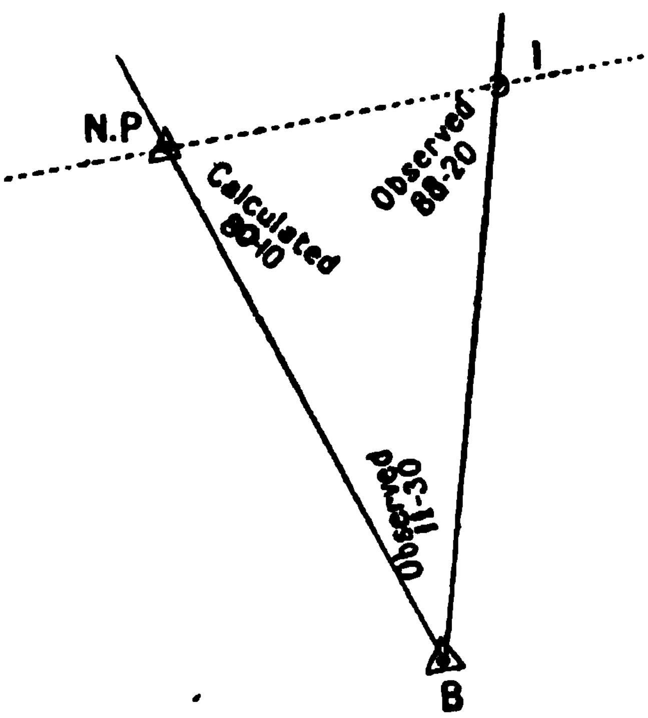


FIG. 152.

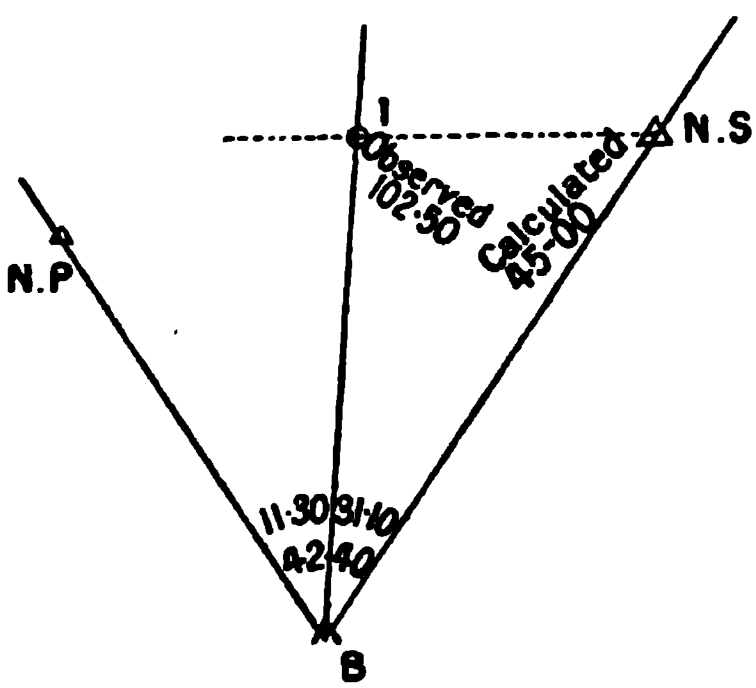


FIG. 153.

marks; in fact, it is worse than (1). At (3) he finds the same bad 'fixes,' though he takes the angles.

At (4) and (5) they are sufficiently satisfactory, and these are plotted with station pointer.

Having walked on to B, he now takes the necessary angles to fix (1), (2), and (3), by *calculated angles.*

At (1) the fix was N.S. 103° 50′ B 88° 20′ N. Point. Dealing with the right angle first.

At B, N.P. 11° 30′ fix (1). Therefore, in fig. 152, the angle at B is 11° 30′. Project this line.

At (1) the observed angle between N.P. and B is 88° 20′.

Therefore the angle at N = 180° − (88° 20′ + 11° 30′) = 80° 10′. Project the angle 80° 10′ at N, from the line N B, and where it cuts the line projected from B, will be the position of (1).

Now dealing with the left angle (see fig. 153):

Referring to the main triangulation; at B, N.P. 42° 40′ N.S.
and since at B, N.P. 11° 30′ fix (1),

then, at B, fix (1) to N.S. $= 42° 40' - 11° 30' = 31° 10'$.

And since the angle at fix (1), between B and N.S., is 103° 50′, then,

the *calculated angle* at N.S., between B and fix (1), $= 180° - (103° 50' + 31° 10')$
$= 45° 00'$.

At N.S., from the line N.S. to B, project the angle 45° 00′, and this line should intersect, where the others did before, at fix (1).

This is fixing by calculated angles; and, as previously stated, every one must select and fix his own marks.

The edge of the protractor is used for projecting these calculated angles; and, always provided the main or secondary points are accurately fixed, there will be no 'cocked hats' with the subsidiary marks that lie 'within' them. Fix (2) and fix (3) are plotted in an exactly similar manner.

Fixes (4) and (5) need no explanation; the angles are placed on the legs of the station pointer, applying the necessary corrections for the errors of the legs.

Notice that at each fix, the last fix back was shot up, as a check; and, if the plotting has been carefully done, there will be but a very small error in the positions—too small, probably, to affect the subsequent operations of fixing sounding positions.

Fix (6) was fixed by tracing-paper. Take a sheet of tracing-paper about the size of the plotting-sheet.

With a rough idea of one's position, make a pencil dot, on the tracing-paper, at the spot it will occupy on the plan.

Through that, rule a line roughly in the direction of N.P.; because all the angles were taken with N.P. as the initial point.

Place the centre leg of the station pointer along this line; and with the right leg set off each reading, and rule along the edge of the leg.

The first angle is 39° 50′ N.S.; move the right leg to 39° 50′, and rule along its edge for the whole length; label the end of the line N.S.

The next is 93° 50′; label that line A: and finally rule the line S. Point with the left leg of the pointer laid off 48° 10′, and label it S.P. Remove the station pointer, and manipulate the tracing-paper so that all the lines drawn shall go through the objects named on them. Prick through at the position of the dot.

This completes all the positions; the coast-line has been, on this occasion, sketched in between the fixes, and the nature of the coast inserted (see Plate II.).

There remain the summits, and the topography of the hills.

Three sketches are obtained of the summits; and the artist making them must, when he is plotting, be able to identify the summits in each view; the angles are laid off to the various hills, and the valleys are sketched in.

This is not altogether so easy as it looks; but the sketches accompanying this example are very simple ones.

Two of them are taken from nearly the same point of view, and from these views there can be no doubt which summit is which.

The third sketch, from the boat, is a side shot at them, and, unless very carefully plotted, will create a chaos of lines.

Finally, there is the elevation taken from N.S. of Peak to the shore H.W. line in line with Peak.

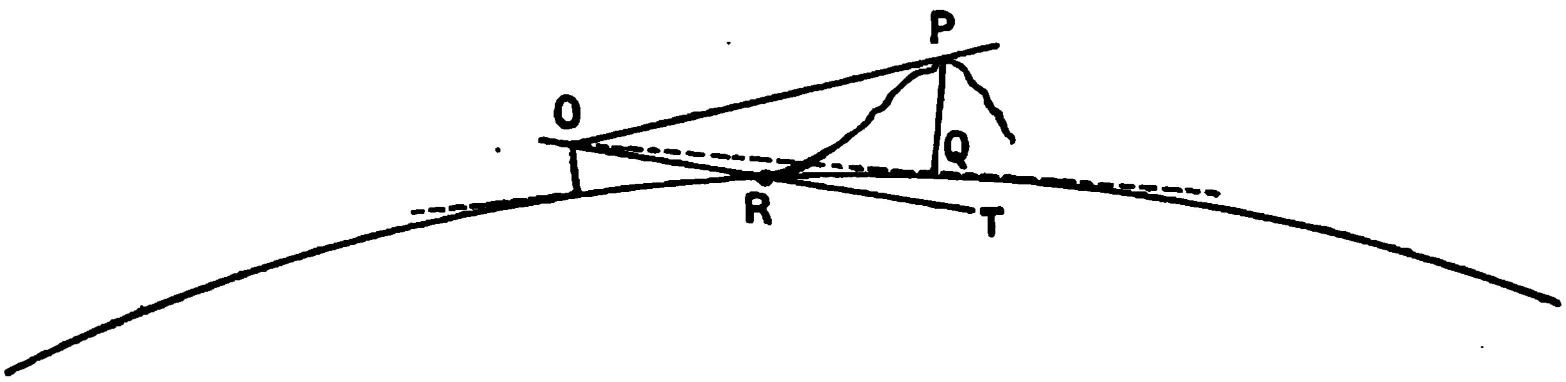


FIG. 154.

To calculate the height:

See fig. 154—P Q (height) = Q O . tan P O Q = O Q . tan (P O T − Q O T).

P O T is the observed angle of elevation.

Q O T is the correction for dip, for shore horizon; this, for a height of 5 feet, and for a distance O R, as measured off the chart, $= 3'$.

$\therefore$ P O Q $= 2^\circ\ 53' - 3' = 2^\circ\ 50'$.

Q is the point perpendicularly under P, and O P measured on the chart represents the distance O Q.

With the above quantities, find P Q.

P Q is the height above O; and, since O is 5 feet above H.W., the height of P above H.W. = P Q + 5.

P Q will be found = 350 feet; therefore the height of the Peak above H.W. is 355 feet.

The sounding was carried out the next day.

Everything now being in readiness, the tide-pole readings being registered from hour to hour, and the 'points' on the plotting-sheet all marked and named, having been pricked through to another sheet that is pasted on to a board: pasting is suggested, because drawing-pins will always be just where the station pointer leg is required to be. To complete the one day's work, the magnetic meridian may be drawn on the paper; dis-

TO [illegible]

[illegible]

tinguished by the usual symbol, half a spear-head; copy one from a published chart.

When all is complete, soundings and all, then the points plotted on the original sheet are pricked through on to a clean sheet of paper underneath; the coast-line is transferred from where it was plotted, to the fair sheet, by tracing-paper, and the details filled in from the original; the same is done with the soundings, as also the hills and topography.

Everything is inked in, and, if desired, painted according to the usual custom, as laid down in Wharton's book.

Both true and magnetic meridians are inserted; do not forget the scale; and, lastly, the title (see p. 189, par. 381).

Then submit all matter called for, according to the regulations. Anyone must see the necessity of these bona fides, if the work is to be authorised by the Admiralty.

Adjustment of angles taken for the purpose of extending the base to the plotting side. (For observed angles, see p. 219; for method of correction, see p. 232.)

(1) Δ N C W.

Obs. Angles.	Corrn. for Parallax.	Corrd. Angles.	Propl. Adjt.	Corrn.	Corrd. Angles.
N 4° 29′	0′	4° 29′	9(100 + 100) = 18	+ 1′	4° 30′
C 55 35	0	59 35	100(100 + 9) = 113	+ 5	55 40
W 119 50	− 6	115 44	100(100 + 9) = 113	+ 6	119 50
		179 48			180 00

(2) Δ B C W.

Obs. Angles.	Corrn. for Parallax.	Corrd. Angles.	Propl. Adjt.	Corrn.	Corrd. Angles.
B 1° 29′	0′	1° 29′	9(180 + 180) = 3	0′	1° 29′
C 30 09	− 2	30 07	180(180 + 9) = 34	− 4	30 01
W 148 36	0	148 36	180(180 + 9) = 34	− 6	148 30
		180 12			180 00

By rough calculation, N W nearly = N C = 1000 yards, and B W nearly = B C = 1800 yards.

Note.—The discrepancy in the length of the side N B, calculated through N W and B W, arises from the smallness of the angles at N and at B; wherein an error of observation makes a considerable error in the length of the sides calculated from these angles; they should be observed with the greatest possible care, or *additional* triangles can be created, through which N B will be calculated—not forgetting that the more triangles there are, the more the errors accumulate.

CHAPTER III.

ERRORS IN OBSERVING ANGLES AND ADJUSTMENT OF THE ANGLES OF A TRIANGLE.

434. Correcting the Observed Angles to make them up to 180°.—When it is practicable, and also when all possible accuracy is aimed at, all three angles of the 'primary' triangles should be taken. Every angle observed will be in error (see *Errors of Observation*, Part I., p. 82), and by taking all three angles, it may be possible to reduce the probable error in each angle.

For example, take an equilateral triangle A B C (fig. 155). The lengths of the sides are all equal, and the angles are all equal.

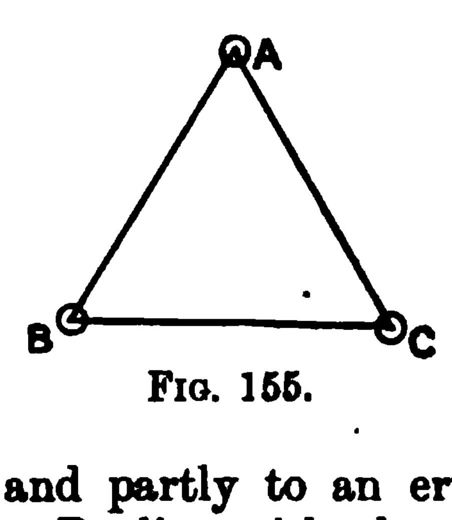


FIG. 155.

The same observer going to A, B, and C with the same instrument, will not make his three angles, when added up, equal to 180°; they may be so much short or so much over. The error is partly instrumental (see *Errors in Theodolite*, p. 121, par. 261, and p. 151, pars. 323, 324; and *Sextant*, Part I., p. 82); in a sextant it is partly due to a 'false station' (see Part I., p. 2), and partly to an error in observing.

Dealing with the observation error in any one angle, this may arise from the theodolite not being level; or to a slight displacement of it while observing, or that the observer is either not over the point of the triangle, or has not taken the other point correctly. Supposing the theodolite error to be consistent, it is then a question of observation only.

If the points of the triangle are represented by a spot of the smallest conceivable dimensions, then from each such spot, each of the other spots should be observed over the centre of the cross wires of the telescope.

But, in practice, the points of the triangle are not infinitely small and well-defined spots, and not always very distinguishable and definite large ones: the centre of the cross wires encloses

a definite area, even with the finest web, and it is in locating, and in placing the centre of the cross wires over this spot, that the error in observing creeps in: moreover, the angles are not taken exactly *from* each spot. Δ A B C, fig. 155, represents such points, and the observer is supposed to have stood anywhere within any one ring, and taken the angle to any part of the other rings.

A large spot observed in error at a long distance, will subtend a smaller error than the diameter of the same large spot if observed from a shorter distance (see p. 122, par. 265). Therefore, referring

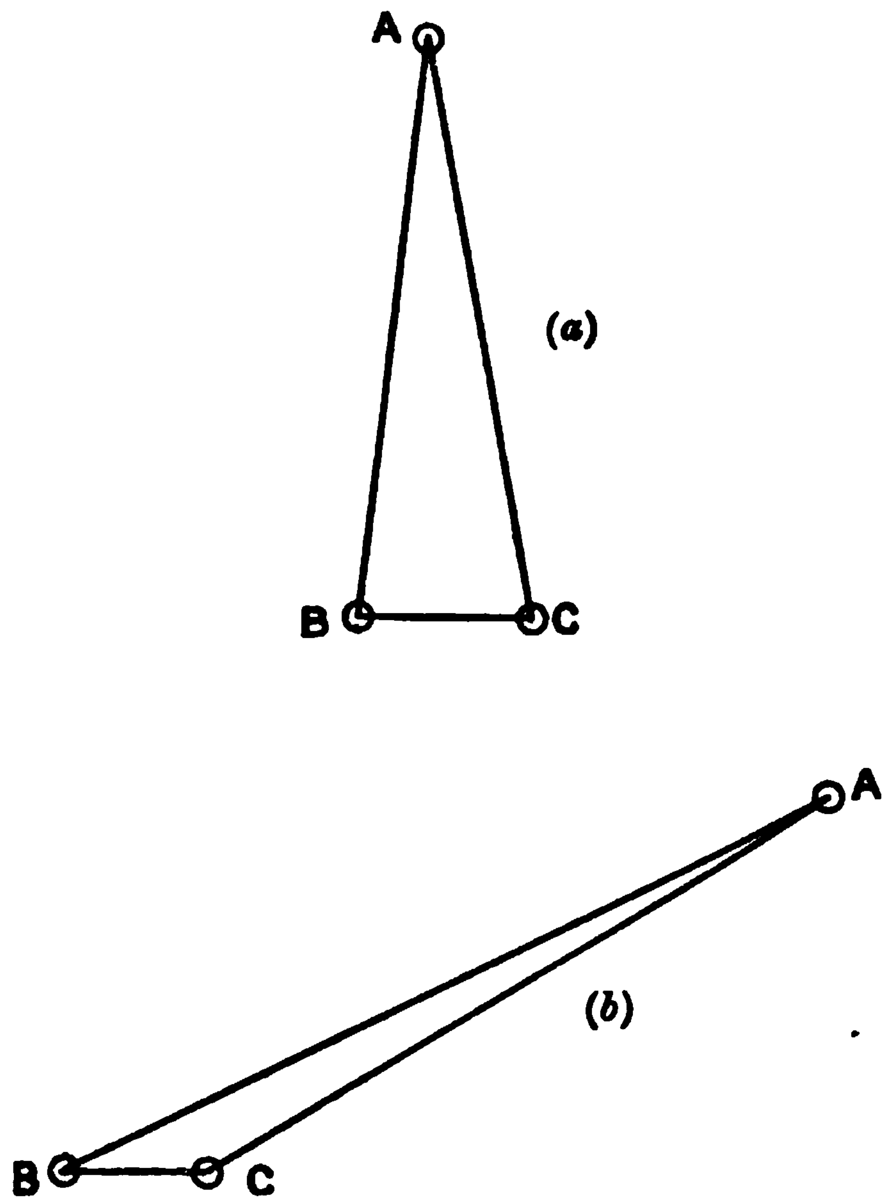


FIG. 156.

back to the equilateral triangle, if the instrumental errors are the same in all three angles, *the distance apart of the spots the same* (that is, A B = A C = B C), and the personal error of observation, and of position, the same, then each angle will possibly, and even probably, be equally in error; and ⅓rd of the total error could be applied to each angle to correct it.

Take the extreme case of an acute-angled isosceles triangle, as shown in fig. 156 (*a*), or of an obtuse-angled triangle such as is shown in fig. 156 (*b*).

Suppose the same observer, with the same instrument, with the same consistent error, makes the observation. Then the angle B A C will probably be less in error, *i.e.* more correct, than the

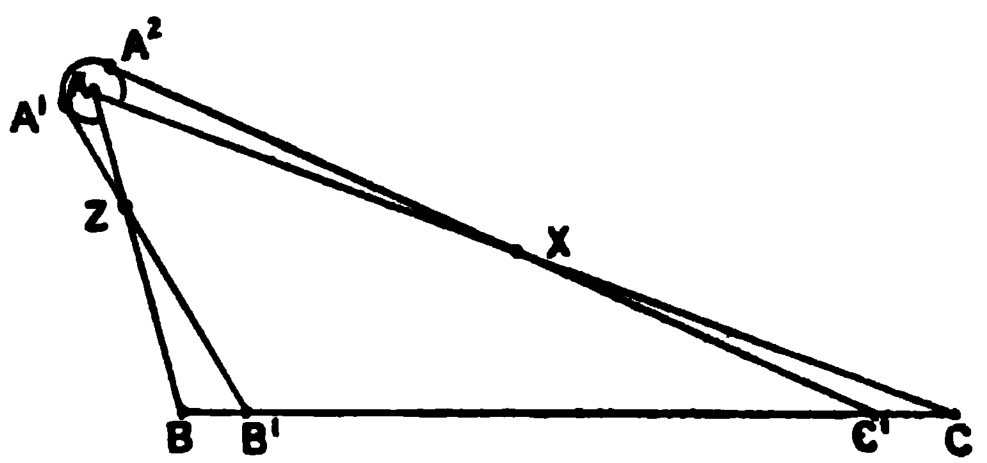
angle at B or at C, on account of the *distance* of B or C from A, as against the distance between B and C.

Suppose the angle at A to be five times the value—merely an arbitrary valuation—of those at B or C, reckoning it in the proportion to distances A C and B C, and that the correction applied to each is in the inverse ratio of their values, the $\frac{1}{11}$th of the total error should be applied to A, and $\frac{5}{11}$ths to each of the larger angles, since they are equal, in fig. 156 (*a*).

The distances A C and B C will vary according to the magnitude of the angle opposite to them, the greatest distance being opposite the greatest angle; and the greatest angle of any triangle is nearer 90° than any other, therefore that angle which is nearest to 90° will require the largest correction; but the amount of the correction is not exactly proportional to the reading of the angle, though near enough when the total error is very small (see p. 232).

When the angles of a triangle are 'cooked' so that they will add up to 180°, then each erroneous position is slightly moved from that which the observer supposes it to be.

In Δ A B C, fig. 157, B is the largest angle, and therefore subtends the longest side, A C.



Let A =	67°	25′
B =	83	15
C =	29	25
	180	05

FIG. 157.

Suppose the whole error to be in angle B, and that A and C are correct, then A Z B¹C is the figure created.

Since the observed angle at B is too large, then B¹ is the observer's position, and A¹B¹C is the measured angle. A¹B¹C = A B C + B Z B¹.

The angle at Z, *i.e.* B Z B¹, is therefore the error of observation; and for an error of observation of this dimension, the position of B has been transferred from B to B¹.

Conversely, applying this error to the observed angle would transfer B to B¹.

In the same triangle, if the whole error is in C, then A B C¹X is the figure created; and, if the error was all in A, there would be another four-sided figure that can similarly be constructed.

Having already distributed the other errors—*i.e.* those due to the difference of quality of the instruments, the personal equation

of each observer, the exposed position in each case, and the reading of the zero when referred back, that is whether it was correct or not, and even taking into account the good or bad zero that was used, for one observer may have selected a better one than the other observer (see *Zeros*, p. 122)—*after* taking all the above into account, it must be granted as probable, that to account for the remaining error, at each point of the triangle a similar form of error was made: that form has already been described in this par., p. 229, and will be perhaps better understood by referring to fig. 156. It is now required to shift the position of all three points an equal distance; and the question is, What is the corresponding proportion of the total error that shall be applied to each angle, so that each position will be equally affected in distance?

Thus, in fig. 157, if CC^1 is made $= BB^1$, and the line A^2C^1 is drawn cutting AC^1 in X, then angle C^1XC is the error to be applied to the angle observed at C^1 that will shift C^1 to C; that distance being $= BB^1$.

In $\Delta\ BZB^1$, $BB^1 : B^1Z :: \sin BZB^1 : \sin BB^1Z$.
BZB^1 is the error at B; call it b.
$B^1BZ =$ observed angle at B (practically).

1. Then $BB^1 : B^1Z :: \sin b : \sin B$.

In $\Delta\ CC^1X$, $CC^1 : C^1X :: \sin CXC^1 : \sin C^1CX$.
CXC^1 is the correction to be applied to C; call it c.
C^1CX is the observed angle at C^1.

2. Then $CC^1 : C^1X :: \sin c : \sin C$.

$BB^1 = CC^1$; and when BB^1 and CC^1 are small, and AA^1 or AA^2 are very small, then B^1Z is practically $= C^1X$ in *any well-conditioned triangle*; where the sides AB and AC are equal, $B^1Z = C^1X$ exactly.

3. Therefore $\sin b : \sin c :: \sin B : \sin C$.

And if b and c are very small, then $b : c :: \sin b : \sin C$; and hence, to make the same displacement in the positions, the angles must be adjusted in the proportion of their sines, or the sines of their supplement.

In a practical form, let

(1)				(2)		
A = 67° 28′	nat. sin	·92		A = 122° 32′	nat. sin	·84
B = 83 18	„	1	(nearly)	B = 9 26	„	·16
C = 29 26	„	·48		C = 48 14	„	·74
180 12		2·4		180 12		1·74

(1) Total error is 12′; divide 12 proportionally to ·92, 1, and ·48.
(2) „ „ „ „ ·84, ·16, and ·74.

(1)	(2)
Corr. at 83° = 5′ 00″	Corr. at 122° = 5′ 43″
„ at 29 = 2 24	„ at 9 = 1 06
„ at 67 = 4 36	„ at 48 = 5 11

and therefore the corrections should make the probable triangle

(1)	(2)
A = 67° 23′ (nearly)	A = 122° 26′
B = 83 13	B = 9 25
C = 29 24	C = 48 09
180 00	180 00

To correct for the error due to the single cause suggested above, and especially in ill-conditioned triangles, there is a more rigid and probably more accurate formula; the mathematical investigation is beyond the scope of this work. Let a, b, c be the sides of any triangle opposite the angles A, B, C: then the proportion of the error is:

For $A = a(b+c)$, for $B = b(a+c)$, for $C = c(a+b)$.

In the examples above, the proportions and corrections will be:

(1)

	Propn.	Corrn.	Corrd. Δ.
A = 67° 28′	5·7	− 4′ 10″	67° 24′
B = 83 18	6·8	− 5 0	83 13
C = 29 26	3·9	− 2 50	29 23
180 12			

(2)

A = 122° 32′	28·3	− 5′ 10″	122° 27′
B = 9 26	9·7	− 1 45	9 24
C = 48 14	27·9	− 5 05	48 09

It is generally accepted, however, as being near enough for all practical purposes, with instruments that read off to the nearest minute, and when the triangles are well conditioned, if the angles are corrected in proportion to their sizes, or to their supplement if above 90°.

By selecting the most distant zero (see p. 122) the error of the initial line is reduced to a minimum.

By carefully plummeting the theodolite, the error of the observer's position is eliminated.

The definitiveness or otherwise of the exact centre of the point observed, and the fineness of the telescope wires, have an influence on the error that creeps in; and all of such errors as have been enumerated, combine with the quality of the instrument and its adjustment, which, in fact, is the first factor in the value of the angles observed.

CHAPTER IV.

COAST-LINE.

435. To Fix the Coast-line.—In example, par. 430, the coast from A to Cliff was described as cliffy, and so on; but the coast-line is not often so simple.

436. Coast-lining from Boat Fixes.—The outline of a coast is the line marked by H.W. springs. When it is impossible or inconvenient to land, a number of 'natural objects' may be 'shot up,' and the coast sketched in between them.

In fig. 158 the portion from A to B is inaccessible; it may be a cliff open to a heavy swell, or a coast-line fronted by unwholesome mud flats; A and B are two fixed *marks* at the ends, or two *objects* which have already been 'fixed.'

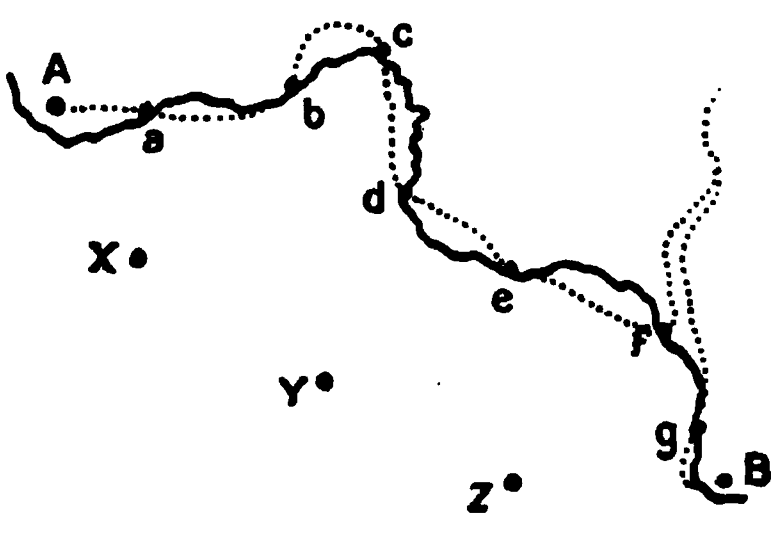


FIG. 158.

It is required to put in the coast-line between them.

Let a boat anchor at X.

X is 'fixed' by station pointer, from three objects beyond the part of the coast shown.

At X, angles can be taken to a number of objects situated on the H.W. line along the coast, shown as *a*, *b*, *c*, *d*, *e*, *f*, etc.

Small letters are here assigned to them, so as not to confuse the figure; in practice, they would be given suitable short names.

The angles would be written down thus

At X, P 52° 40′ Q 70° 50′ R.

This is the 'fix' of X from objects not here shown.

Then follows

A	10°	00′	black stick
	20	10	bush
	25	20	tree branch
	27	30	dog (dead)
	etc.		etc.

It is better to plot these angles right away.

Then up anchor, and shift to Y; so that you can just see c, and be able to 'shoot' it up again.

So at Y, P 70° 50′ Q 52° 40′ R; this is the 'fix.'

A	5°	00′	stick
	10	10	bush
	29	50	tree
	40	00	dog
	etc.		etc.

All these angles are laid off from the ends of the *plotting side* XY, and the intersection of the lines gives the position of the object. Then up anchor again, and move on to Z; 'fix,' and take the angles to the same objects as before, and this will 'check' their positions. Probably some of the objects are only visible from two of the 'fixes'; in that case one must be satisfied with two lines through them instead of three.

With the points plotted, the intermediate coast-line which lies between them is sketched in: the more fixed points there are, the less is the room for speculation.

If it is possible to land at A or at B, from either or from both, many of the above objects might be seen and 'shot up.' There would then only be occasion for perhaps one position of the boat; probably that of Y.

In the case of a shore fronted by mangroves, it is very hard to so 'shoot up' the coast on account of the monotonous and indefinite detail along the edge; but it is the only means available, except waiting for high tide and holding on to a branch, to 'fix it' from a boat.

But this form of 'doing' the coast does not always turn out very satisfactory: it is possible to miss many details, and it is not always easy to see whether the line joining any of the two points 'shot up' curves towards or away from you; this is illustrated by the dotted line round the coast in fig. 158.

In a case personally known to the author, a creek sufficiently large for a small steamboat to proceed some way up it, was

entirely missed; the positions corresponding to X Y Z in fig. 158 on that particular occasion were about a mile from the land.

In the case of a ship steering round an island, this may, perhaps, be the only way in which the details of the coast can be put in. (See *Example*, p. 381, and p. 360, par. 584.)

And, in the days gone by, a great many coasts of continents were 'shot up' in much the same manner, and rivers sometimes escaped notice. This was called a 'running survey'; for an example, see pp. 366, 381, or Plates XI. and XII.

437. 'Walking in' the Coast.—So, when it is possible to land, the coast-line is better 'walked in,' and plotted as one goes along. On an ordinary coast, four miles, in a day of about ten hours, is a good average day's work.

Armed with a sextant, a 'station pointer,' a protractor and pencil (a 10-foot pole in reserve), and a sheet pasted on a board, to plot on (all the points necessary to 'fix' with are pricked on to this sheet; they are 'ringed' in ink and named in the same way as on the original plotting-sheet).

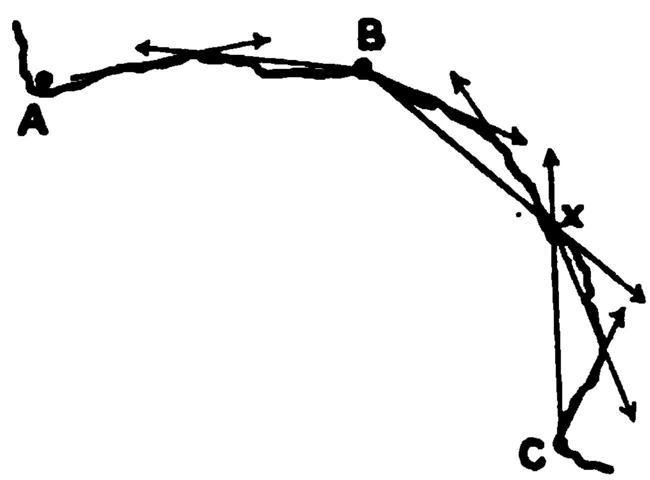


FIG. 159.

The surveyor begins, say, at some station already fixed on the paper; and, since he will require to fix his position at about every $\frac{1}{4}$ inch of paper, one or more flotsams or natural peculiarities, according to the nature of the beach, lying about, at or near the H.W. line, between himself and the probable position of his *next* 'fix,' will be 'shot up.'

If there are no objects or things on the beach just where he would like them, some one can put one stone on another, or stand a stick up here and there. Fig. 159 illustrates a case in point.

Starting at A, the 'trend on' is shown by the line with the arrow from A.

B is 'fixed' by station pointer angles, from fixed objects on the plotting-sheet.

The 'trend back' is shown by a line from B, and so is the 'trend on'; x is shot up from B, then angles taken at it, 'trend back' and 'trend on' which are not plotted until x's position is fixed by another angle to it from C, *after arriving at C*; and so on, 'fixing' again at C and shooting up B, x, and 'trend back' and 'on.'

The dots at the intersection of the arrows show where the thing 'shot up' might be.

Another way is as follows:—

Where there may not have been the necessity to 'fix' at B,

the same two things are 'shot up' from A and C (fig. 160), A and C being 'fixed' from the marks elsewhere on the field board.

But practice and ingenuity, as well as the surveyor's eye, may be required in nooks and corners, where the objects on the board to fix with are out of sight.

Take fig. 161 as an example.

A and B are the only places at the entrance of this embryo cove from which outside marks can be seen to 'fix' with.

Then from A and B, things at *a*, *b*, and *c* are 'shot up.' This fixes *b* by two lines, but gives one line only to the objects at *a* and *c*. Now by moving on to *b*, angles can be taken there, giving lines of direction to *a* and *c*, as well as angles giving the direction of the

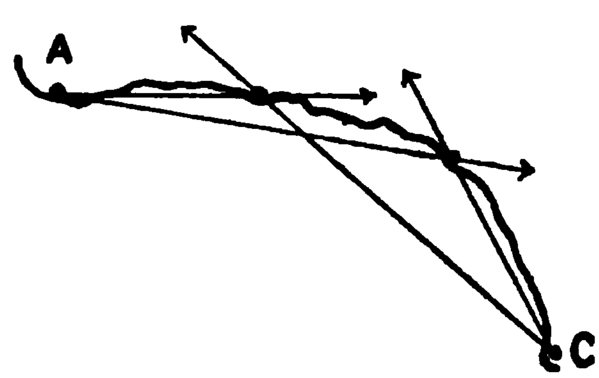


FIG. 160.

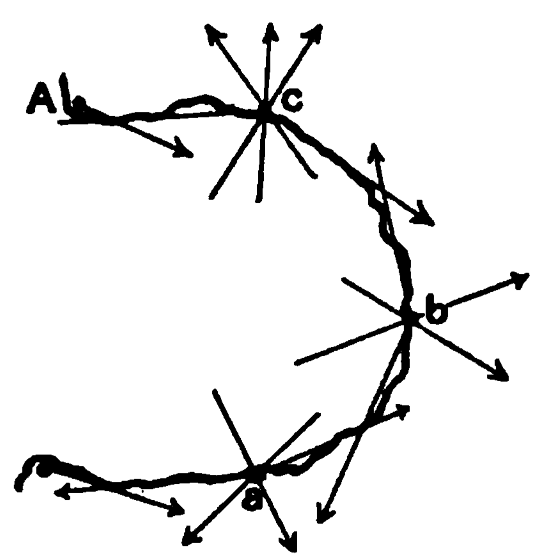


FIG. 161.

'trend' for so many yards on each side of *b*; and at *a* and *c* similar angles can be taken to give the 'trend' on each side of them. At *a* an angle is taken to *c* and to *b*, as a check on itself and on *c*. Starting at A, walking round through *c*, *b*, and *a*, the angles are taken in turn, though picked out for plotting afterwards, as shown. Angles to the 'trend' are always necessary; do not trust to the eye.

If a flick of whitewash is made on a stone, or stick, at the positions *a*, *b*, *c*, there will be better definition to fix their positions by.

The details, as before, are sketched in between the points, and will not extend outside the lines of 'trend.'

438. 'Coast-lining' an Inlet by Small Triangulation.—When the nook partakes of a more or less extensive inlet, it will be necessary to make a small triangulation, such as is shown in fig. 162.

The whitewash brush here is compulsory. A and B are supposed 'fixed' from the points outside.

Unless there is a well-defined object already there, some sort of distinguishing mark must be made at *a*, *b*, *c*, and *d*: a pyramid of whitewashed stones is eminently the best, and seen the furthest.

From A and B, the ends of the plotting side A B, 'shoot up'

a, *b*, *c*, and *d*, if they should be visible from both A and B; in any case, one must be so visible. Suppose *a* and *b* can be seen from both, then an angle to them from A and from B will plot their positions by two lines.

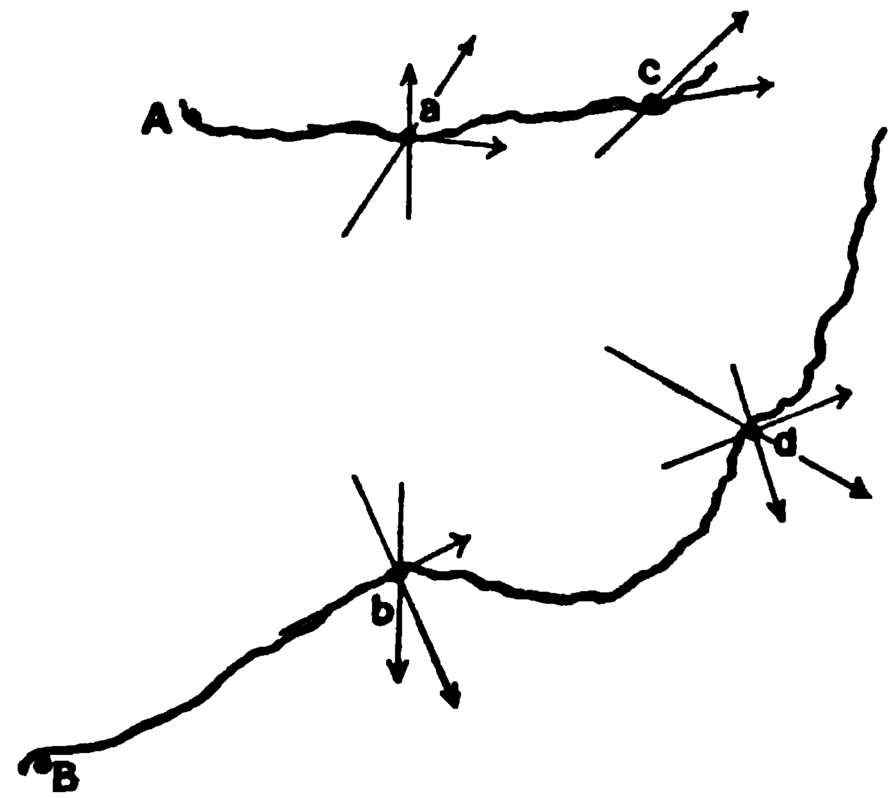


FIG. 162.

See fig. 163; accept *b* as correct; then, going to *b*, take angles to all the others; moving on to *c*, do the same from there to all the other marks, both backwards and forwards; and so on. It is, in fact, one triangulation on another; the plotting side of one

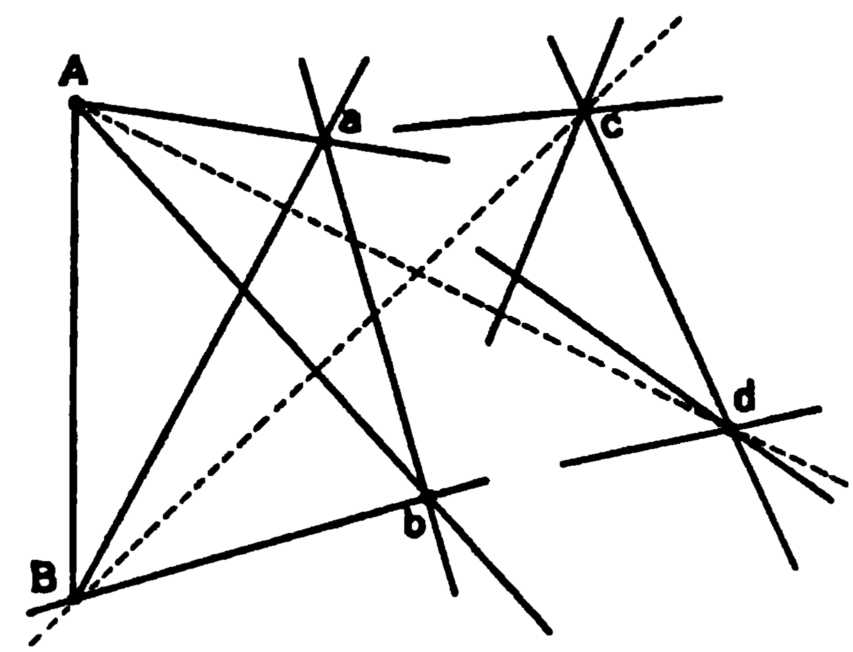


FIG. 163.

triangulation being the last line of the one further back, *but if possible connected with points still further back* (see dotted lines A*d* and B*c*). Obviously, errors are increasing with each triangle added; hence the necessity first alluded to, of defining each point distinctly by whitewash marks.

A river survey is carried out in precisely the same way. But

the original line A B is obtained in a somewhat different manner; and, moreover, when the end of the river is reached, means should be found for re-adjusting the triangulation, both as to total distance as well as to its slue. (See further on, p. 244, and Appendix V.)

439. Coast-lining with 10-Foot Pole.—In the foregoing it has been assumed that the coast-liner has been able to find sufficient objects on the open coast to 'fix' himself by; and that when he lost sight of them, as he would do in a small cove or a creek, he was obliged to make a small triangulation.

But when he reaches the confines, or border, of the plan, such a triangulation as shown above is no longer possible; and he has to put in the outer coast of the harbour. Such a case is illustrated in fig. 164.

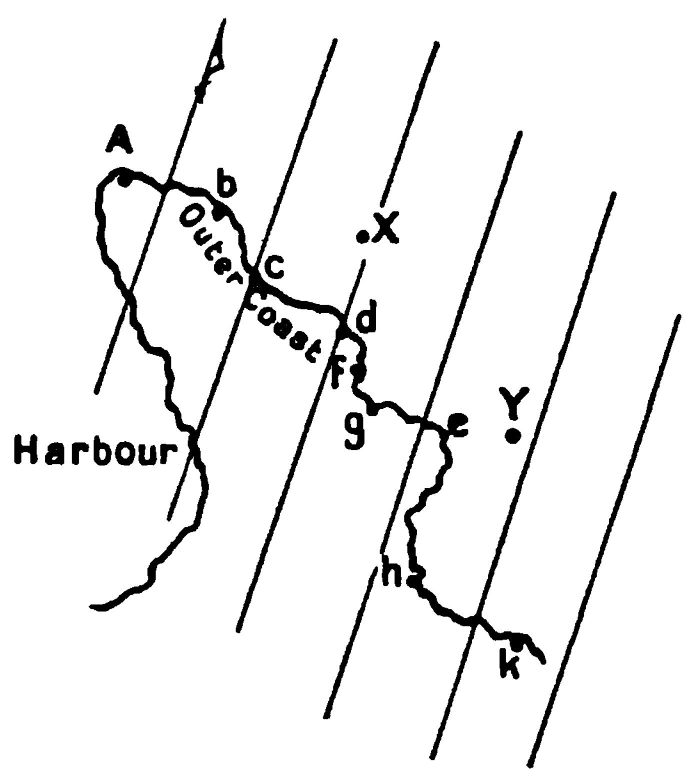


Fig. 164.

From A to the south-eastwards represents the outer coast either of a harbour or of an island.

Here recourse must be had to the 10-foot pole described on p. 154, par. 329, which, as previously stated, has been carried in reserve; a pocket or prismatic compass will also be required.

The general principle is that, A being the last and outer point that could be 'fixed,' the points *a*, *b*, *c*, etc., along the outer coast will be 'fixed,' by their bearing and distance from each other, starting from A.

The bearings taken with the prismatic compass being magnetic, to simplify the laying off of the bearings with the protractor, a number of magnetic meridians are ruled on the paper in the vicinity of the coast, as shown in fig. 164.

A scale will be required for the measurement of distances, and will be obtainable with the aid of the 10-foot pole.

This scale will be in the form of a table, showing the distance corresponding to the angles subtended by the pole, calculated from 1° to 5°.

The table as shown in Appendix VIII. can be used. But, if desired, one can be made in the following way, which will probably be as useful.

Since dist. in feet = length of pole l . cot angle (see p. 153), then

dist. in inches of paper, approximately $= l \,.\, \dfrac{\cot a \,.\, \text{scale in inches}}{6000}$.

To further simplify it; if $\frac{l\,.\,\text{scale}}{6000}$ is a whole fraction, such as $l = 10$ feet and scale $= 6$ inches, then $\frac{l\,.\,\text{scale}}{6000} = \frac{60}{6000} = \frac{1}{100}$. Also $\frac{\text{nat. cot}\,a}{100}$ = length in inches of paper for angle a.

A part of the nat. cot. table only need be copied out for such angles as are desired.

If the scale = 5 inches, and if l is made 12 feet, then the scale is exactly the same as before.

If the scale is 4 inches, and l is made 15 feet, it will still be the same as the cot. table, and this does away with any special table. The same results are obtained if $l\,.\,\text{scale} = 60$, and, knowing the scale, the distance of the discs on the pole can be adjusted so that it will $= \frac{60}{\text{scale}}$, or, more correctly, $\frac{60{\cdot}8}{\text{scale}}$.

Then, again, if the $\frac{1}{4}$-inch scale on the edge of the protractor is used, as it undoubtedly should be, then $\frac{\cot a}{4{\cdot}000}$ = distance by the $\frac{1}{4}$-inch scale, and this does away with the use of dividers.

The '10-foot' pole can be made into 12-ft., 15-ft., or any other length of pole, by sliding the discs in or out. The odd fractions of the scale can be neglected in such work as putting in the outer coast.

As the total distance usually required is under a mile, the total error will probably be about 80 feet in a mile (see p. 153).

And, since the 'work' in the present case is a plan of the *inside* of the harbour, the portion of the coast-line outside, for 1 mile on either side of the entrance, is not of very great importance—it is more to give a finish to the work. On a 6-inch scale, 80 feet = ·103 of an inch; such an error is negligible.

Now, referring back to fig. 164, A is the outer point fixed; then, starting from that point, the pole is sent on to b; a man holds it there horizontally, swaying the ends in a horizontal movement to and from the observer, or vertically, as shown in fig. 165, so that the maximum angle subtended by the length of the pole is measured with the sextant. It would be better to remove the index error of the sextant.

At the same time the magnetic bearing of it is taken from A. There should be no need to mention again, that steel cigarette-cases and pencils, knives, and keys exert a 'local attraction' on the compass, and had better be given to someone else to take care of.

The distance, in inches of paper, corresponding to the length of the pole with the angle, measured along the bearing-line set off, gives the position of b relatively to A.

The position of *b* is marked by anything handy: one stone on another, a stick, or, temporarily, a man. The pole is then sent on successively to *c*, *d*, *e*, etc., and as one progresses onwards, the bearing and distance are taken to the poles ahead, and a bearing taken to the marks back. The mean of the bearings fore and back is accepted.

Or, X and Y can be fixed by bearing and distance from A and from each other; and from each of them, angles may be taken to various things along the coast; in fact, it is just the same as fixing the coast-line from X, Y, Z, as shown in fig. 158; but here each fix depends on the one back.

This last method probably produces less total error than the first suggested, which involves a greater number of successive

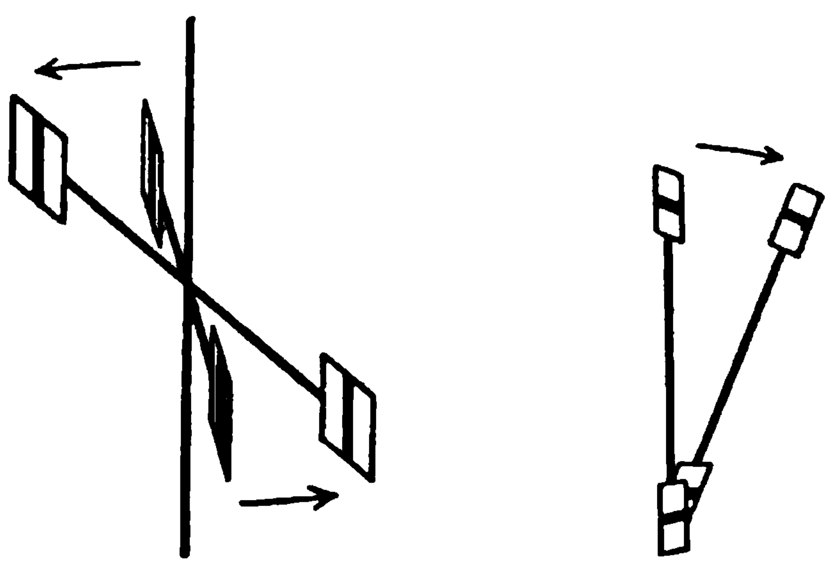


FIG. 165.

measurements along the coast, each depending on the other; but if there is no foreshore, there is no room for X, Y, etc.

Precisely the same system is carried out in the case of the outer coast-line of an island. Starting from a point which can be 'fixed' by angles, the surveyor works round the coast, eventually arriving at another part of the island where again a 'fix' can be obtained by angles; this is not necessarily the same point as he started from, but must be a point correctly related to the first through the same triangulation.

440. 'Squaring in' Errors of Coast-lining by 10-Foot Pole.—It will be found that the last bearing and distance that should bring him to this point, does not tally with what it should be.

In the following sketches, fig. 166, the hard line represents the true shape, and the dotted, the erroneous; each indicates one of the four things that has happened:—(1) the bearing is right, and the distance is too short, $B B^1$, or too long, $B B^2$; (2) the distance is right and the bearing wrong, represented by angle $B A B^1$; (3) the bearing is in error, shown by B A B, and the distance is too short, B X being the error; (4) the bearing is in error, shown by $B A B^2$, and the distance too long, B X being the error.

If time permits and the matter is important, the most correct solution is to go round both ways, and take the mean of the results of each tour.

If there is no time, then the total error in *distance* must be re-adjusted in the proportion of each piece measured. This suggests that, if the distance between each pole measurement is about equal, and if the coast is put in as given above, *i.e.* from X, Y, etc., then the calculations necessary for readjusting the distance are simplified.

If the error in bearing is *considerable*, which, if the bearings have been taken forward and back, is not probable, then that will

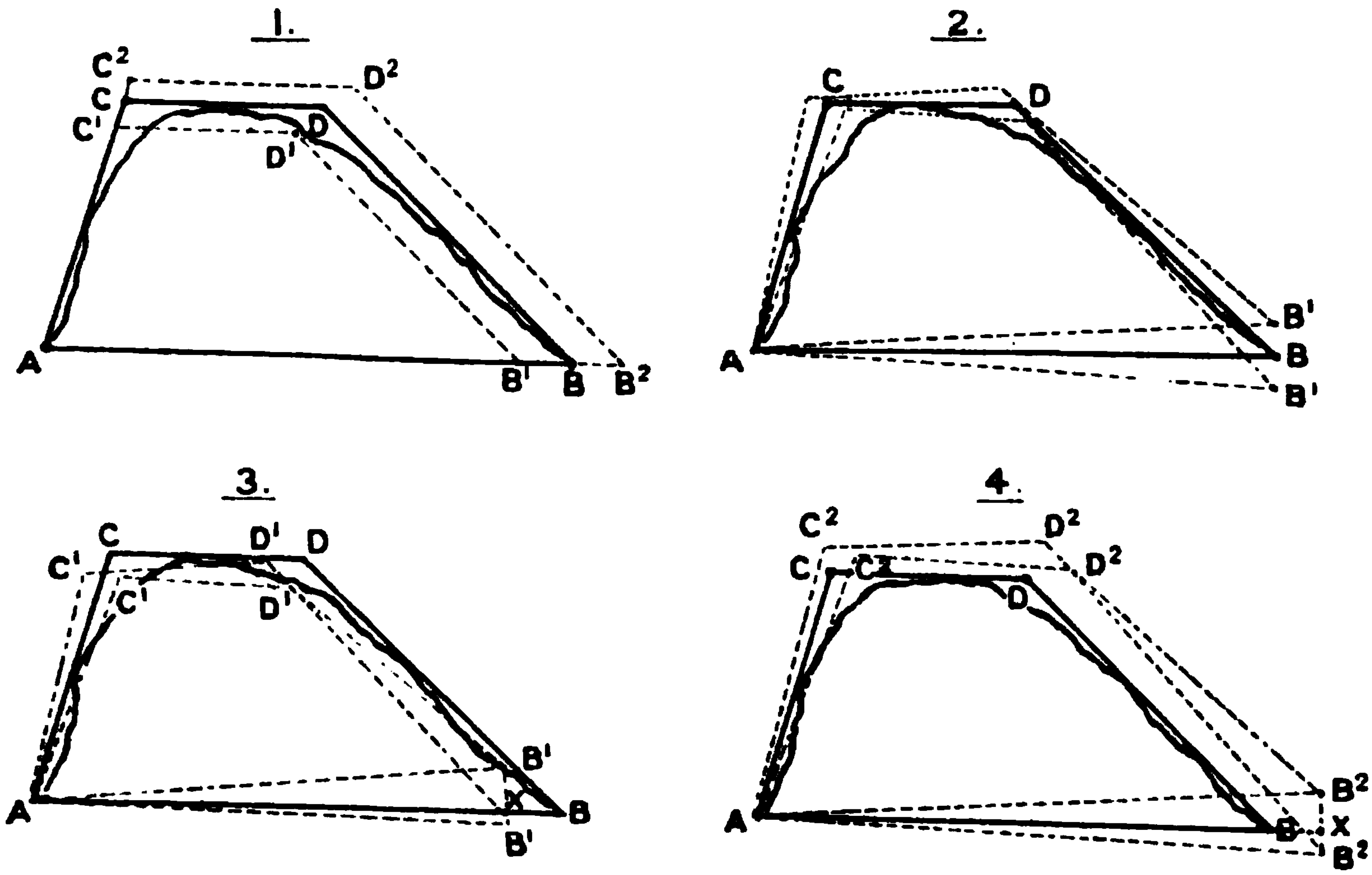


Fig. 166.

also have to be re-adjusted, by applying the error shown by B A B¹ or B A B² to all the bearings taken; but in the work above noticed, this degree of accuracy is not called for; if the error is under 4 or 5 degrees, in a coast-line 1 mile long, it can be neglected; one has to bear in mind that already 2½ hours have been occupied in putting in this outer coast, and, disregarding the error in bearing, another hour or two would be occupied in re-plotting the positions to tens of feet; whereas the total error is possibly only 200 feet, represented by ·2 inch on a 6-inch scale.

In many of the running surveys in the past decade, when two surveys undertaken from two opposite directions did not quite meet, an imaginary sandy beach was interposed; this was by no means justifiable; but, a matter of a few feet on an outside and

unimportant coast, especially when the scale is small, will hardly endanger a vessel.

The whole thing is a diminutive form of the error that exists in every piece of work, even in an extended triangulation. (See the example given on a large scale in a river survey, Appendix V.)

Conceive the whole distance laid out as a zigzag line hinged at the points, and take the case of 3 in fig. 166. A B in fig. 167 represents the true position of A and B on a straight line; B^1 is the observer's position, his distance being too short by B X, and his error to the right looking from A towards B. B A B^1 is the error in bearing; this angle can be applied to all the bearings, and it pushes the zigzag line on to B; in a distance of one mile this is probably a negligible quantity.

As regards the distance B X, it must be admitted that the error was a constant one, and the total amount of the error B X, *i.e.* the difference between A B and A B^1, must be divided up in pro-

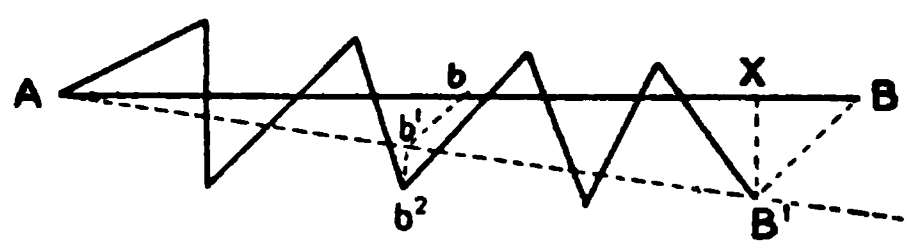


FIG. 167.

portion to each distance measured, and the proportional error applied to each distance. In other words, the zigzag line will have to be pulled out to B, and, in doing this, all the bearings would be altered; but the same result is obtained if the distances of each apex are altered instead.

Along the line A B^1 tick off each distance that has to be re-adjusted, as, for instance, $Ab^1 = Ab^2$; this should be done on the same scale of the work. Join B B^1, and through b^1 draw a line parallel to B B^1; where this line meets the line A B will be the adjusted distance A*b*. Each point would have to be treated in the same way, and they can be measured from this line, and be re-plotted.

A river bank may be surveyed with the aid of a pole, 12, 15 or even 20 feet long, in exactly the same manner.

441. Coast-line of a River Bank from Two Boats and 10-Foot Pole.—Two boats may be used, if necessary; and, conjointly, they literally stride up and across the river.

As in the case of an outer coast, which has to be connected with the main work, so, if this river forms part of a coast survey, one point on it must be connected with the remainder of the survey.

If it is an entirely *separate* piece of work, its position must be fixed astronomically.

In this, greater accuracy will be required than for an outer coast-line; and it will be always necessary to put up some mark at each spot, and, as before, take the bearings both forward and back to the pole. In fact, it may be carried out to any degree of accuracy that is wanted. For example, two poles can be used, one in each boat and an observer in each; this would give two bearings and two distances, the mean of which would be accepted. Now, an error of 1′, when a 10-foot pole subtends 1°, = 10 feet nearly, if 2° it is 2 feet; so that the nearer the pole or the larger the angle measured, the less is the error in distance for the same error of observation.

At the mouth of a river, its distance across may be 600 feet, and the angle subtended by the pole will be about 1°, so that the measurement would have to be taken very carefully.

It is probable that, owing to instrumental and other errors of observation, the angle will be 2′ in error, which means an error of about 20 feet in 600 feet, or about 200 feet in every mile. Owing to this, a 10-foot pole cannot be expected to provide accurate results beyond 300 feet; to avoid this error, it will be found better to use a 15-foot or longer pole.

Each distance will be in error proportionate to the angle, and, since there will be many distances measured, and each portion ahead depends upon the one immediately behind, the errors in position will accumulate rapidly.

It is, in fact, an exaggerated form of the creek shown in fig. 158; and of that with the zigzag, fig. 167, but now it will resemble a trestle-work, each point being hinged.

Some means must therefore be found for checking the work from time to time—that is, pull out or push back the trestle-work; or, failing this, some point at the head of the river must of necessity be connected up, in both bearing and distance, either directly with a point at the entrance, or with something else that is connected with that point directly or indirectly. In fact, it is required to locate the pivot and the end of the piece of trestle-work.

In the extensive triangulation of a large and important river, there will be a network of such connecting points, each being linked together by *large* triangles; while the triangulation of the river bank falls into its place, 'inside' these main triangles.

442. Connecting up a Triangulation with a 'fixed' △. —Where, however, there is no such large triangulation, the last position on the banks fixed by the 10-foot pole or by any other means, must be connected up with one or two or more objects in another triangulation, such as one in an Ordnance Survey, and also with the point you started from.

In fig. 168 let the point A or B, at the entrance of the river, be a point belonging to the main triangulation; or let one of

them be connected to M, which forms part of the coast triangulation. See the following example:—

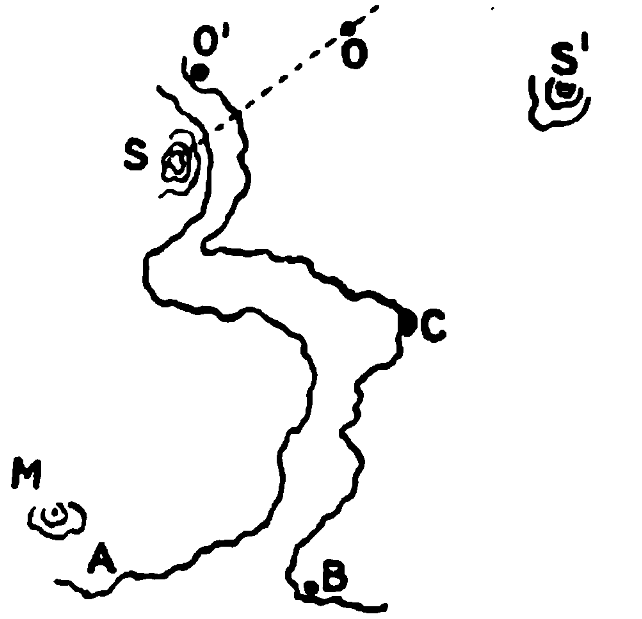


FIG. 168.

Let S be a summit, also fixed in the triangulation, or a position derived from an Ordnance Survey. (See *Scales* and *Transferring Positions*, p. 176, par. 364.)

Position of S can be transferred to the work by its bearing and distance from M, or by its diff. lat. and diff. long. from M. Suppose the observer arrives at O′, and he is visible from S. Then from S a true bearing of O′ may be obtained, or an angle between M and O′. This will give the direction of O′'s position; in this case nearly N.

443. 'Squaring' in a River Survey.—If S O is the line of position drawn through S, then O′'s position is out in bearing, and the total error O M O′ gives a constant correction to the bearing of each point of the work: in fact, the trestle of triangles must be slued, pivoting from M.

In the above case, the general trend of the river is north and south, therefore the line which best 'checked' his position in *bearing*, is one in a north and south direction.

Had he taken S′ to 'check' his position from, the bearing of O from S′, giving an east and west line along any part of which the observer might be, does not keep him straight in bearing. But a true bearing of S′ would check distance M O; and with the assistance then of such objects as S and S′, the observer 'checks' his last position; and, as before explained, he must deduce a percentage error to apply to each distance found by triangulation, and either apply this separately, or he may do the whole work over again, working from the head of the river to the mouth; depending upon the mode of triangulation used and accuracy aimed at; that is, whether by angles only, by pole and bearings, or by patent log and bearings.

The proportional error is not applied to each actual distance, but to that part of the distance which is directly on the line joining one end of the work with the other, just as is shown in the zigzag line, fig. 167.

For example: M O is the line joining the beginning with the end position.

M to *a*, fig. 169, is the first line from M toward O; from *a* draw *ax* at right angles to M O; then M*x* is the proportional distance of *a* directly toward O; M*y* is the proportional distance of M*b* towards O; M*z* is the proportional distance of M*d* towards O.

Then, if desired, and the importance of the work calls for it, the whole error represented by X O′ (the difference between M O′ and M O) in fig. 169, will have to be divided in the proportions of Mx, My, Mz, etc., and each position pulled back—for instance, z to z', and d will be at d'. If the bearing is much in error, each angle, or, in this case, each bearing, will have to be corrected; but this may be a needless refinement of adjustments: if it is done, apply the angle X M O to all the angles, cM O, dM O, etc. There is a time and there is not a time when such adjustments are necessary; and adjustments of errors are in the exact proportion to the degree of accuracy the instruments are capable of.

Noting the fact that the three angles of a triangle cannot, when the angles are taken with a sextant, be made to equal 180 degrees, even supposing that the observer does stand exactly over each point of the triangle; when the survey has been entirely carried out with sextant angles, starting from a base also derived through angles taken with a sextant, the degree of accuracy may not be very great, and such adjustments as have been alluded to would be an affectation.

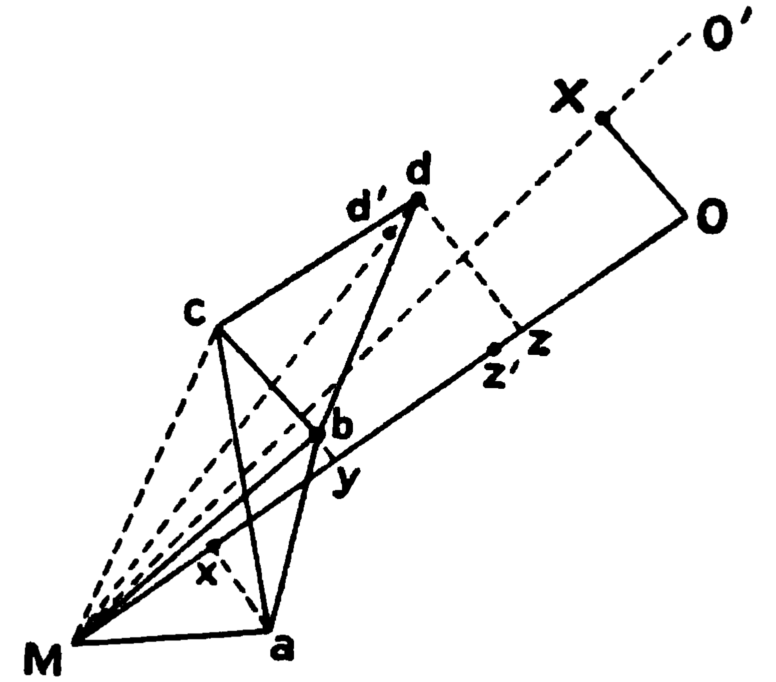


FIG. 169.

When, however, such a river is surveyed as a separate piece of work, on a larger scale than the triangulation of the coast, and with the most accurate instruments, then some point on it must be fixed astronomically, either directly so, by observations for latitude and longitude, or by a true bearing and distance from some other point which is astronomically fixed; and all refinements of adjusting, are made with reference to the astronomical position of each separate point of the triangulation. (See Appendix V.)

Referring to fig. 168, C may be so fixed.

444. 'Connecting up' from Astronomical Positions.— Or, if the astronomical positions of S and S′ (fig. 168) are transferred from an Ordnance triangulation, relatively to the latitude and longitude of C, on to the same sheet as that on which the river is plotted, then the true position of the observer can be obtained by plotting the intersection of the two true bearings, taken at O, of the objects S and S′; but, before actually plotting these bearings, they must be converted into Mercatorial bearings. Or, without transferring the positions S and S′ to paper, the astronomical position of O can be calculated, and plotted relative to C: or at O

it can be an 'absolute' position in latitude and longitude, the observations at C not being necessary.

Example.—S and S′ (fig. 170) are two distant summits about fifteen miles apart, their positions being too distant to plot on the paper.

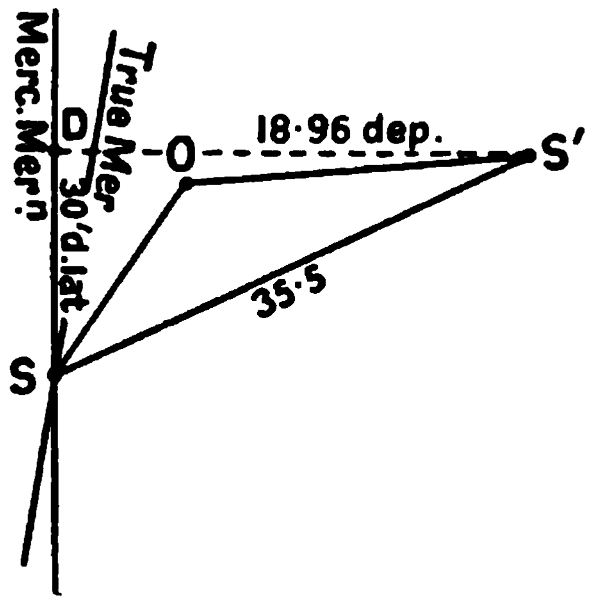


Fig. 170.

It is ascertained that:

lat. S.	50° 30′ N.	long.	15° 30′ W.
lat. S′	51° 00	long.	15 00 W.
d. lat.	0 30	d. long.	0 30

$$\text{dep.} = \text{d. long.} \,.\, \cos \text{mid. lat.}$$
$$= 30 \,.\, \cdot632 = 18\cdot96.$$
$$\frac{\text{dep.}}{\text{d. lat.}} = \tan \text{Mercatorial bearing.}$$
$$\frac{18\cdot96}{30} = \tan \text{D S S}'.$$
$$= \text{N. } 32° \ 18' \text{ E.}$$
$$\text{Dist. (S S}') = \text{D S} \,.\, \sec \text{D S S}'.$$
$$= 30 \,.\, 1\cdot183 = 35\cdot5 \text{ miles.}$$

Therefore the bearing and distance of S′ from S is N. 32° 18′ E., 35·5 miles.

At O the T.B. of S is observed and found to be S. 8° 14′ W. and the angle between S and S′ is 127° 42′.

So as to be able to convert the T.B. into a Mercatorial bearing it is required to find the distance from O to S.

At first this will have to be done by using the T.B. as a Mercatorial bearing, then, after correcting it, repeating the calculation.

Then, taking D S O = 8° 14′, and since D S S′ = 32° 18′, then O S S′ = 24° 04′. Since S O S′ = 127° 42′, and O S S′ = 24° 04′,

then $OS'S = 28° 14'$ $(180° - 127° 42' + 24° 04')$, and dist. $OS = SS' . \sin OS'S . \operatorname{cosec} SOS'$,

log SS'	1·550228
log sin 24° 04'	9·609313
cosec 127° 42'	10·101701
	1·261242 = 18·25 miles.

Convergency = dist. sin Merc. bearing . tan mid lat.

To know the mid lat. we must find the diff. lat. between O and S.

diff. lat. = dist. . cos bearing.
18° 25'. cos 8° 14'.
18° 25'. ·99 = 18° 07'.

Then mid lat. = 50° 39' (50° 30' + ½ diff. lat.)

log 18° 25'	1·261242
sin 8 14	9·155957
tan 50 39	10·086213
	log ·503412 = 3·187 = 3' 11" + to T.B.

Therefore Mercatorial bearing = 8° 14' + 1' 35" (½ convergency) = 8° 15' 35" = Mercatorial bearing. Then recalculate all the above with this Mercatorial bearing; and convergency = 3·2' = 3' 12"; and the corrected Mercatorial bearing equals N. 8° 15' 36" E.

Recalculating S O S' gives O S practically the same as before, viz. 18·25 miles, and also diff. lat. the same = 18·06' = 18' 3".

And since the lat. of S	50°	30'	0"	N.
diff. lat. to O	0	18	3	N.
then lat. of O	50	48	03	N.

diff. long. = dep. sec lat.; and dep. = diff. lat. tan bearing.

Then diff. long. = diff. lat. tan bearing . sec mid lat.

log d. lat.	1·256687
log tan bearing	9·163123
log sec mid lat.	10·197872
	log ·617682 = 4·146 = 4' 08"

Since long. of S is	15°	30'	0"	W.
diff. long. to O	0	4	08	E.
then long. of O	15	25	52	W.

Should there be no friendly summit such as S or S' to check the results, then the positions near the mouth, and at the head, must be ascertained astronomically (see *Observations for Latitude and*

Longitude, pp. 40, 37, and Appendixes III. (*a*) and IV. (*b*)); and, as before, the details must be made to fit in between them, by re-adjusting, in latitude and longitude, each point that has been found, by a triangulation either with a theodolite, sextant, or with a pole and compass. (See complete example worked out in Appendix V.)

It has already been pointed out that with only one object visible, its bearing will, or may, only check either the bearing or the distance, depending upon whether it is in the line of the general direction of the work, or whether it is at right angles to it.

But the length of the river must be considerable, in order that an astronomical check can be practically utilised.

445. When Astronomical 'Check' can be Used.—For admitting an error of 20 feet in every mile of triangulation with the best instruments, or 300 feet per mile, using a 10-foot pole and prismatic compass; by the difference of astronomical observations with the sun or with stars, a position can only be practically obtained within about 600 feet between *any* difference of latitude and longitude, however near together the places may be; but if the river is 60 miles in distance as the crow flies, or in diff. lat. or in diff. long., the distance in error might be from 1200 to 18,000 feet, depending upon the means adopted, whereas the astronomical diff. of latitude and longitude might, and should, only be the same as before, that is, 600 feet. Taking the other extreme, a river two miles long, even with a 10-foot pole and compass, would be within 600 feet of being correct, and difference of astronomical positions would not therefore be a check. Hence, then, in a river thirty miles long, the *difference* of the astronomical positions at both ends of the river will check the 'field' work when *the best* instruments are used.

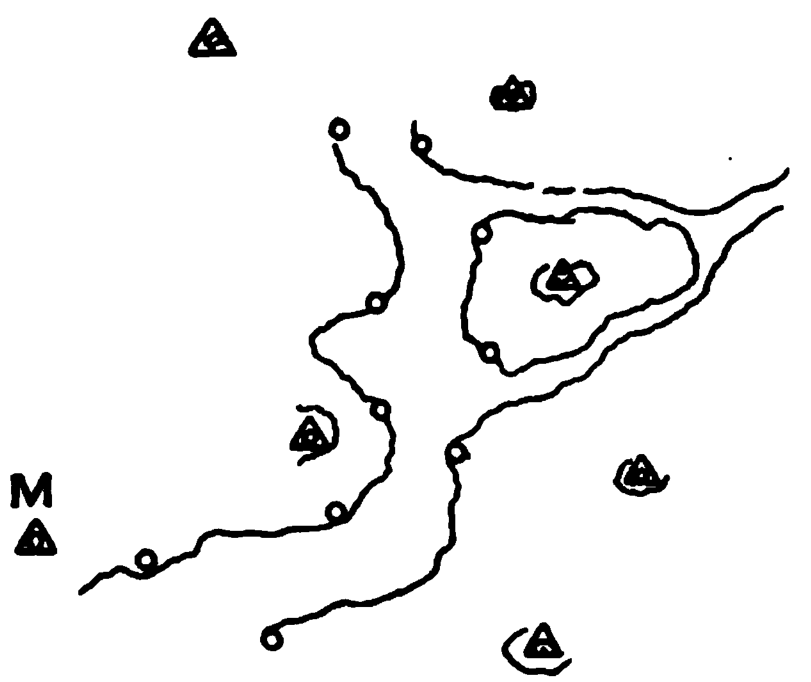


FIG. 171.

A river of such a length would have, probably, a wide enough entrance, and be wide enough for the greater part of its course for a systematic triangulation with the best instruments; and there would also possibly be such high ground sufficiently far back from the banks to enable a 'main' triangulation to be made, while the triangulation of the banks would be secondary to it (see fig. 171, in which the △'s belong to the main outer triangula-

tion, the ⊙ to the secondary triangulation of the bank); whereas the pole and compass is only intended for an offshoot to such a river, extending probably at no great distance, and astronomical positions could hardly be resorted to in such a case, though there may be occasions when there is nothing else to go by.

446. Survey of a Creek by Patent Log and Compass.—In such creeks or offshoots a rougher and quicker method of obtaining the bearings and distances from point to point, is by a patent log and a compass bearing. The boat is run in mid-stream from off one point to abreast of another, the distance run between the two being registered by patent log, and the direction of the boat's course noted. And on the return journey, the operation is repeated, the distances and bearings going and returning are meaned, and the line of the banks sketched in.

Find the error of the boat's compass, before starting, by noting the direction it points when going between positions fixed by sextant angles—the difference between the course by compass and the true direction will be the total error on that course. If the means are at hand (see par. 467), it will lessen the labour if the strength of the stream is ascertained from the boat anchored at the beginning and end of the work, and its rate allowed for in the patent log distances; but this method cannot be so accurate as when these distances are run both going and returning, though it may be more expeditious.

The distance across at any part will have to be estimated. The simplest, and perhaps least deceptive scale of measurement to go by, would be the length of one's own boat.

As before explained, if it is possible to reach a point of vantage at the head of the river, or near it, or in fact at any stage so that other objects of the general or 'main' triangulation are visible, then a 'fix' by them should if possible be obtained; or, if not an absolute fix, a single line checking either the bearing or the distance, and the previous work readjusted.

CHAPTER V.

SURVEY OF A RIVER MOUTH.

447. *Example of a survey of the entrance to a river, and of a triangulation along its banks.* (See Plate III.)

The following are the details of how the work was started:—

Mound is known as an established point of the main triangulation, either Ordnance or Hydrographic, and it will be the connection between either of these and the work which follows.

If it has no existence in either, its latitude and longitude must be found independently, or some other spot connected with it must be so found.

It will be best to take a preliminary look round; make a rough sketch, and see what is visible and from where; then arrange the general idea, with a view to the position of the main △ (see *Plotting Sides*, p. 194, par. 394). Think out the scheme with regard to what is to follow, then decide what places the marks shall occupy, and which marks *must* be visible from each other.

In the present example, a mark is erected at Sand Point, also at Tree, and at the other places shown on Plate III..

Spit is visible from Mound, and also from Sand Point; it is purposely placed so.

Sand Point to Spit is adopted as the plotting side, for reasons that have been well considered.

The scale adopted was 6 inches = 1 mile = 6050 feet.

It was a somewhat large scale; but let us suppose there were sufficient reasons to justify it, there was no time pressure held over the surveyor, and there would be ample opportunity and room on the paper to put in distinctly every detail; also bear in mind that the eventual publication will be on a reduced scale to the one he adopted; he has, moreover, first-class instruments, including a good theodolite, and a good observing sextant, and chain, and all the assistance he may require.

While the marks were being put up—and the party so doing must have a clear conception of the general outline of the tri-

angulation—search was made for a suitable place to measure a base.

On the foreshore, along the outer coast, to the southward of Sand Point, there was a sandy beach; and it was found possible, by slightly moving the position of Sand Point mark from where it was originally erected, to chain a length along this beach in a direction with Mound in line with Sand Point; and an angle of elevation of Mound was taken with a theodolite, both at S and at B (see Plate III.) at the end of the line S B (see *Elevations*, p. 133). The distance chained from S to B was 567 feet. This was all the length that could possibly be obtained.

The mean of the elevations, obtained with the telescope reversed, of M from S was 2° 50′ 30″; and from B was 2° 2′ 30″. At Sand Point the T.B. of Spit was found, by theodolite angle and an altitude azimuth with a sextant, to be N. 72° 14′ E. (see *Altitude Azimuth*, p. 147, par. 317). The variation is 15° W.

The following angles were observed from the various marks:—

At Mound ⊕,	Spit	360°	00′
	Round	4	30
	Stone	18	20
	Mid S	42	02
	Sand Pt.	77	34
	Tree	96	10
		Z.O.K.	

At Sand Point,	Elbow	16°	50′	Spit	3°	50′	Round.
	Mid N	27	55	,,	5	50	Stone.
	Mound	68	20	,,	18	50	Mid S.
	Tree	89	20				
At Spit,	Mid S			Sand Pt.	16	50	Tree.
	ϕ Stone	11	02	,,	18	30	Mid N.
	Round	58	02	,,	28	00	Elbow.
					34	15	Mound.
At Elbow,	Mid S	21	55	Sand Pt.	21	05	Mid N
	Stone	45	05	,,			
	Round	125	40	,,			
	Spit	135	00	,,			
At Mid S,	Elbow	16	10	Spit			
	Mid N	77	55				
	Mound	92	50				
	Tree	118	28				
	Sand Pt.	57	08	Mound.			

From those observations the following complete triangles were picked out :—

Δ S P M				Δ S P E			
Sand Point	68°	20′	− 4	Sand Point	16°	50′	+ 2
Spit	34	15	− 1	Spit	28	00	+ 3
Round	77	34	− 4	Elbow	135	00	+ 5
	180	09			179	50	

In Δ S P Mid S,				In Δ P M Mid S,			
S	18°	50′	+ 2	P	45°	17′	− 2
P	11	02	+ 1	M	42	02	− 2
Mid S	149	58	+ 7	Mid S	92	50	− 5
	179	50			180	09	

The + and − were the adjustments, to make the sum of the three angles in each triangle = 180°. (See *Adjustment of Angles*, p. 232, par. 434.)

The continuation of the triangulation up the river is a repetition of the above, and is not given; this part of it can now be plotted. The first thing will be to find the bearing and distance of the plotting side, Sand Point to Spit.

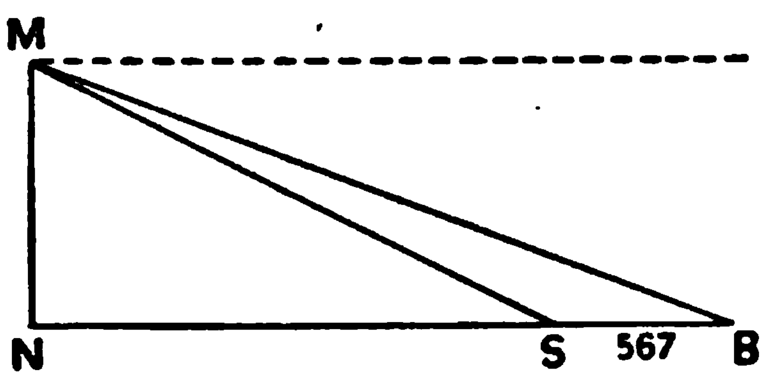


Fig. 172.

It is really not necessary to know *exactly* this true bearing and distance—the triangulation can be plotted without—but anyway it is better to know both approximately, before anything is placed on paper, so that the eventual scale will be near the one desired (see an example of this on p. 376, par. 591). For illustration's sake, the true bearing and distance will be admitted as having been calculated first; the T.B. being N. 72° 14′ E.

Referring to fig. 172.

448. Plotting Sides found from Elevations for Two Points.—N S, the distance from Sand Point to Mound (see fig. 172),

$= \mathrm{MS}\,.\,\cos \mathrm{MSN}$

1. $= \mathrm{MS}\,.\,\cos 2°\ 50'\ 30''$.
2. $\mathrm{MS} = \mathrm{BS}\,.\,\sin \mathrm{MBS}\,.\,\mathrm{cosec}\ \mathrm{BMS}$ (difference of elevations) $= 567\,.\,\sin 2°\ 29'\ 30''\,.\,\mathrm{cosec}\ 0°\ 21'\ 00''$.
3. $\mathrm{NS} = 567\,.\,\sin 2°\ 29'\ 30''\,.\,\mathrm{cosec}\ 0°\ 21'\,.\,\cos 2°\ 50'\ 30''$.

log N S	2·753583
log sin 2° 29′ 30″	8·638230
log cosec 0 21	9·994466
log cos 2 50 30	12·214057
	log 3·605337 = 4033 feet.

Note here the cosec of the small angle, wherein a small error will make a considerable difference to the value of N S; and the utmost care would have to be taken in obtaining a length by such means; it is only given here to show what can be done under such conditions. To supplement the angles of elevation, another angle should be taken at Mound, viz. the angle S M B; and with a sextant would give better results than with a theodolite, because the radius of the sextant is much larger than that of the *vertical* arc of the theodolite, and it is more closely graduated: the angle would necessarily be measured both 'on' and 'off' the arc.

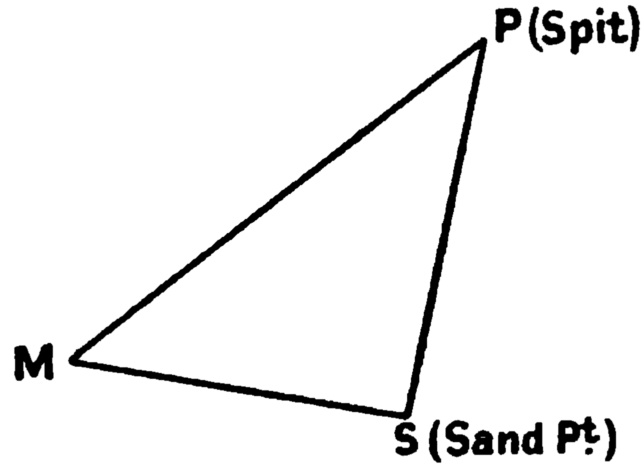


Fig. 173.

The angle B M S is the difference of depressions or elevations on which the value of N S so much depends; and if it does not agree with the difference of elevations as found with the theodolite, these angles would have to be adjusted to it—in fact, in Δ M S B, the three angles must make 180°.

In Δ M S P, fig. 173, M to S corresponds with the N S previously calculated = 4033 feet; and there is the complete triangle (see above)

Mound	77° 30′
Sand Pt.	68 16
Spit	34 14
	180 00

Find the distance Sand Point to Spit.

Let Sand Point be S, and Spit be P.

Then S P = S M . sin M cosec S.

$= 4033 . \sin 77° 30' . \operatorname{cosec} 68° 16'.$

log 4033	3·605337
sin 77° 30′	9·989581
cosec 68 16	10·031522

log 3·626440 = 4231 feet = S P.

The scale adopted is 6 inches = 6050 feet.

Therefore 4231 feet $= \frac{4231}{6050} . 6 = 4{\cdot}196$ inches of paper.

The observed bearing of Spit from Sand Point is N. 72° 14′ E., and the distance is 4·196 inches; lay down the two positions on the paper as described in plotting on p. 221, and from each, project the angles observed; and since it is on so large a scale, each angle should be projected by the chord method (see p. 56, par. 151);

and the objects as shown in the sketch are plotted. At this stage of the work there should be practically no 'cocked hats.'

From the points plotted, the triangulation continues onwards up the river, built up on the original side, Sand Point to Spit, and continued on the side Spit to Elbow, just as has been explained in the triangulation of a plan.

Or, if the river here narrows to a creek, or the banks are inaccessible, etc., the continuation can be carried out by compass direction and patent log for distance, or with a pole 10 feet or more long. (See *Coast-lining with a 10-Foot Pole*, p. 238, par. 439.)

Some way up the river, suppose there is another convenient Mound, X, that can be connected with the work (see Plate III.): use may be made of this, and an attempt made to 'fix' it, from some point back on the first part of the triangulation, by a true bearing and angles.

If, however, Mound M only is in sight, then a true bearing should be taken of it; this will not 'fix' Mound X, but will only give a line of direction to it; laid off at Mound M from the true meridian, it will be a check on the slue of the last bit of work, but no check as to distance.

The survey having proceeded as far as is intended, there is more or less a vast accumulation of errors—errors of observation and of plotting, and therefore in the bearing of the end of the work from the beginning, and also in the total distance.

What is required then is, if possible, to check the total errors.

Either there is nothing available, or, if there is, it is not visible from the original Mound △; or only Mound △ can be sighted; or Mound and one, or perhaps two, other of the original marks can be sighted; or, again, it may be possible that the other Mound, X, alluded to can be seen from Mound M.

In the first case, if nothing is visible, there is nothing for it but to 'fix' astronomically, that is, find the latitude and longitude of the halting-place.

This will give an 'absolute' position—that is, a position not depending on any other results obtained by the same means. This 'absolute' position can be compared with the latitude and longitude of Mound and the bearing and distance from it deduced: there is a limit to the distance that makes an astronomical fixing effective (see p. 248, par. 445).

But, if the latitude and longitude of Mound has been derived from an outside triangulation, Ordnance or Hydrographic, and the observations made are with artificial-horizon sights, the difference of latitude and the difference of longitude of the two positions, will not be so satisfactory as when the same observer, with the same instruments, makes his own, and probably equal errors, at both places; for, in this last case, since the errors are much the same, the *difference* of results will be far more accurate than if

the difference is deduced from a comparison between the observations made by the surveyor and those made by someone else.

Hence it will be better, under the conditions, for the same observer to take observations for latitude at both ends, and to find the difference of longtitude by meridian distances. (See *Meridian Distances*, p. 415.)

If Mound △ only can be sighted, a true bearing must be obtained of it; and, since in the present case this will give a line nearly east and west, it could make amends for omitting to take observations at Mound; for then, admitting the astronomical position of mound, the difference between it and the position found by observation will be 'checked' by that true bearing line; or, again, with such an east and west line, astronomical lines of position near north and south might be sufficiently satisfactory. This entails equal altitudes, for longitude only.

Lastly, if Mound △ and another, or any two others in the triangulation, are to be seen, then the position can be determined by a true bearing to one of them, and angles to the others.

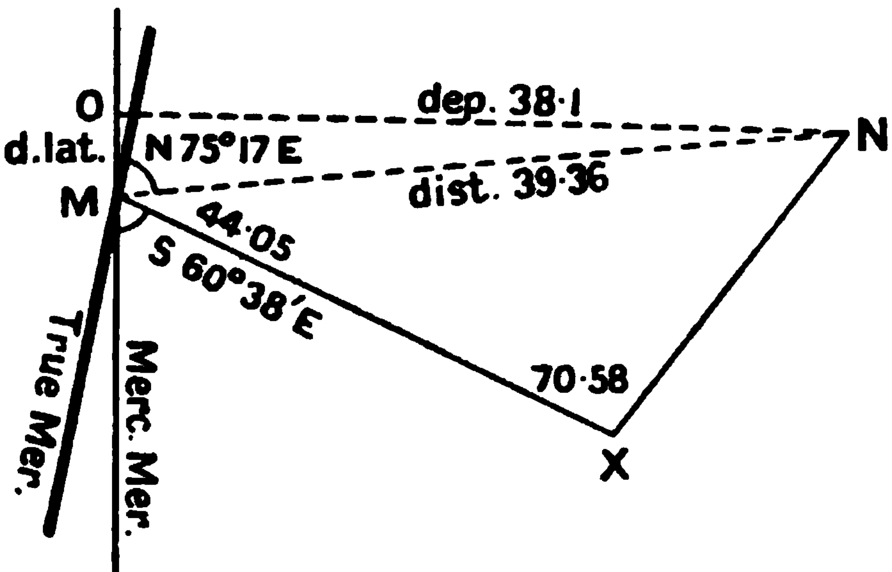


FIG. 174.

For example, in fig. 174 let M be Mound, the Δ be fixed on the paper, and X the last point on the river arrived at. Let N be another point whose geographical position can be ascertained from the Main Hydrographic, or the Ordnance Survey of the coast. Its position is not plotted on the paper we are working on. We then have the latitude and longitude of both M and N; and hence diff. lat. and diff. long., from which the Mercatorial bearing and distance can be deduced.

Let lat. M be	50° 30′ N.	long. M	15° 00′ W.
lat. N	50 40	long. N	14 00
diff. lat.	10	diff. long.	60

$$\text{Dep.} = 60'.\cos 50^\circ\ 35'\ (\text{mid. lat.})$$
$$= 38{\cdot}1 \text{ miles O N (see fig. 174).}$$

$\frac{\text{O M}}{\text{O N}} = \tan \text{O M N}$ (the Mercatorial bearing of N from M),

O N $= 38{\cdot}1$ miles, and O M (diff. lat.) $= 10'$,

$\frac{\text{O N}}{\text{O M}} = 3{\cdot}81$ (log natural tan) $= 75^\circ\ 17' = $ O M N,

and O N M $= 90^\circ - 75^\circ\ 17' = 14^\circ\ 43'$.

To find the distance between M and N.

$$\begin{aligned} NM &= OM \,.\, \sec OMN \\ &= 10 \,.\, \sec 75^\circ\ 17' \\ &= 10 \,.\, 3{\cdot}9363 = 39{\cdot}363 \text{ miles.} \end{aligned}$$

449. Calculating Bearing and Distance between two Places by Spherical Triangle.—The result may be calculated by spherical triangle instead of by traverse; for (see fig. 175)

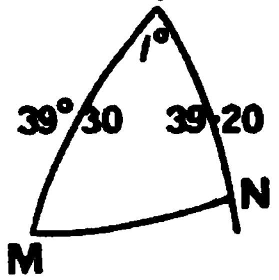


FIG. 175.

M P N = diff. long. = 1°; P M = co-lat. of M = 39° 30′; P N = co-lat. of N = 39° 20′; P M N = true bearing, and M N = distance.

To find M N—

log hav	1° 0′	5·881684
log sin	39 30	9·803510
log sin	39 20	9·801973
difference	0 10	5·487167 hav = 0° 38′ 06″
vers. log	0° 38′ 06″	0000061
vers. log	0 10 00	0000004
vers.		0000065 = 0° 39′ 22″ ≒ 39·36,

which is the same as before.

To find angle P M N—

log cosec	39° 30′ 0″	10·196490
log cosec	0 39 22	11·941065
	38 50 38	
	39 20 0	
log ½ hav	78 10 38	4·799700
log ½ hav	0 29 22	2·630668
hav		9·567923 = 74° 54′ 07″.

The true bearing of N from M is therefore N. 74° 54′ 07″ E.

450. Application of Convergency.

$$\text{Convergency} = \text{diff. long.} \,.\, \sin \text{mid. lat.}$$
$$= 60 \,.\, 772 = 46{\cdot}32 = 46'\ 19''.$$

True bearing		N. 74° 54′ 07″ E.
½ convergency	+	0 23 09
Merc. bearing		N. 75 17 16 E.

And this agrees with the bearing found by traverse.

Let the true bearing of M from X be N 60° 15′ W, and the angle at X between M and N be 70° 55′; then

1. *To find the Mercatorial bearing of* M *from* X, the diff. long. being 60′, and the mid. lat. = 50° 35′.

$$\text{Convergency} = \text{diff. long.} \,.\, \sin \text{mid. lat.}$$
$$= 60' \,.\, \sin 50^\circ\ 35'$$

log 60′	1·778151
log sin 50° 35′	9·887926
log	1·666077 = 46·35′ = 46′ 21″

The true bearing of M from X is	N. 60° 15′ W.
½ convergency	+ 0 23 W.
The Merc. bearing is therefore	N. 60 38 W.

Therefore the Mercatorial bearing of X from M is S. 60° 38′ E. And, since the Merctaorial bearing of N from M is N. 75° 17′ E., therefore the angle at M between N and X is 44° 05′. The angle at X between M and N was 70° 55′.

Then the angle at N between M and X is 180° − (70° 55′ + 44° 05′) = 65° 00′; and

in M N X,	M = 44° 05′
	N = 65 00
	X = 70 55
	180 00, M N = 39·36 miles.

It is required to find the distance M X.

$$MX = MN \,.\, \sin 44^\circ\ 05' \,.\, \operatorname{cosec} 70^\circ\ 55'$$

log M N	1·595340
sin 44° 05′	9·842424
cosec 70° 55′	10·024548
log	1·462312 = 29·0 miles.

This is the distance of X from M, and can be laid off from M on the bearing of S. 60° 38′ E. and fixes X.

As this position of X is sure to differ from that found by triangulation, all the other points in the triangulation will have to be readjusted. If rigid accuracy is aimed at, this involves a considerable amount of calculation, since every side of every triangle would have to be corrected.

For example, suppose there has been a constant error made, and the whole distance to be 1 inch of paper (roughly 1000 feet) too long, and $\frac{1}{2}°$ out in bearing, equivalent to 1·5 inches at right angles to M P.

In fig. 176 let T be the true position, and O the observed.

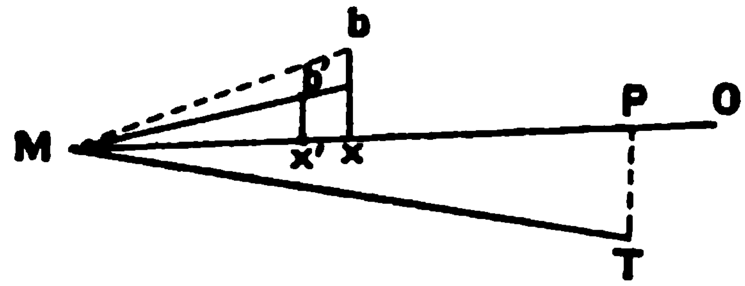


FIG. 176.

From T draw T P at right angles to M O, where M O is 29 miles and M P T is $\frac{1}{2}°$. M P can be considered equal to M T, and P O equals 1 inch, or 1000 feet, the amount in error in distance, while P T is the amount in error at right angles to P M or P T.

Let b be a plotted point, distant 12 miles from M.

From b draw bx at right angles to M P; $\mathrm{M}x = \mathrm{M}b \cos b\mathrm{M}x$.

Let $b\mathrm{M}x = 30°$; then $\mathrm{M}x = 12 . \cos 30° = 10{\cdot}4$ miles.

$$\text{Error in M}x : \text{error in M P} :: \mathrm{M}x : \text{M P or M T};$$

$$\text{Error in M}x = \frac{\text{error in MP}}{\text{MT}} . \mathrm{M}x = \frac{10{\cdot}4}{29} \text{ inches} = {\cdot}36 \text{ inch};$$

and Mx will have to be moved towards M ·36 inch.

For the error in bearing, project at M the angle $b\mathrm{M}b' = \frac{1}{2}°$. From x' draw $x'b'$ parallel to bx intersecting Mb' in b'; then b' is the corrected position of b.

If necessary, all the other stations can be similarly moved, and the coast-line either re-plotted or otherwise fitted in. If the positions M and O are connected astronomically, the same process is carried out, only the positions are shifted in latitude and longitude—that is north or south, east or west, in their proportions of the total error in latitude and longitude between the astronomical and triangulated positions. For a complete readjustment of this kind, see Appendix V.

It may be suggested here, that if from X, fig. 174, more objects than M and N are visible, then a large 'outer' triangulation could have been made from M; and from some of the points

of this triangulation, that of the river bank could have been connected to it from time to time; and this is in fact what will occur when practicable (see fig. 171, p. 248).

There would then be an 'outer' triangulation, with the sides of the triangles some considerable distance apart, perhaps 10 miles or so; and inside of these the 'secondary' triangulation of the river banks would have its place, with far better results than if carried out independently, and afterwards squared in.

To put in the coast-line between the points :—

Commencing on the north side at Tree, Twig	8°	Mid. N.
Rock	19	
At Rock, trend on 50 yards	18° 0′	Mid. N.
At Twig, Tree	9°	Rock
	15	trend back
At Mid. N., Tree	8°	Rock
	21	Twig

From Tree to Mid. N. was a sandy beach.

At Mid. N. ⊣ mangrove 5° Elbow.

At Elbow, Mid. N. 5° ⊢ mangrove.

In a similar manner the remainder of the north side was done, and the south side in much the same way; or by a combination of fixing, by station pointer from the marks on the opposite shore; and as described in *Coast-lining*, p. 235, par. 437.

For the plotting of these see fig. 177.

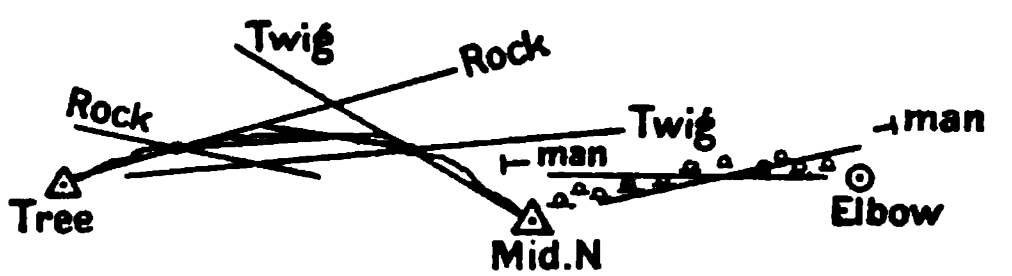


FIG. 177.

For the outer coast :—

451. Example of Coast by 10-Foot Pole.—North side with a 10-foot pole and prismatic compass.

At Tree sent pole on,

pole (1)	2° 40′	bearing	324½°
pole (2)	1 05	„	313

The angles are those subtended by the pole, measured 'on' and 'off' the arc of the sextant, and the bearings are all read from magnetic north, the variation being, as stated, 15° W.

A 10-foot pole table is required (see p. 238).

A number of magnetic meridians are drawn, from which the bearings are laid off with the protractor, as explained in *Coast-lining*, par. 439. Pole (1) and pole (2) leave a small mark, such as one stone standing on another.

The observer goes to the mark made at pole (1).

	pole (2)	1° 40′	bearing 308°.
At pole (1)	pole (2)	31 30	grass,
	reef Rock	38 00	pole (2).

Then the observer goes to the mark of pole (2) and sends the pole on to (3).

At pole (2)	grass	33° 00′	pole (1)'s mark ; bearing 128½°
	pole (1)	86 30	reef Rock
	pole (3)	1 16	bearing 325°
etc.	etc.	etc.	etc.

This was the system by which the poles were 'fixed' on the foreshore, and objects along the high-water line 'shot up' from them, as explained on p. 240.

The south shore was done in a somewhat different manner: the 'fixes' were obtained by leaving the pole at Sand Point, and taking a bearing of and a distance from it.

'Fixes' (1), (2), and (3) are on the outer edge of the dry foreshore, or in a boat aground on the edge of the mud.

At (1)	pole at Sand Point	1° 00′		bearing N.
	Sand Point	29 00	*a*	
		90 00	*b*	mangrove bushes.
		118 00	*c*	
At (2)	pole at Sand Point	0 35		bearing 333°.
	Sand Point	2 00	*a*	
		9 00	*b*	
		0 00	*c*	
	position (1) back	20 00		Sand Point.

Then the pole was sent on to (3).

pole at (3) 1° 06′ bearing 122°.

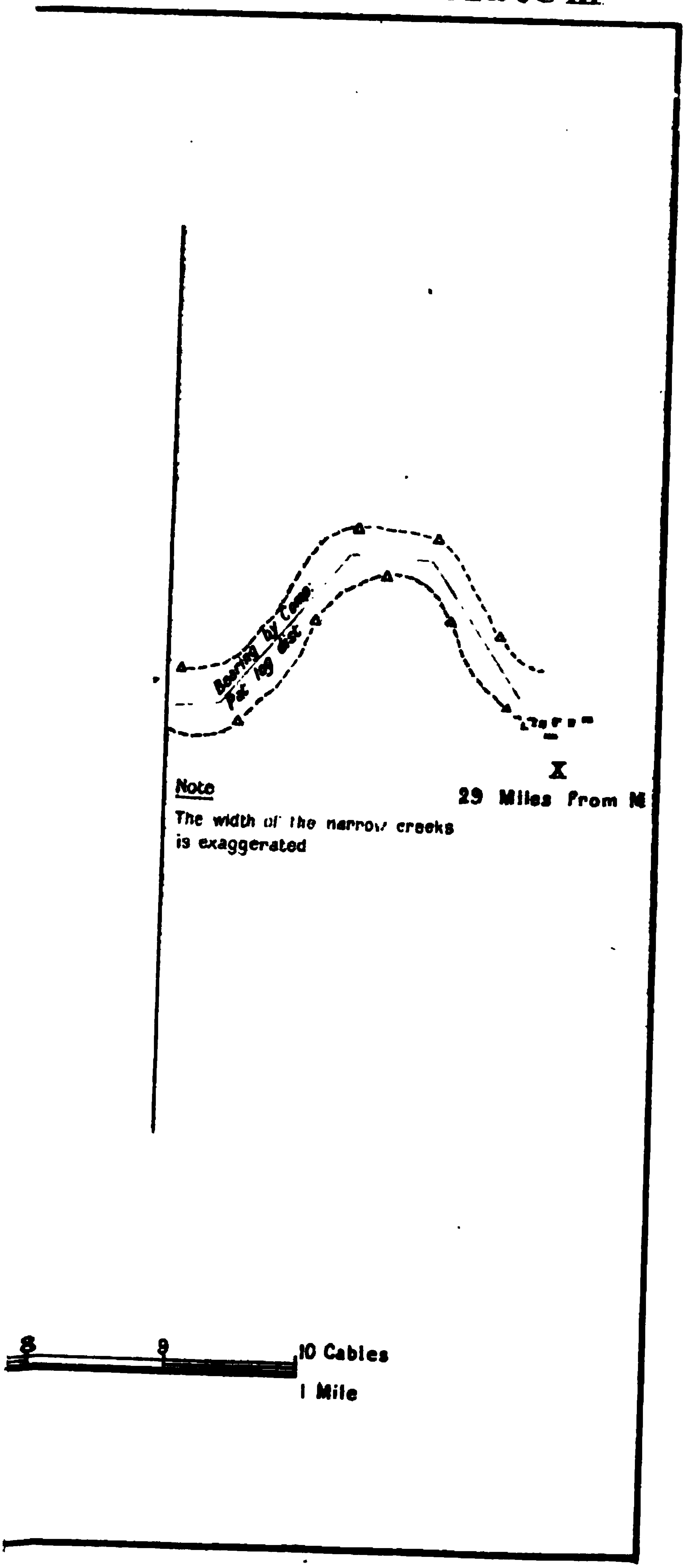
At pole (3)	position (2) back	11° 00′	Sand Point.
			a ϕ *b* ϕ Sand Point.
	Sand Point	15 00	*c*

And so the outer coast-line was put in.

Sounding inside the river:—

Since each of the stations Tree, Mid. N., Elbow, Sand Point, Mid. S, Stone, Round, etc., were marked, they would be used for 'fixing' the position of the boat when sounding; in this case both for inside the river as well as for outside.

Plate III


X
29 Miles from M
Note
The width of the narrow creeks is exaggerated
8
9
10 Cables
1 Mile

A tide-pole was erected at Sand Point.

(As this example was to illustrate the plotting of the triangulation and of the coast-line, the tide-pole readings are omitted.)

The soundings given are plotted to show what part the coast line marks played in 'fixing' the boat while sounding.

Soundings in feet :—

IX. 00 A.M. At ship,	Mound	83°	00′	Tree	28°	00′	Sand Pt.	45
Shoved off. Steered towards tree.								
41 39 27 35 33 31 29								
	Mound	17	40	Tree	50	10	Sand Pt.	25
22 19 16 13 10 10 16								
IX. 30	Mound	42	30	Mid. N.	40	00	Spit	23
		38	30		44	30		36
30 30 28 24 24 28								
	Mound	12	50	Tree	50	40	Sand Pt.	33
X. 15		6	40		52	40		37
33 29 27 27 29 31 37								
	Mound	34	30	Tree	48	00	Spit	44
X. 40		32	00		52	30		42
36 32 29 29 29 32 34								
XI. 00	Mound	2	00	Tree	52	40	Sand Pt.	39
etc. etc.	etc.			etc.				

The tide reductions were

IX. 00	9 feet.
IX. 30	11 feet.
X. 00	12 feet 3 inches.
30	14 feet.
XI. 00	16 feet.

452. Extending the Soundings Seaward ; Marks Necessary, and how to Fix them.—It will be noticed that the extent of the sounding outside the bar is limited; this is due to an insufficiency of marks to 'fix' by; and it might be advantageous to extend the work, both further out, and further to each side of the bar. But there are not sufficient objects, suitably placed, to 'fix' the boat; Mound can still be used as a left object, and Tree or Sand Point as middle objects; but means must be found to find, or erect, a right object, and to 'fix' its position.

If, so far, the subject has been carefully followed and understood, some means will be devised to 'fix' a natural object (if there is one), or one erected for the purpose, on the shore south of Sand Point, or on the summit of a rise or hillock; or, failing these, a floating mark, boat, or beacon moored in the required position.

To 'fix' any of them, it may be necessary to take an angle to it from Mound, in which case it must be visible from Mound. (See *Example* 1, p. 198.)

Taking the very worst contingency, if the erected or natural

mark is not visible from any of the 'fixed' stations, it will have to be 'shot up' (see p. 233) from three boat positions, each position being 'fixed,' so far as is practicable, by a 'direct' angle from one of the stations, and 'calculated' angles from Mound and two stations. (See *Example* 1, pp. 203, 204.)

453. *Example.*—To 'fix' the position of the supplementary mark X, placed on the coast for the purpose of extending the soundings beyond the limits of the entrance.

1st case. Suppose X is visible from M only (see fig. 178).

M will take angles P to X,
P to boat.

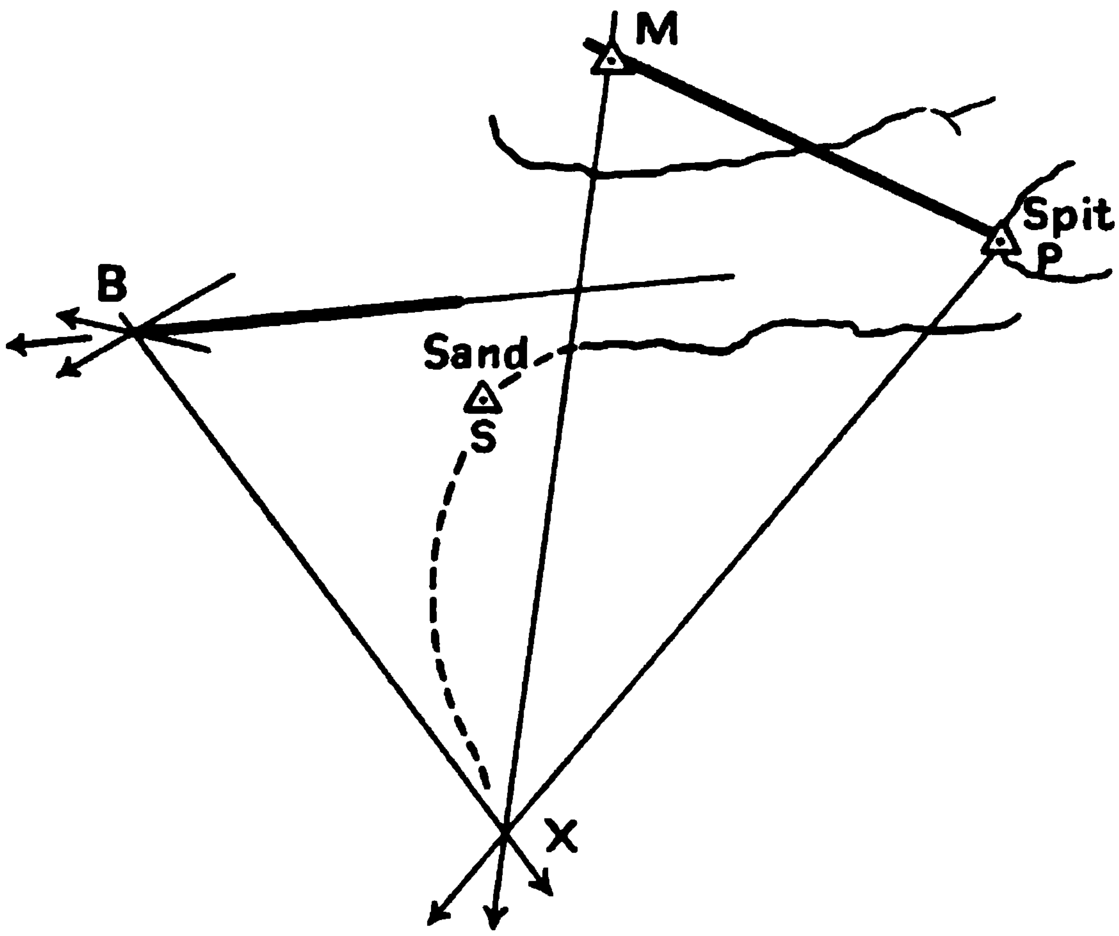


FIG. 178.

Let B be the position of the boat placed in that position for the purpose of fixing X: angles to B are also taken from S and P.

1. The angles projected from these places (M, P, S, see arrows) fix B.

The following angles are taken at B:—M to Spit,
M to X.

2. There will now be two lines, as shown in fig. 178, projected through X—one from Mound, and the other from B.

At X take the angle between M and B.

In Δ M P B we have the three angles M, P, B, and the length of side M P is known from the triangulation; then find P B and M B.

In Δ B M X the three angles have been taken, and, knowing M B, find B X.

3. In Δ B P X, knowing B P and B X and the angle P B X (observed from the boat), find angle B P X. Project this angle from P; the line should go through X. *Notice that* P *and* X *are not*

visible from each other, yet a line of reference has been calculated. The thick lines in the figure show from what line of reference each angle should be projected.

It may be mentioned here, that by erecting a pole, or a high staff at M or at X, they may be made visible from each other over such obstacles as trees.

454.—The second case alluded to is when X is not visible from any of the previous marks (see fig. 179).

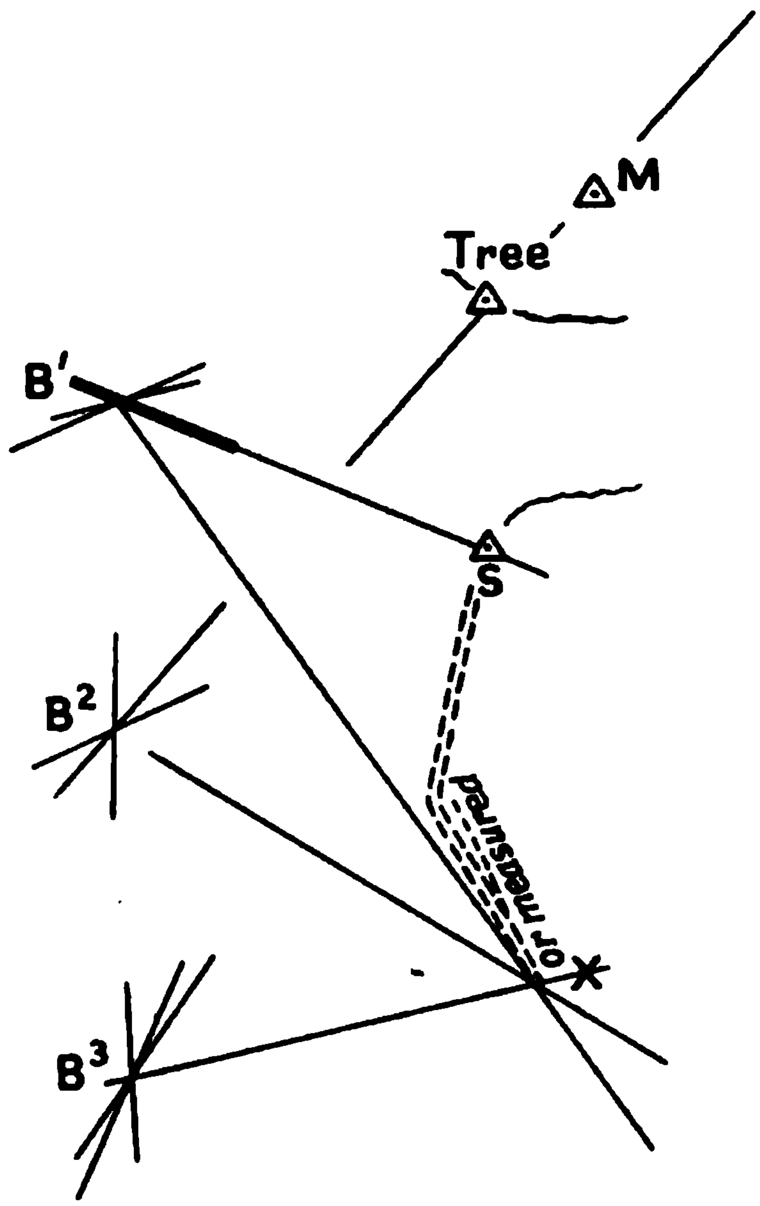


Fig. 179.

At S take B^1 to M
B^1 to T
At B^1 take M to S, also S to X
T to S

Then fix B^1 thus: In Δ S B^1M, S = observed
B^1 = „
M = calculated
180° 00′

In Δ S B^1T, S = observed
B^1 = „
T = calculated
180° 00′

Project the 'direct' angle from S; this gives the line SB^1; the calculated angle from M gives MB^1, and the calculated angle from T gives TB^1.

From B^1 project the angle observed S to X.

The boat moves to B^2, leaving a small buoy at B^1 with a flag on it, so that it shall be visible from B^2.

At S, B^2 to T, At B^2, T to S
B^1 to S
S to X

$$\text{In } \Delta\ SB^2T,\ \begin{aligned} S &= \text{observed} \\ B^2 &= \quad ,, \\ T &= \underline{\text{calculated}} \\ & \quad 180^\circ\ 00' \end{aligned}$$

Project the 'direct shot' from S, and the calculated angle from T.

1. They intersect (badly) at B^2.

At S, the angle between B^1 and B^2 can be deduced; it equals the difference between TSB^2 and TSB^1.

$$\text{In } \Delta\ SB^1B^2,\ \begin{aligned} S &= \text{observed} \\ B^2 &= \quad ,, \\ B^1 &= \underline{\text{calculated}} \\ & \quad 180^\circ\ 00' \end{aligned}$$

2. Project B^1 (calculated), and it should go through the position of B^2: B^2 is now 'fixed' as well as one can possibly do so.

3. Now project the line B^2X, and this 'fixes' X by two 'shots.' Leave a small buoy with a flag on it at B^2, and proceed to B^3.

4. Do the same there as at B^2.

Or the distance from S to X might be measured with a 10-foot pole, or chain, or levelling staff, with a theodolite, both from S to X, and from X back to S, in bits shown by a double pecked line in fig. 179; the bearing being taken with a compass, and X probably 'fixed' with the same accuracy.

A moored beacon, or a boat flying a large flag at her mast, could be 'fixed' in the same way as described, at a position indicated by B^3 in fig. 179, p. 263.

Beyond this, any further extension of the work must be undertaken by a vessel fully equipped for the purpose.

CHAPTER VI.

TIDES AND TIDE REDUCTION.

455. Information Relating to Tides Necessary in a Chart.—An attempt will here be made to offer a very short, elementary and simple explanation of the movement of tides; bearing in mind, that it is in order to obtain the datum necessary for the reduction of soundings from hour to hour, as well as to provide the information relating to the 'establishment of a port' and the 'rise' there of the tide at 'springs' and 'neaps,' all of which are required on a chart.

For an easily digested theory of tides, the student should by all means refer to *The Tides Simply Explained*, by the Rev. J. H. Moxly; and for other details on the subject see *Admiralty Tide Tables.*

456. Reference to Datum.—The operation of sounding has necessarily to be carried out at all states of the tide; and on every chart there is, or should be, a statement of reference to a mark on shore, serving as the datum level for all the soundings shown; hence the soundings from hour to hour have to be referred through the tide-pole to that datum.

457. Datum, that of L.W. Ordinary Springs. — This level or datum is, as near as can be arrived at, the level of low-water ordinary springs, and the mark on shore referring to it is the bench mark.

Without some elementary knowledge of tides and their apparent vagaries, such a datum would be the wildest guess-work.

458. Effect of Wind and Height of Barometer on Tides.—With the rise and fall of the barometer, there occurs a greater or less pressure of the atmosphere on everything. With a strong wind blowing 'with' the direction of the tidal stream, and a low or small atmospheric pressure, the level of the tide in a harbour might be raised several feet; whereas with a wind from the opposite direction, and a reduced barometric pressure, the two influences might possibly counteract each other.

A high barometer, with a strong wind blowing out of the harbour, would lower the level of the tide; but if opposed, the two are counteracting each other.

The effect of the atmospheric pressure is about 1 inch of level of the water, for $\frac{1}{20}$th inch of change in the reading of the barometer.

459. Meteorological Effects not to be Confounded with a True Tide.—These are meteorological disturbances of the tide; they may considerably modify the true tide, and must not be confounded with it.

460. Definition of a True Tide.—The change in the level, and daily rise and fall of the water at any place, which is caused by the attraction of the sun and moon, and which produces a tide 'cone,' is called a true tide.

461. Tide Wave.—This movement takes the shape of a 'wave,' which forms at different places at different times, and recurs at each place at more or less regular periods.

The wave or cone produces a gradual rise on its approach, and a gradual fall as it recedes, though neither are uniform.

462. Tidal Stream.—As the tide cone or wave approaches shallower depths, or recedes from them, the effect produced is a horizontal movement, known as a tidal stream.

463. Effect of Tidal Stream over Shallower Water.—At earliest childhood one has noticed, that as waves reach the shore, they produce a rush of water up the beach, which reaches further when the tide is rising than when it is ebbing.

As the tide 'rises' over 'shallows,' or over a 'bar,' there is a stronger stream over them than over the deeper water.

On a large scale, the same effect is produced by the position of the tidal 'cone' from deep to shallower water.

Conceive a large shoal surrounded by deep water; as the tide rises or falls, the tide cone is passing over it, and the tidal stream flows over this shoal from all directions, north, east, south, or west; so that during the one vertical movement, a tidal stream may be running in more than one direction.

464. Tide Wave and Tidal Stream not Synchronous.—A tide may be rising at any place, and owing to the configuration of the land the tidal stream may be running east for the first hour, south the next, and perhaps west for the remainder of the time the tide is rising.

Particularly in the case of a river, it may have been noticed that the tidal stream continues to run out at the sides of the river, while the tide has actually commenced to rise; and later, the tidal stream will commence to run up the river; in this case the tidal stream has actually been running in two opposite directions during the same vertical movement.

465. Effect of Opposing Tide Streams.—When two tidal streams meet in the open they will cause 'eddies' or a 'tide race'

or 'overfalls'; and, under certain local conditions, if they are confined by land immediately near, such meeting streams will produce a temporary change or cessation in the vertical movement of the tide.

466. 'Stand' in the Tide.—For example, two such tidal streams, as shown in fig. 180, meeting at any part of the channel, would not only create eddies, but the whole body of the water would 'stand,' or there might be a slight rise; and, supposing the west-going stream to be the main body of the tide, while the south-east-going stream is an interference with it, then the progress of the dominating stream will be temporarily stopped; the vertical movement of the tide will, in fact, be stopped for a time, until the main stream overpowers the other.

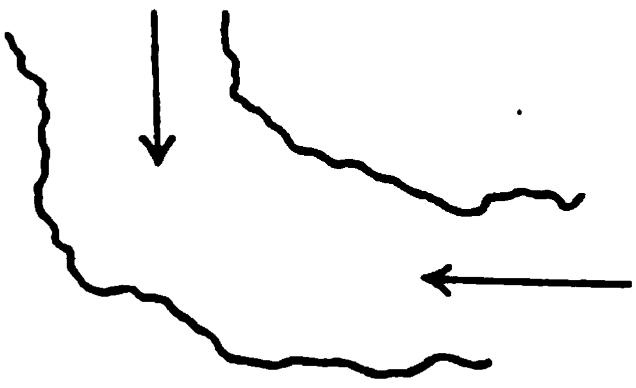


FIG. 180.

467. Method of Observing Tidal Streams.—The force and direction of the 'tidal stream' is found by observations made at different positions, and inserted in the chart. (See Plate IV.)

A boat anchors at a 'fixed' position, and, by means of a 'log-ship' and 'hour-glass' (just the same as a ship's, but the log-ship is on a larger scale), ascertains the strength of the tide in knots; the direction being taken by the angle of the line or by its bearing.

The tidal stream will, as has been pointed out, be accentuated by friction of the land, by passing over an uneven bottom, or by shallow water, and is always greater in a river than in the open sea or in deeper water.

The log-ship is, however, under the influence of wind and of the consequent surface drift, and the line drags; for more accurate data, more elaborate means are employed, such as a body wholly immersed, with an indicator of its position (see Wharton).

But at the entrance of a port, at the periods both of springs and neaps, it may be calm enough on many occasions to allow of the use of either a tidal log-ship, or a float with an indicator above water, or of drifting in a boat, the position of which is fixed at intervals of time.

468. Definition of a Current, as Distinguished from a Tidal Stream.—A current, on the other hand, is a surface stream; and either a more or less constant ocean current stream, or a local current drift stream, such as would occur at places under the influence of trade winds or of monsoons or of periodic winds; in any case, it is in no way attributable to the same causes which produce tides, although the total strength of the

tidal stream is influenced by it, and it may mask the *true* tidal stream.

469. Information Required in a Chart, H.W.F. and C., and Rise of Tide.—The information required in a harbour or river for the purpose of sounding reductions, is a datum of the vertical movement of the tide.

Vertical movement is separated into:—

(1) The time the tide wave reaches its daily maximum and minimum, as well as its periodic maximum and minimum; hence a time datum.

(2) The amount of its daily rise from hour to hour; and hence a height datum.

470. Daily Recurrence of High and Low Waters.—The interval between the daily maximum (high water) and the daily minimum (low water) is, in the majority of places within the tropics, about 6¼ hours; and high and low water occur at these places twice a day.

The time between high water and the succeeding high water is about 12 hours and 24 minutes.

471. Normal and Abnormal Daily Tides.—Under normal conditions, the movement of the tide is as in fig. 181 (*a*). In places it may be either as in (*b*) or (*c*) of same fig. In the last two cases, it is due to the interferences explained in par. 466; or it may be owing to the geographical position, and the interposition of continents between the place and the position of the tide cone (*vide* Moxly).

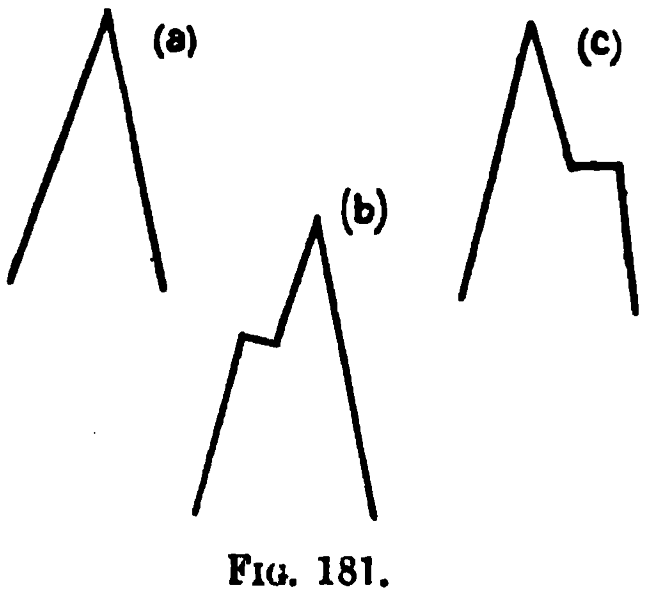


FIG. 181.

Usually the interval of time between H.W. on one day and that on the next day is roughly 24 hours 48 minutes.

472. Movement of Tides Referred to Movement of Moon.—Now the average *difference* of the moon's passage across the meridian from day to day is about 48 minutes (it ranges from 40 to 65, depending on the distance of the moon from the earth), and, since this corresponds with the daily retardation of the tide wave, the moon can be adopted as a rough clock for the time of tide, calculating from her time of meridian passage.

473. Actual Tide Wave not under Moon.—But the actual daily tide does not reach its maximum at the same instant that the moon is on the meridian, at all places.

474. Lunitidal Interval.—The interval between the time of

the moon's meridian passage at any place, and the following H.W. there, is called the lunitidal interval.

475. Age of Tide; Lagging.—A tide can be said to belong to, or to be associated with, the moon at a particular date; and it comes about that, instead of exactly following closely in the wake of it, the tide may be some days behind: in that case the tide is said to have an 'age.' For example, the 'derived' tide wave in the English Channel, on its arrival, is 1¼ days later than the moon with which it is associated: in its progress through the Irish Channel, and round the North of Scotland, the wave reaches the east coast of England 2½ days *old*, or 2½ days after the age of the moon who owned it, she having passed the meridian twice in the interval. A tide, for instance, belonging to the moon whose age is 3 days, may follow 2 days behind her; and since she is 3 days beyond the date of her maximum effect, and the tide is lagging 2 days behind, the tide is only 1 day beyond 'springs.'

476. Tides Attributed to Sun and Moon.—A tide wave is attributed to both the sun and moon; that caused by the sun recurs every 12 hours, that by the moon every 12 hours 24 minutes—that is, at both the 'superior' and 'inferior' transits.

477. Zero for the Measurement of Time of Tide 'Sizigy' Establishment.—The zero for the time of tide at any place is when the sun and moon are on the meridian at the same instant, *i.e.* in sizigy; and the interval (lunitidal), to the H.W following, is the *establishment* of that particular place; and hence is the time datum. This is *calculated* for the time the moon's meridional passage was $0^h\ 0^m\ 0^s$; for, this hour is not necessarily the time she actually passes the meridian on the day of 'new' moon.

The establishment of the port (H.W.F. and C.) is required on all charts and plans; it is the time established at the place on the day of F. (full) and C. (change or new).

478. Establishment only Occurs at Sizigy.—The establishment is only constant on that occasion of coincidence.

479. Observations for Finding Time of H.W.—To find the exact minute of H.W., take observations for about ½ hour before H.W. and L.W., at intervals of about ten minutes, noting the times of the highest and lowest. For anything more elaborate, refer to *Admiralty Tide Tables*. Then draw a figure, as shown on p. 275.

480. Observations for Reducing the Time of H.W. to Sizigy.—The passage of the sun and moon over the meridian at the same instant may not occur during the stay at any place; in such a case, it is better to draw a curve of the lunitidal intervals, for each time of the moon's meridian passage, for the days on which the time of H.W. is obtained, and to deduce the position

of the lunitidal interval when the moon's passage is 0 hr. (see fig. 182).

This entails a period of at least several days before and after the new or full moon.

Transit.	H.W.	Lunitidal Interval.
h. m.	h. m.	h. m.
10 33	1 07	2 34
11 21	1 51	2 30
0 09	2 29	2 20
0 55	3 03	2 08

Fig. 182.

481. To find Time of Establishment from Tables.—But when the stay does not include these dates, then (see *Admiralty Tide Tables*, or Tables 1 and 2 in App. XIII.), given the hour of the moon's transit, from the *Nautical Almanac*, and having the time of H.W. as before, the lunitidal interval is deduced; then, depending upon the age of the tide, which must be accepted from the surrounding geographical position, the necessary corrections given in Tables 1 or 2, App. XIII., are applicable.

Example, to find the time of moon's meridian passage:—

On the 6th June 1908, meridian passage (upper transit) is $5^h\ 56^m$ at Greenwich: the difference between meridian passages on 6th and 7th is 43^m. Longitude is $15°$ W. $= 1^h$.

Correction for longtitude (see *Inman's Tables*) $= +2^m$, therefore time of moon's transit $= 5^h\ 56^m + 2^m = 5^h\ 58^m$.

482. 'Spring Tides.'—When the sun and moon were on the meridian together, their combined tide wave produced a 'spring' tide, though, as before, the actual tide wave associated with them does not arrive for some time after, usually days (see *Age of Tide*, p. 269). This periodic tide, like the daily one, may be said to be dominated by the time of the moon's meridian passage and her declination, etc.

This retard of the tide is explained by Moxly as not being the tide cone under the moon, but a 'derived' tide wave, or one retarded by 'interference' (see further explanation).

At this conjunction, then, of the sun and moon the tides will rise in most places higher above the mean water-level, and fall lower, than those both before and after the coincidence.

If, in fig 183, M be in the position of the moon and S that of the sun, both with declination 0°, *i.e.* over the equator, and L O L′ a meridian passing under them, then their joint action produces

the maximum tide wave or 'cone' at O and O′ (see App. XII.), but not necessarily on the same day that they were in this position; it depends on the geographical position of the place, which is indicated by the establishment, and age of tide. If the earth is moving in the direction shown by the arrow, then a place in latitude O′ will pass through L.W. at C, $6\frac{1}{4}$ hours after; and $12\frac{1}{3}$ hours after will be at O′, again at H.W.

483. Lunar and Anti-lunar Tide at 'Superior' and 'Inferior' Passage.—In fig. 184, if M be the position of the moon, and S that of the sun, then a place in latitude O will, after passing through a L.W., arrive at O^1 and again have a H.W., but not so high as it was at O; in this case the day tide, which occurred on the side of the 'superior' passage of the sun, will be higher than the night tide. Again, a place O^2 will have on arriving at O^3 the night tide larger than the day.

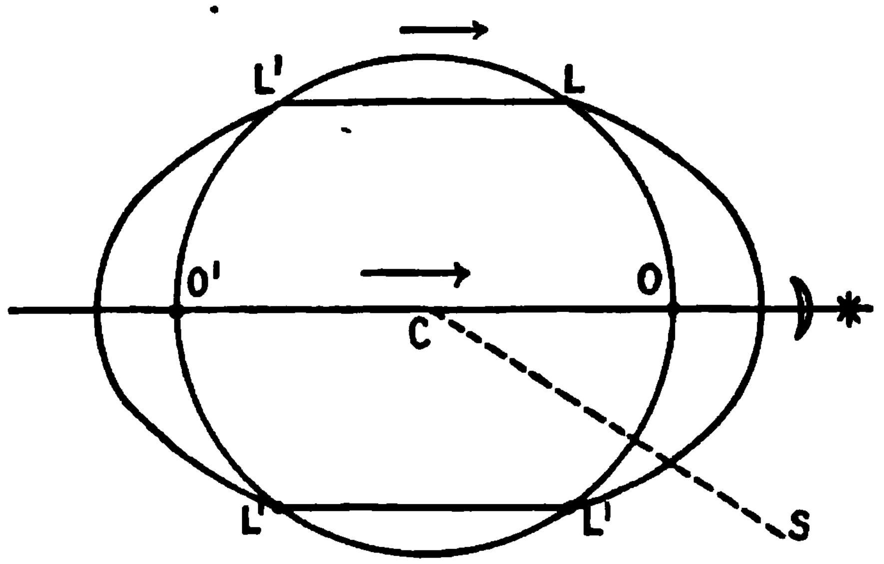


Fig. 183.

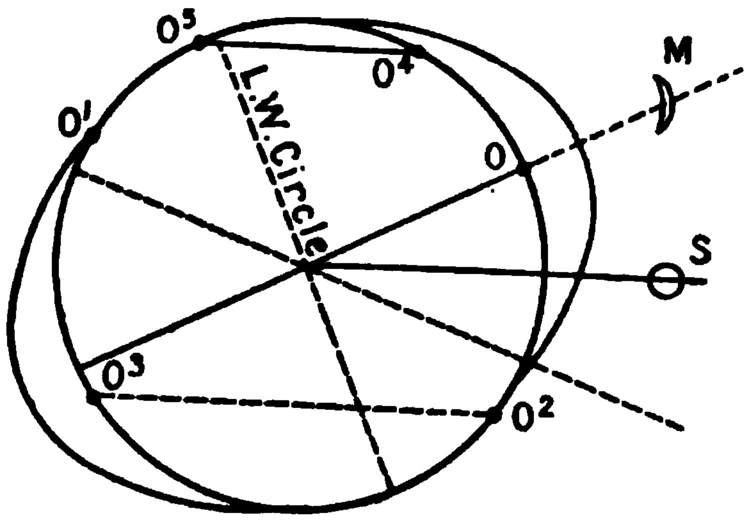


Fig. 184.

484. Places with One Tide a Day.—And it is possible to conceive places in much higher latitudes, such as at O^4 to O^5, where from high to low is 12 hours, and there is no high water at all at the inferior transit; and from a low high at O^4 to a L.W. at O^5 is 12 hours.

485. Low High Tides and High Low Tides.—From the above figure, the position of the sun and moon can be placed according to their declinations, and the wave measure graduated to the distance of either luminary from the earth, showing the various relative heights that are obtainable on different occasions of 'spring' tides.

For example (see fig. 185), take the extreme cases: one is shown dotted. M and S have the same declination: in the one case it is north for both, and both are at their nearest to the earth; in the other case, both are in south declination, and at their furthest from the earth.

At a place O, the spring tides in the first case will be much higher than they would be in the second, or dotted positions,

where the luminaries are at extreme south declinations; the tide cone they produce being at the furthest distance from O, and their distance from the earth being the maximum.

By shifting the position of S in the dotted line from south to north, leaving M as it is, in extreme south declination, both now being furthest from the earth, and in extreme opposite declination, the 'spring' tide at O will be very small for a 'spring' tide; and at O′, smaller still.

486. **Neaps.**—Seven and a half days after the conjunction of M and S, as shown above, owing to the moon's meridian passage being 48 minutes later day by day, they will have separated $7\frac{1}{2} \times 48$ min. $= 6$ hrs.; and then, while the sun is trying to create its H.W. at 6 P.M., for instance, the lunar tide crest is at noon. At these periods the tides will not rise so much above M.W.L., nor will they fall so low as at 'springs.' These particular periods are called 'neaps.'

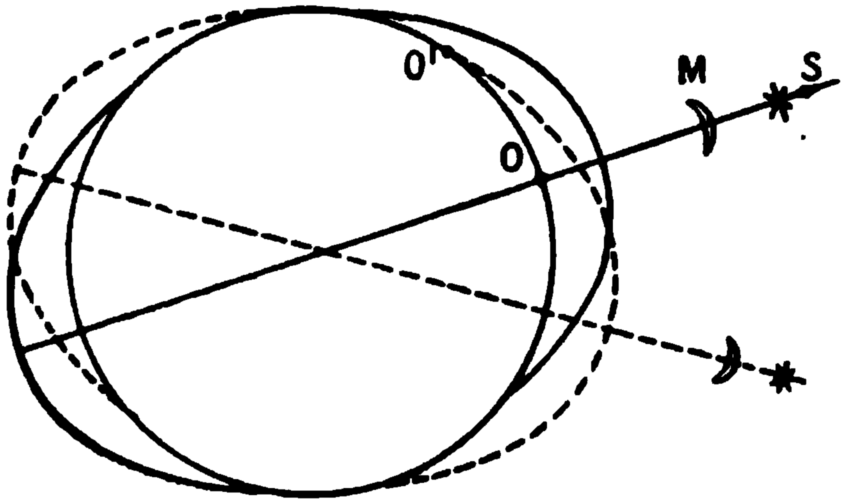


FIG. 185.

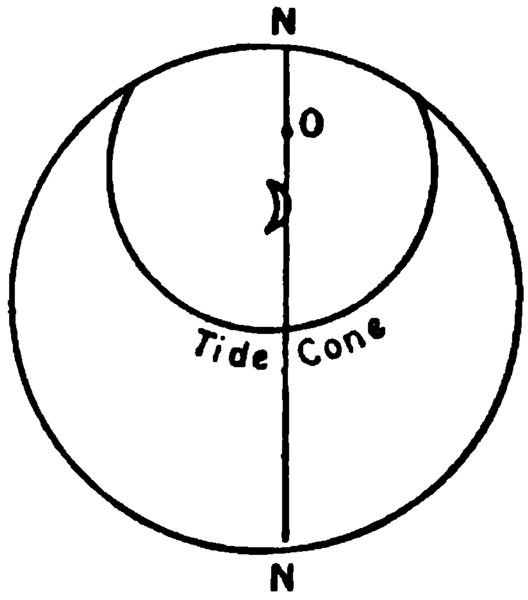


FIG. 186.

In fig. 186, N N represents the meridian, and the moon is over it, either at its nearest or at its furthest distance from the earth. S^1, S^2, S^3 are the positions of the sun, in respectively north, 0°, and extreme south declination, also nearest or furthest from the earth.

Let O be a place on the earth's surface.

Then at those positions of the luminaries 'neap' tides will occur.

487. **High Neaps and Low Springs.**—It is conceivable that by shifting about the positions of M and S (fig. 185), as, for instance, if M is at extreme north declination, and nearest the earth, and S at extreme declination and also nearest the earth, the 'neap' tides at O will be very large for neaps, and possibly be equal to, or even higher, than the very small 'springs' spoken of before, or perhaps that might occur at any indefinite 'springs.' As Moxly states, "the tide may gain more from the approach in declination than is lost by the decline towards 'neaps.'"

And it is equally conceivable that, by selecting the geographical

position and adjusting the position of the sun and moon, such 'strange' facts as the highest tides occurring before new or full moon are sometimes realised, as, for instance, at Hong-Kong.

488. Proportion of Neaps to Springs Differs at Different Places.—Therefore the rise of 'springs' fluctuates, and so also does the rise of 'neaps.' No definite proportion exists between the heights of 'springs' and 'neaps' that holds good for all places; it is, in fact, different in different places at different times, and no *scale* table can be laid down, as is possible, however, with the times of their occurrence.

489. Long Observations Necessary to Deduce a Correct Datum.—The explanation given, shows the great difficulty there must be in deducing a very accurate level of the datum (L.W.O.S.) at a great many places. The tides there, may be regular enough in their turn, but their irregularity lies in the relative positions of the luminaries' tide 'cones,' and in their irregular resultant; and it is only once in eighteen years that the same relative positions recur.

490. Rise of Tide Stated on a Chart.—Besides requiring the establishment (H.W.F. and C.), *i.e.* the time of H.W. at full and 'change' of moon, at its 'upper' transit, a chart should indicate the 'rise of springs' as well as the 'rise of neaps'; and also the position of the 'datum' with reference to a 'bench mark' , a mark accepted as a permanent record of reference to either high- or low-water ordinary 'springs.'

The tide-pole is required for these matters; and, since all the soundings have to be reduced to the adopted datum, a tide-pole must be erected to obtain the necessary reductions hour by hour during the period of sounding, to reduce them to what they would be when the tide is at the level of the datum.

491. Position of Tide-pole.—Now the tide is later up a river or a creek than at its mouth, and the rise and fall is less than at the entrance.

A tide-pole exposed to a 'swell' would be difficult to register, and so would one exposed to the open, where a small wind makes a big sea.

In a harbour, then, the happy medium must be found. Then there is the question of who is going to read it—whether from the ship or from the shore—so that no definite statement can be made as to exactly where it shall be, though word has been said about where it should not be.

If it can be nailed to the side of a pier, or a pile, so much the better as regards the trouble of putting it up (fig. 187 illustrates the trouble often involved). If not, then, given a sheltered spot, means of access, and a boat, stand it and 'stay' it upright.

Any plank or baton will do for a tide-pole; it should be about 20 feet long, marked in feet and inches, and, if desirable, painted like a barber's pole.

Fig. 187 shows how it may look when erected and secured,

when required to stand for months; a lantern is attached for reading after dark.

Dead low water should be the chosen hour, and the party erecting it, should be ready to wade up to the armpits.

It should be needless to say, that every consideration must be given that at *no* ordinary period of the tide should the pole dry, unless, of course, the rise of the tide is more than 20 feet, *i.e.* above the top of the pole, it will not do so; if it remains up through 'spring' tides, it must not dry at L.W. springs. If the pole is set up at L.W., showing 3 feet, this will only allow of it registering a further fall of 2 feet to L.W. springs—allowing

FIG. 187.

1 foot to be in the bottom—and allows only of a rise of not more than 19 feet at 'spring' tides.

492. Occasion for two Tide-poles.—If the 'spring' tide rises more than this, two tide-poles will be necessary, one standing further inshore than the other, and marked from 18 feet upwards, so that when one gets covered by the tide, the pole further inshore 'picks up' the register; and *vice versa*, as the inshore one dries the outer pole continues the register. There must be a coincident reading on the two poles at one level of the water.

If the tide-pole is required for sounding for a few days only, a less elaborate arrangement will suffice, though the method of setting it up is much the same.

493. If Necessary, use an Approximate Datum while Sounding.—The tide-pole will, of course, have to be read during the whole period during which sounding is taking place; but before any reductions can be made to the soundings, the 'datum' must be decided on (see p. 277). If there is any reason for hesitating, some approximation to it should be temporarily made

use of, otherwise the soundings taken convey nothing, because they are taken at different states of the tide. This leads to the suggestion that a tide-pole should be erected the very first thing.

494. Tidal Diagrams.—Before proceeding any further, it will be better to explain the meaning of the expressions that will be made use of.

Suppose the tide-pole readings on the day of the highest and lowest tide to be :—

	ft.	in.		ft.	in.
XII. 30 A.M.	19	6	IV. 00 P.M.	8	0
46	19	8	30	5	10
I. 00 P.M.	19	2	V. 00	4	00
30	18	3	30	2	7
II. 00	16	10	VI. 00	1	8
30	14	11	30	1	2
III. 00	13	7	VII. 00	1	1
30	11	4			

19 feet 8 inches was the highest obtained, and 1 foot 1 inch the lowest.

Seven days afterwards the following were the pole readings :—

	ft.	in.		ft.	in.
VI. 30	16	3	X. 00	9	8
VII. 00	16	1	30	8	3
30	15	5	XI. 00	7	0
VIII. 00	14	6	30	6	1
30	13	3	XII. 00	5	5
IX. 00	10	0	30	4	3

From each of these a daily curve of tide can be formed: by joining their heights plotted on 'squared' paper they would show as in fig. 188. The two curves are here shown on the same figure, the 'hard' line being that of the above readings plotted, while the dotted line shows that of the probable continuation. The highest to lowest curve is that of 'springs'; the other is of 'neaps.' As a rule, the curve at 'neaps' is more irregular than that of 'springs.'

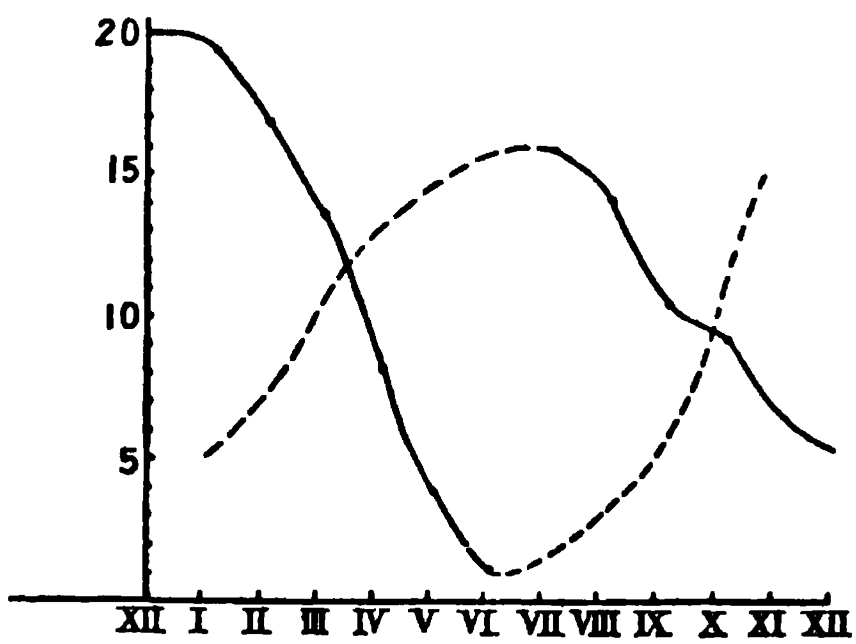


FIG. 188.

The vertical line is graduated in feet and inches, and the lower horizontal line is in hours and minutes, time of day.

The combined curve from 'springs' to 'neaps' may be plotted in one diagram thus:—

Fig. 189.

In fig. 189 the vertical line is graduated in feet and inches. The lower horizontal line is graduated in hours and minutes, but they will now represent the time of the moon's meridian passage. The upper horizontal and dotted line is the level of H.W.O.S. The lower horizontal line is the level of L.W.O.S., the accepted 'datum.'

495. Mean Water Level.—Midway between these lines, and here accepted as a straight line, is the mean water level of the sea (M.W.L.).

A 'true' tide will fall nearly equally below the M.W.L. as it will rise above it.

The movement of the tide from L.W. to H.W. is plotted on this diagram, using the 'ordinate' (perpendicular) for height and the 'abscissæ' (horizontal) for the time of the moon's transit. In the previous example given, the moon's meridian passage was 0 hr. 26 min. at superior transit, and 12 hrs. 56 min. at 'inferior' transit. The one is the time of transit at one H.W., the other that at the next H.W.; and midway between them, is the time of L.W. in this case, VI. hrs. 41 min. Starting then at H.W., the height of the highest tide is marked on the ordinate. In the example given this is 19 feet 8 inches; and, supposing the graduations on the abscissæ commence at 0 hr., then a height of 19 feet 8 inches will be marked 0 hr. 26 min. to the right of the vertical. At L.W. on that day the height registered 1 foot 1 inch, and the time of L.W. is VI. 41; this height will be plotted at VI. 41.

At the next H.W., XII. 56, which may be less or more than the day tide just plotted, its height, and the corresponding moon's meridian passage, taken from the *Nautical Almanac*, will be set off as an ordinate and abscissa, and so on for the whole lunation.

496. Fifty-seven Tides in a Lunation.—There are 57 tides in a lunation, and from 'springs' to 'neaps' there will be 14 high waters and 14 low waters; there will consequently be 14 dots on

the upper half of the figure, and 14 on the lower half. If the upper dots are joined and a curve derived from them, this will represent a curve of high waters, while that joining the lower dots will be 'smoothed' into a curve of low waters.

497. **Rise of Tide.**—The 'rise of the tide' is the distance from the L.W.O.S. to H.W. on any day.

498. **Range of Tide.**—The 'range of tide' is the distance between low and high water on the same day.

499. **Diurnal Inequality.**—The zigzagging of the upper dots is called 'diurnal inequality.'

500. **Rise of 'Springs.'**—From the lower horizontal line (L.W.O.S.) to the upper dotted line (H.W.O.S.) is the 'rise of springs.'

501. **Rise of 'Neaps.'**—From the same lower line to H.W. at 'neaps' is the 'rise of neaps,' and it is these two quantities that are given on every chart.

502. **Bench Mark.**—If 'amendments' only are being made to a chart, the pole is required for a few hours only, and a record has been left of the position of the 'datum' from which all the previous soundings are shown, this 'datum' must be referred to a reading on the tide-pole as set up.

The statement on the 'title' of the chart will read that "the 'bench mark,' , is chipped on the north and east corner of the sign of 'the Maintop' public-house, and is 10 feet above the level of H.W.S. (or it may be given above L.W.O.S.)," the rise of 'springs' being 15 feet.

503. **Referring the 'Bench Mark' to the Tide-pole.**—To simplify the illustration, assume that a place was found for the tide-pole at the side of a jetty, and nailed there. With the aid of a levelling staff, or any marked baton, and a theodolite, the level of the 'bench mark' above the highest reading on the pole is noted. Suppose it is 5 feet above the 20-foot mark; then H.W.S., being 10 feet below the bench mark, would read $10 - 5$ feet below the 20-foot mark on the pole $= 15$ feet.

And since the rise of springs is 15 feet, then the pole has accidentally been placed so that the 0 of its reading is the level of L.W.S., as adopted in the previous survey; and whatever the reading will be on the tide-pole at the hour of sounding, is the reduction to the soundings at that hour.

504. **Finding the 'Datum' without the 'Bench Mark.'**—Taking another case, suppose there is no record or bench mark; the rise of springs is given as 15 feet.

At H.W. on this particular day the tide-pole read 12 feet, and at L.W. it read 3 feet; then the 'range' of the tide is 9 feet, and the M.W.L. reads $7\frac{1}{2}$ feet.

At 'springs' the range or rise was 15 feet, or $7\frac{1}{2}$ feet was the rise from L.W.O.S. to M.W.L.; and since the tide-pole registers

7½ feet at M.W.L., then it must read 0 at L.W.O.S., and the consequent readings at each hour are the reductions at that hour.

By this last method of deduction the tide-pole may be set up anywhere quite away from the position of the bench mark, since that mark is not referred to.

505. To Establish a 'Datum' by Pole at L.W.S.—The next example will be where the survey is an original one. The surveyor will now have to establish his own datum.

The method of finding the 'establishment' has been explained in par. 480.

It is now required to establish the rise of 'springs.' He must either stay at the port through a period of 'spring' tides, and make an allowance of about a foot or so below, or perhaps above, the L.W.S. tide-pole reading; it is not possible to lay down any hard-and-fast rule which will apply in all cases—so much depends, as has been already pointed out, on the moon's declination and parallax, as well as the sun's declination and parallax, and the geographical position.

506. By 'Floatsam.'—Or, what may turn to be almost as accurate, is to find, if it exists, the mark of 'floatsam' or of heaped-up weeds on the beach, or of drift-wood; this will probably be very near the position of the last high-water springs, though it may have been an extraordinary spring tide that has placed the debris there. The level of this would have to be referred to the pole; or some definite height above it levelled to the pole, and 'referred' to any reading on it.

507. Where no Level Available; by Sea Horizon.—Failing a theodolite or a level, the line of the sea horizon can be substituted; if the pole lies between you and it, stand over the debris, and where the sea horizon line cuts the pole will be the height of your eye above H.W.S.; of course, the height of the observer's eye must be accurately measured.

508. Obtaining a Horizontal by 'Shore' Horizon.—Or a shore horizon would serve; for if the distance of the shore horizon in line with the pole is known, then, given the height of your eye, and the distance of the shore horizon from Table 14, *Inman's Tables*, the correction given there can be placed on the sextant, to the right of the index (off the arc), and where the reflected image of the shore horizon that is in line with the pole, intercepts the pole, will be the horizon of the observer.

Example.—The observer's eye is, say, 5 feet above the H.W. line. The distance to the shore on the other side of the pole is 450 yards. From Table, Appendix IX., the 'dip' of shore horizon for a height of 5 feet and a distance of 450 yards is 12′ 40″. Set 12′ 40″ on the right of the index on the sextant: look at the pole through the 'clear' portion of the 'horizon glass,' and, where

the reflected image of the shore line cuts it is the horizontal line; read it off the pole.

509. Substitute for a Level.—The question may be put, What if you have no level or theodolite?

The following are suggestions:—

Given a boat's bucket brimful of water and a pencil.

Someone in the boat will probably have four needles or four pins of equal length; and a new pencil—if not an uncut one, a used one cut off square at both ends will serve; but the longer it is the better.

Push the four pins or needles into the pencil, as in fig. 190, equidistant from each end; float the arrangement, pins 'up and down,' and there will be a level pencil with two sights on it. Place the bucket at H.W. line, measure the height of the surface of the water from the ground, look along the pin heads, and see what they point to on the pole.

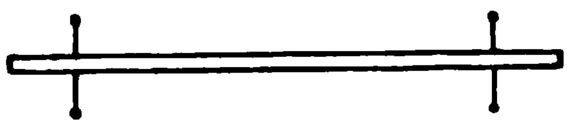


FIG. 190.

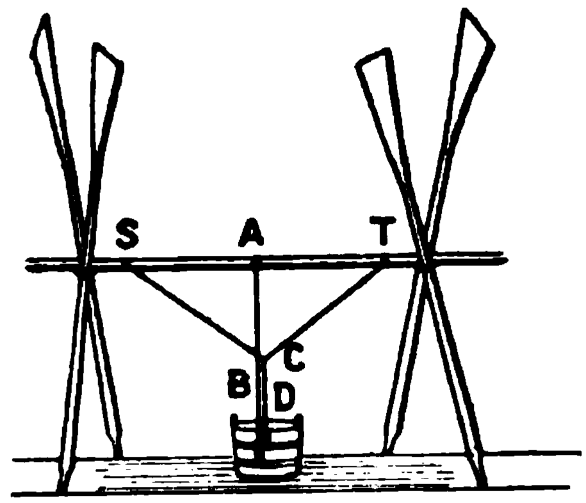


FIG. 191.

The probable error when the pole is about 30 yards off, is about 6 inches.

Or, here is a more elaborate idea.

Cross, and lash together, four oars, as in fig. 191; rest the longest 'stretcher' in the boat on each pair of crossed oars.

Take a piece of twine about 5 feet long; make a loop at each end, sufficiently big to go over the 'stretcher'; halve the string from end of loop to end of loop; at the half mark tie another piece of twine about 18 inches long, and put a weight on the end, such as a marline-spike.

Make two notches on the stretcher about 4 feet apart, or as far apart as the 'stretcher' will allow of; put the loops of the string over the notches, as in S T of the figure.

Halve the distance between the two notches on the 'stretcher,' and cut another notch.

Take another piece of twine about 4 feet long, tie a weight at the end of it, a hammer for instance; put a loop on the other end of the string and hang it from the centre notch on the stretcher. The two strings with weights on are labelled A B and

CD in fig. 191. When these two lines are coincident the stretcher is horizontal.

To make it so, either raise the crossed oars at either end or, better, put a small stone under the end of the stretcher that requires to be raised.

The probable error is about 6 inches in the reading on the pole at about 30 yards.

The bucket of water shown in the sketch is for the purpose of steadying the weights.

By whatsoever means employed, having found the reading on the tide-pole corresponding to the presumed H.W. spring mark, and if the reading on the pole is taken at high and low water on that day, then if, for instance, the H.W. spring mark reads 20 feet on the pole, and H.W. for the day reads 17 feet, and L.W. reads 10 feet, the H.W. springs mark would be 3 feet above the H.W. mark for the day; and, assuming that the rise above H.W. to H.W. springs would equal the fall below L.W. to L.W. springs, then L.W. springs should read $10 - 3$ feet $= 7$ feet, and therefore the mark of 7 feet on the pole represents L.W.S.; if this is accepted then it follows that the corresponding rise of springs is 13 feet.

510. Deducing a Datum by means of Rocks marked Awash.—The following methods for deducing a datum for reductions are each, in the order given, rougher than the other. If it should happen that there is a rock marked on the chart as 'awash' at L.W.S. (⁜), a pole, or marked baton can be placed on it, and the height from the water's edge to the bottom of the stick, subtracted from the tide-pole reading at that moment, would give what the reading on the tide-pole would be when the rock was awash, *i.e.*, L.W.S. *according to the previous survey.*

511. And a similar idea can be carried out from a rock that is marked 'awash at H.W.S.'; its height above the level of the water at that moment, added to the reading on the tide-pole, will give what the reading would be at H.W.S. as accepted in the previous survey, and in this case, given the rise of springs, then the reading of the tide-pole at L.W.S. would be deduced.

512. Application of Tide-pole Readings.—In both of the above it is assumed that good judgment was exercised in allowing the statement to exist.

Heretofore it has been taken for granted, that it is understood that the tide-pole was used not only for the purpose of obtaining a datum, but, when that was found, that the reading on the tide-pole was continued from hour to hour, and the difference between the datum reading and the reading of the pole at any moment was the reduction to the soundings taken at that hour.

But supposing that, though the tide-pole is used, it is only read off at high and low water.

513. Tide Clock or Tables for Tide Readings.—In Table 10, *Inman's Tables*, or in the 'tide clock' in the *Admiralty Tide Tables*, also see Appendix XI., Plate XIX., a scale is given showing the rise above, and fall below, M.W.L. It is simply a possibility of what may have occurred, and is a substitute for what the tide-pole readings were, when it was inconvenient to read them in the ordinary way.

Fig. 192 illustrates the idea of the scale better that it can be described.

Let 2 feet be the reading at L.W. VIII. A.M., and 20 feet the reading at H.W. II. P.M. It is required to make out a table for the probable tide-pole readings, or reductions, from hour to hour. Now, the range of the tide for this day is from 2 feet to 20 feet =18 feet, that is 9 feet above M.W.L. and 9 feet below it.

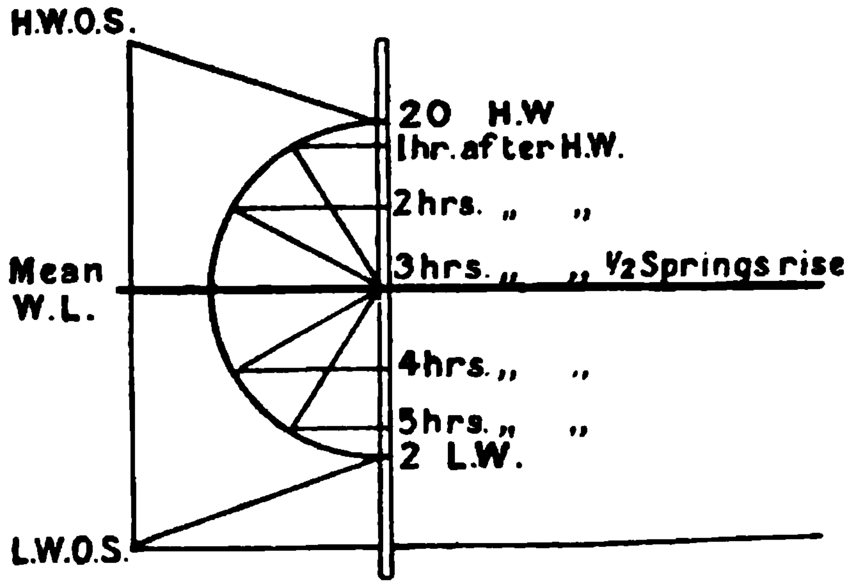


FIG. 192.

The table gives the amount of the rise above M.W.L. for each hour, *when half the range is* 9 *feet*.

Half an hour after H.W. it is 8 feet 8 inches *above* M.W.L., 1 hour after H.W. it is 7 feet 9 inches, 2 hours after H.W. it is 4 feet 6 inches above M.W.L., at 3 hours it is at M.W.L.; 4 hours after H.W. it is 4 feet 6 inches *below* M.W.L., 5 hours it is 7 feet 9 inches below, etc.

Now from the chart, the rise of springs is given as, say, 24 feet, then the height of M.W.L. or ½ springs rise, is 12 feet above L.W.S.

If, then, 12 feet be added to the proportional rises *above* M.W.L. as given in the tables for each ½ hour, and those proportions which are *below* M.W.L. subtracted from 12, then we shall have the amount of the rise of the tide from hour to hour, to apply as reductions. It will read thus:—

		ft.	in.	
H.W.	VIII. 00	9	0	+ to 12 feet (M.W.L.)
1 hour after „	IX. 00	7	9	
2 hours „ „	X. 00	4	6	
3 „ „ „	XI. 00	0	0	− to 12 feet.
4 „ „ „	XII. 00	4	6	
5 „ „ „	I. 00	7	9	
6 „ „ „	II. 00	9	0	

514. Tide Readings obtained by Lead Line.—There is a slight modification of this when, instead of a tide-pole, a lead line is used on a *hard and even bottom.*

The reading on the lead line is obtained at H.W., and later a similar reading taken at L.W.; from these two readings the range of the tide is deduced. Then, as before, a table of reductions is concocted for the proportional rise from hour to hour, and applied to the $\frac{1}{2}$ springs rise.

This last is the roughest of all methods, but, still, might be called into requisition.

515. Empirical Reductions.—It applies in such a case as when the ship is out of reach of land, and the rise of springs is an unknown quantity. As before, deduce the range from a sounding obtained over the same spot at H.W. and at L.W.; then allow $\frac{1}{10}$th of the range below the L.W. reading on the line, as a datum for reductions, when the observation is made 1 day on each side of springs; allow $\frac{8}{10}$ths of the range below the reading if the soundings are taking place at neaps; and interpolate at the rate of $\frac{1}{10}$th for each day from springs to neaps, each day being each day of the age of the moon, supposing the tide to have no age.

For the intermediate hours between high and low water, use, as before, the 'table' or 'tide clock.'

515a. The reverse case is to reduce a series of soundings to L.W.S. without the use of a tide-pole or lead line, but where the rise of springs is known.

If the period is springs,

reduction at L.W. . .		0.
"	1 hour after W.L.,	$\frac{1}{12}$th rise of springs.
"	2 hours " "	$\frac{3}{12}$ " "
"	3 " " "	$\frac{6}{12}$ " "
"	4 " " "	$\frac{8}{12}$ " "
"	5 " " "	$\frac{11}{12}$ " "
"	6 " " "	$\frac{12}{12}$ " "

If the period is neaps, *i.e.* when the moon is $7\frac{1}{2}$ days old

At L.W. neaps, reduction,	$\frac{4}{12}$ths of rise of neaps.
1 hour after " " "	$\frac{5}{12}$ " "
2 hours " " " "	$\frac{6}{12}$ " "
3 " " " " "	$\frac{8}{12}$ " "
4 " " " " "	$\frac{10}{12}$ " "
5 " " " " "	$\frac{11}{12}$ " "
6 " " " " "	the whole of the rise of neaps.

Rough estimate for the period between springs and neaps.

First deduce the rise of the tide for the day.

The rise of springs and the rise of neaps can be obtained from the

chart. Supposing the rise of springs to be 20 feet, and that of neaps 14 feet, then the rise of the tide diminishes 6 feet in $7\frac{1}{2}$ days, or at the rate of $\frac{6}{7\frac{1}{2}}$ feet in one day.

If the age of the moon is 5 days, and the age of the tide is 2 days; or, since when the moon was 0 days old (springs) the tide was 2 days behind it, then when the moon was 5 days old the tide was $5 - 2 = 3$ days beyond spring tides.

In 1 day the rise of the tide diminished $\frac{6}{7\frac{1}{2}}$ feet, therefore 3 days from spring tides it will have diminished in rise $3 \cdot \frac{6}{7\frac{1}{2}}$ feet $= 2\frac{1}{2}$ feet nearly; and the rise of tide for the day is 18 feet roughly.

Then at L.W. reduction is	$\frac{2}{12}$ths of the rise for the day	=	3 feet.
1 hour after ,, ,,	$\frac{3}{12}$,, ,,	,,	$4\frac{1}{2}$,,
2 hours ,, ,, ,,	$\frac{5}{12}$,, ,,	,,	$7\frac{1}{2}$,,
3 ,, ,, ,, ,,	$\frac{7}{12}$,, ,,	,,	$10\frac{1}{2}$,,
4 ,, ,, ,, ,,	$\frac{9}{12}$,, ,,	,,	$13\frac{1}{2}$,,
5 ,, ,, ,, ,,	$\frac{11}{12}$,, ,,	,,	$16\frac{1}{2}$,,
6 ,, ,, ,, ,,	$\frac{12}{12}$,, ,,	,,	18 ,,

QUESTIONS AND EXAMPLES ON TIDES.

1. What is the zero for the establishment of a port? What data are necessary to find it?

2. What is the zero for the soundings on a chart? What data are necessary to find it when the stay in a port will be only for a few days, and those during the period of neaps?

3. What data are required when amending the soundings on a published chart?

EXAMPLES.

516. To Find Reduction to Soundings from Data Supplied on Chart.—*Example* 1.—At a position abreast of the Start, fixed by sextant angles, it is required to determine the reductions necessary to soundings obtained at 4 00 P.M., at 7 00 and at 8 00 P.M.

Rise of springs at the Start is 15 feet, and of neaps 11 feet.

H.W.F.C. given on the chart is 6 hours 16 minutes.

Age of moon is 6 days, and age of tide 2 days.

The moon's *meridian passage*, taken from the *Nautical Almanac* and corrected for longitude (see Table 33, *Inman's Tables*) is 4 hours 14 minutes.

When the moon's meridian passage is 0 hour, the time of H.W. is 6 hours 16 minutes; when the moon's meridian passage is 4 hours, the correction to the establishment is 1 hour 2 minutes,

(see App. XIII., Table 2); then the lunitidal interval between the moon's meridian passage and the following high water on the day in question is $6^h\ 16^m - 1^h\ 2^m = 5^h\ 14^m$.

And, since the moon's meridian passage is 4 hours 14 minutes, then $5^h\ 14^m + 4^h\ 14^m$ = time of H.W. = $9^h\ 28^m$ P.M.

Allowing for a 6-hours' tide, it will be L.W. at about 3 hours 20 minutes.

The age of the moon is 6 days, that of the tide 2 days; therefore the tide is only 4 days from springs.

Spring tides rise 15 feet, neaps 11 feet, hence the rise of tide diminishes at the rate of $\frac{4}{7\frac{1}{2}}$ feet per day; and in 4 days will diminish $\frac{4}{7 \cdot 5} \times 4$ feet = 2 feet nearly.

Therefore the rise of the tide on this particular day will be $15 - 2$ feet = 13 feet.

And (1) by the rough method shown above:—

At 4 00 it is approximately 1 hour after L.W., or 5 hours after H.W., and the reduction = $\frac{3}{12}$ths of 13 feet = 3 feet nearly.

At 6 00 it is 3 hours after L.W., reduction = $\frac{7}{12}$ths of 13 = 7 feet, or $1\frac{1}{4}$ fathoms.

At 8 00 it is 5 hours after L.W., or 1 hour before H.W., reduction $\frac{11}{12}$ths of 13 = 11 feet, or 2 fathoms.

(2) By another method, using the tables or the tide clock:—

If the rise of springs is 15 feet and the rise of the day is 13 feet, then springs rise 2 feet above the rise of the day; supposing the tide falls equally below L.W. to L.W.S. as it rises above it to H.W.S., then each of the quantities shown in fig. 193 are 2 feet; from r to r' is the range = $13 - 2$, hence the range of tide for the day is 11 feet. And $\frac{1}{2}$ the range is $5\frac{1}{2}$ feet—that is, the tide will rise $5\frac{1}{2}$ feet above the M.W.L. to H.W., and fall $5\frac{1}{2}$ feet below it to L.W.

FIG. 193.

The table says that when the height above M.W.L. is $5\frac{1}{2}$ feet, the height above M.W.L. will be:—

	ft.	in.	
At 1 hour after H.W. . .	4	4	
2 hours " " . .	2	6	
3 " " " . .	0	0	
4 " " " . .	2	6	below M.W.L.
5 " " " . .	4	4	" "
6 " " " . .	5	0	" "

And, since the M.W.L. is ½ the springs rise, its height above L.W.S. is 7½ feet.

Then the reductions will be :—

	ft.	ft.	in.	ft.	in.	h m	
At H.W.	7½	+ 5	0 =	12	6	9 30	P.M.
1 hour after or before „	7½	+ 4	4 =	11	10	8 30	„
2 hours „ „ „	7½	+ 2	6 =	10	00	7 30	„
3 „ „ „ „	7½	+ 0	0 =	7	6	6 30	„
4 „ „ „ „	7½	– 2	6 =	5	00	5 30	„
5 „ „ „ „	7½	– 4	4 =	3	2	4 30	„
6 „ „ „ „	7½	– 5	0 =	2	6	3 30	„

And this result tallies with the previous one.

Example 2.—Sounding, etc., etc.

Tide-pole showed	6 feet at	VIII. 10 A.M. .	. L.W.
„ „	18 „ „	II. 15 P.M. .	. H.W.

Rise of springs from chart is 18 feet.

Make out a table of reductions.

Here the range of the tide is 18 – 16 feet = 12 feet, and ½ range = 6 feet; and, from the table, for the rise of 6 feet above M.W.L.,

	ft.	in.	
At H.W., height above M.W.L.	6	10	
½ hour after „ „ „	5	10	
1 „ „ „ „ „	5	2	+ to M.W.L.
1½ hours „ „ „ „	4	3	
2 „ „ „ „ „	3	00	
etc. etc. etc.			– to M.W.L.

M.W.L. is 9 feet above L.W.S.

	ft.	ft.	in.	ft.	in.
The reduction at H.W., II. 00 P.M., is	9	+ 6	0 =	15	0
I. 30 „	9	+ 5	10 =	14	10
I. 00 „	9	+ 5	2 =	14	2

and so on.

Example 3.—The following are tide-pole readings :—

	ft.	in.		ft.	in.
VII. 00 A.M., L.W.	5	00	IX. 00 A.M.	8	6
30 „ „	5	4	30 „	10	4
VIII. 00 „ „	6	00	X. 00 „	13	00
30 „ „	6	11	etc., etc.		
			Noon H.W.	18	00

Springs rise 18 feet, as stated on the chart. Make out a table of reductions for the hours between 8 and 10.

In this case, since the rise of springs is 20 feet and the range for the day = 13 feet (*i.e.* the reading of 18 feet − the reading of 5 feet), and allowing that the tide will rise equally above H.W. to H.W.S. as it will fall below L.W. to L.W.S., then the L.W.S. mark would be 2½ feet below the L.W. reading on the tide-pole; and, since the tide-pole reads 5 feet at L.W., therefore the L.W.S. datum mark will be 5 − 2½ feet = 2 feet 6 inches.

If the tide-pole reads 2 feet 6 inches at L.W.S. and 6 feet at 8 A.M. the reductions will be:—

	ft.	in.		ft.	in.		ft.	in.		
VIII. 00 A.M.,	6	00	−	2	6	=	3	6	=	½ fathom.
30 ,,	6	11	−	2	6	=	4	5	=	¾ ,,
IX. 00 ,,	8	6	−	2	6	=	6	0	=	1 ,,
30 ,,	10	4	−	2	6	=	7	10	=	1¼ ,,
X. 00 ,,	13	00	−	2	6	=	10	6	=	1¾ ,,

517. Establishing a Datum from one's own Deductions.—*Example* 4.—Erected a tide-pole at the end of a pier at L.W.

The L.W. reading was 2 feet, the H.W. reading 9 feet 10 inches.

From local information the H.W.S. mark was said to reach a mark which was found by measurement to be 3 feet 6 inches from the top of the pier, and this mark was level with the 11 feet 4 inch mark on the pole.

The following were the tide-pole readings at the hours stated:—

			ft.	in.
At	X. 00 A.M.,	L.W.	2	00
	30 ,,	,,	3	00
	XI. 00 ,,	,,	4	6
	30 ,,	,,	6	3
	etc.,	etc.,	etc.	

What reductions should be made at those hours?

If the H.W. reading for the day was 9 feet 10 inches, and H.W.S. reached the 11 feet 4 inch mark on the pole, then H.W.S. would rise 1 foot 6 inches above the H.W. reading for the day.

In that case L.W.S. would fall 1 foot 6 inches below the L.W. reading for the day; and, since L.W. reads 2 feet, then L.W.S. would read 6 inches.

But here, allowing for a small error in the estimated position of the H.W.S. mark on the pier of, say, 6 inches, it would be safer to assume that the tide-pole would read 0 at L.W.S., and therefore the actual readings on the pole are the reductions necessary at those hours.

Example 5.—A tide-pole was erected at an islet.

The estimated H.W.S. mark on the islet was levelled to the pole and found to read 16 feet 6 inches.

The reading on the pole at H.W. was 13 feet 2 inches,
" " L.W. " 3 feet 4 inches,
but no readings were taken at the intermediate hours except at 10 A.M.

The 'bench mark' was cut on the rocks at a point 4 feet 10 inches above H.W. for the day, and this was, purposely, 6 inches above the apparent mark of H.W.S. on the islet.

(a) What mark on the pole corresponds with L.S.W.?

(b) The tide-pole reads 8 feet 10 inches at 10 A.M., what reduction should be made at that hour?

The bench mark was cut 6 inches above the accepted H.W.S. mark, and it is 4 feet 10 inches above the H.W. reading, therefore H.W.S. is 4 feet 10 inches − 6 inches above the H.W. reading = 4 feet 4 inches.

If the spring tide rises 4 feet 4 inches above H.W. for the day it will fall 4 feet 4 inches below L.W. for the day.

And, since the L.W. reading is 3 feet 4 inches, then at L.W.S. the tide-pole would 'dry' 1 foot—*i.e.* − 1 foot is the datum.

(b) If the reading on the pole is 8 feet 10 inches at 10, and the datum is − 1 foot, the reduction will be 9 feet 10 inches.

CHAPTER VII.

A SURVEY, OR SOUNDING A SHOAL, BY MAST-HEAD ANGLE AND BEARING.

518. M.H. Angle Survey, when Applicable.—This form of survey, of an open roadstead, where no other possible means are available, can only be looked upon as a form of 'reconnaissance.' It may be applied when there is any reason to doubt the accuracy of the charted soundings, either owing to the date of the chart, or to the authority for it, and where any anxiety is felt as to whether the soundings are safe in the circle of swing of the ship.

In the case of a shoal, so far from land that a boat is unable to see objects to 'fix' by, then the boat 'fixes' from the ship by a bearing from the ship and the angle obtained at the boat, subtended by the ship's mast-head to a mark on the water-line. (See *Distances by M.H. Angles*, par. 340.)

By this means the shoal can be sounded, and every fix is relative to the ship's position, dependent on distance by the length of a line represented by the height of the ship's M.H. to water-line, and on the bearing, by the ship's compass or the prismatic compass in the boat.

519. Accuracy Involved.—The degree of accuracy obtained is not very great, because, bobbing about in a boat, with a probable roll of the ship, the angle is likely to be $\pm$ 10′ in error. For a height of 150 feet the corresponding error of position in the boat when the angle is 1° is about 80 yards; but, nearer to the ship the error of position is considerably less.

An error in the bearing of 1°, when the angle at the boat is 1°, will be 50 yards; so a sounding at that distance may be anywhere within 50 yards of where it is placed.

520. Fixing the Ship in the Open, Astronomically.—The ship, being the object of reference from the boat, has to connect her position with the shore. If the land is not visible from the deck or from aloft, the ship must be fixed astronomically

by lines of position; for results obtained with sea horizon, refer to p. 369, par. 589.

When One Object in Sight.—If one object is in sight, then the 'true bearing' of it will give one line of position, and other lines must be found astronomically to intersect the first (see p. 108, par. 235); or if an observer is left on shore, and the ship is visible to him, he can take an angle between her and some other fixed object, which again gives one line of position only.

When Two Objects Visible.—If two objects are in sight, then the T.B. of one, and an angle between the two, will fix the ship by two 'true bearings'; this is explained in *True Bearing*, p. 107, par. 233.

521. **More than Two Objects Visible.**—*Example.*—If more than two objects are visible, then either a fixing station pointer or, if the objects are not suitable for that, true bearings are necessary; or an observer sent to the shore, can take angle to the ship; and the ship take the other angles to the objects visible, and hence be fixed by one direct 'shot,' and two 'calculated' angles, as, for example:—

T is the Tongue lightship.

N.F. is the North Foreland; K, Kentish Knock lightship.

An observer at N.F. takes T 70° 10′ S; K 45° 02′ S.

At ship,

N.F. 56° 10′ T 91° 50′ K.

To plot this: at N.F. project the angle 70° 10′ to the right of the line joining T and N.F.; this will give the line shown in fig. 194 with an arrow.

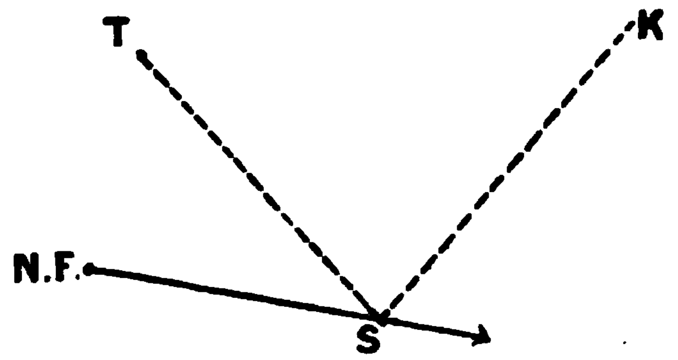


FIG. 194.

Then, in Δ T N S, N.F. = 70° 10′
S = 56 10
therefore T = 53 40 = 180° − (70° 10′ + 56° 10′).

And if the angle 53° 40′ is projected at T and laid off from the line joining T and N.F., the line so projected will intersect the line from N.F. at S;

and in the second triangle N K S, N.F. = 45° 02′
at S = 91 50
therefore at K = 43 08 = 180° − (91° 50′ + 45° 02′);

and the angle 43° 08′ is projected at K from the K to N.F. line; and the three lines should intersect at S.

522. **Sounding a Shoal.**—(1) The ship is moored at or near the shallowest part; at that part because, in the process of sounding in this manner, there are a far greater number of soundings near the point from which the soundings radiate, than

elsewhere; and there will therefore be a greater probability in obtaining a sounding at the shallowest part.

(2) Whitewash a broad band just above the ship's water-line abreast of the foremast; and measure as accurately as possible the height from the fore truck to the upper edge of the whitewash. This, as has been explained on p. 162, par. 340, can be done with a lead line. By lashing a spar to the mast a greater height is obtained; and the greater this height the less effect will an error in the total height, and of observation, have on the calculation of the position; it is very necessary that a flag, of a dark colour, because of the sky-line against which it shows, should be hoisted at the mast-head; or, what is equally good, an inverted cone covered with black canvas.

523. Tide Reductions for M.H. Angle Survey.—If a tide-pole is not erected on shore (see under *Tides*), which is hardly worth while, accept the range of the tide for the day as that of the neighbourhood; or obtain it by means of a lead line (see *Tides*, p. 282), reading the soundings obtained by it, *over the same spot*, at high and low water; and, with this 'range,' and the rise of springs, accepted from the neighbourhood, and the table supplied in the *Admiralty Tide Tables*, form a reading for each hour; or, failing the rise of springs, accept the empirical reduction as given under *Tides*, p. 282, par. 515*a*.

524. The Work.—For the work, mount a piece of cartridge or other paper on a drawing-board, with flour, or starch for choice; pins get in the way.

Draw a straight line up and down through the centre of the paper, and accept this as the true meridian. Prick a mark in the middle of it, and mark this as being the position of the ship.

Accept the variation from the chart, or that found for the occasion, and draw a magnetic meridian through the position of the ship: and also draw a number of similar meridians in different parts of the paper, near to or in the space that will be sounded.

At the bottom of the paper draw a scale; whatever is adopted, remember that the larger the scale, the more details will be required to fill up the space, and the closer can the lines of soundings be, as well as the soundings, and the more time it will take.

525. Drawing the Scale.—Suppose the scale is 4 inches = 1 mile = 6084 feet. Draw a line 4 inches long, and subdivide it in hundreds of yards. If 4 inches = 6084 feet, 1 inch = 1521 feet = 507 yards, and 100 yards = $\frac{100}{507}$ inches = ·197 inch = ·788 divisions on the $\frac{1}{4}$-inch scale on the edge of the protractor; 200 yards = 1·576, 300 = 2·364, and so on: and tick off each hundred yards from the edge of the protractor.

Make out and paste on the corner of the board a table of

distances for every 50 yards corresponding to H, the height of the mast-head, for 1° upwards (distance = H cot angle).

If the height is 150 feet, look out the natural cotangent of an angle from 1° upwards to about 25° for every 20′ of arc; if this cotangent is multiplied by 150 feet, it will equal the distance off in feet.

But since, in our case, every 150 feet is ·8 division (roughly) on the $\frac{1}{4}$-inch scale, then $\frac{150 \text{ cot angle}}{150} \times \cdot 8 =$ distance by the $\frac{1}{4}$-inch scale on the edge of the protractor, for every angle; and hence the cotangent of the angle need only be multiplied by 8 and divided by 10, and the result is a scale of inches of paper, which can be measured right away with the protractor.

With the lead line correctly marked, tallow for 'arming' the lead, a small prismatic compass, sextant, note-book, a board, and a watch, shove off, and pull to a distance of about 50 yards from the ship, in any direction; then stop, sound, and fix.

It will be noted in this manner:—

VII. A.M., 22° 50′ N. 87° E. . . . . 6..

22° 50′ represents the mast-head angle; N. 87° E. the bearing taken from the boat; and 6 is the sounding at the 'fix.'

526. **Bearings taken from Ship or Boat.**—It is better, as a rule, for the bearing to be taken from the boat, so that the worker can fix his position as he goes along, and knows where and what he is doing. If the bearing is taken from the ship, the sounder does not know if he is going straight, or where he actually is, unless the bearings are signalled to him. But in a harbour, as shown by the example that follows, it will be more convenient to use the ship's bearing (see par. 527).

The bearing obtained will be laid off, with a protractor, through the position of the ship from one of the many magnetic meridians drawn, and this gives the direction; while the angle 22° 50′ will give the distance from the ship, and hence the boat's position, and that sounding will be 'fixed.'

Then the boat continues, as well as she can, in the same direction, guided by prismatic compass or the boat's compass; and the leadsman will be called upon to sound, after so many strokes of the oars; the soundings he calls are noted, $6\frac{1}{2}$, $6\frac{3}{4}$, $6\frac{1}{2}$, $6\frac{1}{4}$, $6\frac{3}{4}$. Then the boat stops again, and fixes.

11° 15′. N. 88° E. . . . . . 7.

7 is the sounding at that position, which is fixed as before; since there are five soundings between the two fixes, and they have been obtained at equal intervals, represented by so many strokes of the oars, the space between the fixes is divided so that five

soundings will be placed at equal distances apart—that is, dividing the distance into 6 thus:—

•—|—|—|—|—|—•

This is done by eye, and is quite near enough (by most eyes).

Then on again, more soundings and more fixes, until the normal depth is reached. Suppose the normal depth to be 10 fathoms; then move on to another bearing of the ship—this time perhaps N. 70′ E. Of course, the amount of the change in bearing depends upon the closeness of the work. The distance from the ship, with a height of M.H. of about 150 feet, should not exceed 1 mile; but eight lines in a quadrant should suffice, and the difference between each bearing will then be roughly 11 to 12 degrees.

Starting from the outer end of the line, steer towards the ship, keeping that bearing on as near as possible, allowing, of course, for tide; repeating the fixes; and so continuing the process of starring round the ship.

It must not be forgotten what the degree of accuracy is we are working on; all the errors are large, as has been suggested. ·8 is near enough to ·78 to be used as a scale, and a ½ degree of bearing either way does not affect materially the position at a distance of 1 mile. Anything nearer than that, would be straining at a degree of accuracy which one is unable to plot, and which cannot practically be observed; one must be expeditious both as regards observing and plotting, and, in fact, carrying out the whole investigation: not for the sake of hurrying through the work, but for the sake of the 'fixes' and change of position while fixing.

Here over-accuracy is redundant; the soundings and the leadsman, however, require all one's attention. (See p. 291.)

A great aid to the recovery of the position of a shoal is, as before explained, a sketch of whatever land may be visible, to which angles may be taken. (See sketch, p. 50, par. 136.)

527. Direction of Lines by Transits with the Shore.—The following is an example of the survey of a roadstead.

The general system adopted is the same as for that of a shoal; but in running lines of soundings, it will be more convenient, and better, to steer for some objects on shore, picking up transits or two objects in line to do so; and a more correct bearing can be taken of the point steered for from the ship's compass.

When working on the sea side of the ship, it is only necessary to get some object on the shore in line with the ship, and to take the bearing of this object when again on board.

528. Lines of Direction by Distant Objects nearly Parallel.—The lines so run will be sufficiently straight, so that there remains only the distance to be observed. Again, if there is distant land with a distinct detail, and the ship lies between you and it, and if the ship exposes her length, or part of her

length, to the boat, it may be possible to run lines of soundings almost parallel.

Select a distant spot, and keep this in line with the bows of the ship; run on this line, and take the bearing later, of the distant object. For the next line take the same distant spot and change the part of the ship to keep in line with it, a few feet further aft; run on this line, and the bearing of the distant object from the ship will give very approximately the line of direction of the soundings, and they will be practically parallel, and so on, gradually shifting the spot to a part on the ship brought into line with the same distant object. But, of course, this method can only apply to those lines which lie on the sea side of the ship, and have a distant high land as background.

529. It should be explained that the example here given consists of two distinct parts, each independent of the other. The one is a roadstead, sounded entirely by mast-head angle and bearing; the other part, is the survey of the coast-line of the roadstead. Obviously, the astronomical position of the ship, and the position of points ashore relatively to the ship, connect the roadstead with any point of the triangulation of the coast or of the land.

The example should be plotted.

M.H. Angle Survey of Downham Bay. (Plate IV.)

28th March 1904.

Part I.—At low water erected tide-pole at Smerby Point, the north point of roadstead.

During the sounding, the ship's head remained N. 12° E. by compass.

Approximate variation from chart, 20° W. Deviation, for N. 12° E. is 3° E.

Height of M.H. above W.L., 119½ feet.

Scale adopted, 6·6 inches = 1 mile = 6072 feet.

Latitude 55° 26′ N., by ex-meridian twilight stars, sea horizon.

During the flood, the ship's head remained at N. 12° E., by compass. On the turn of the tide, the ship's head remained at S. 27° W. Deviation, 2° W. Insert tide arrows.

During the sounding the tide-pole reading was:—

VII. 15 A.M.	. . . . .	2′ 3″ (L.W.)
VIII. 00 „	. . . . .	2 6
IX. 00 „	. . . . .	5 3
X. 00 „	. . . . .	8 3
XI. 00 „	. . . . .	9 6
Noon	. . . . .	10 4 (H.W.)

Rise of springs of surrounding coast, 10¼ feet. The datum on the tide-pole is therefore 1 foot.

The following sketch was taken from ship.

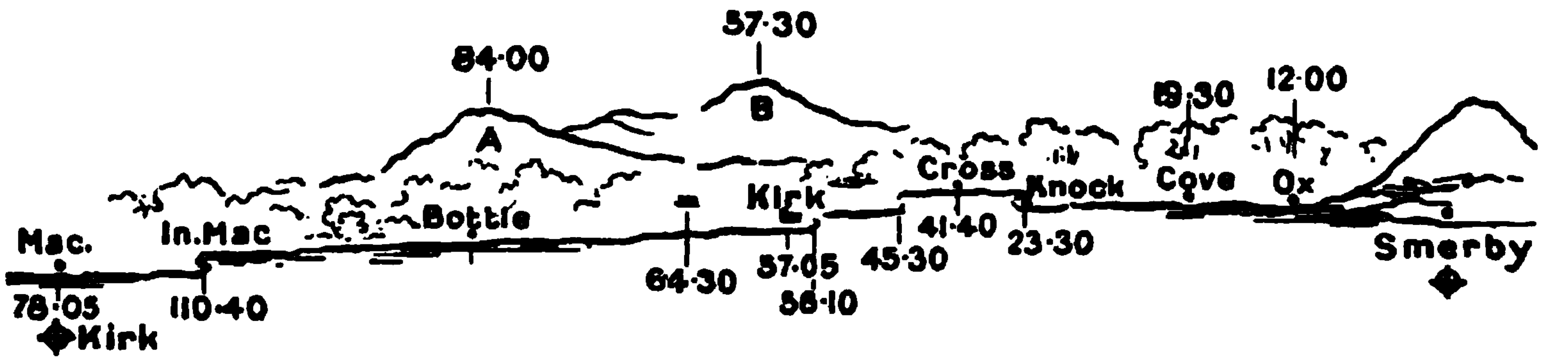


FIG. 195.

Take a piece of drawing-paper about 10 inches square. Draw a true meridian down the centre. Prick on it the position of the ship on about the middle of the meridian line. Draw a magnetic meridian through the position of the ship, and plot the following positions and surroundings. For tide reductions and how to obtain them, refer to Chapter VI. on *Tides*, p. 265, and *Example* 3, p. 285.

8.0 A.M.—Commenced sounding. All bearings are from ship.

S.E. of ship. Ship ϕ a house, bearing N. 14° W.

Line 1.—Ship 150 feet 5½
5½ 6¼ 5¾
9° 20′ 5¼
4½ × 4¾
4° 50′ 5¼
6 7¼ 9
3° 15′ 10

Line 2.—Mound on Smerby Point ϕ ship. N. 2° E.
3° 00′ 10

Keeping ϕ on
7¼ 6¼ ×
4° 24′ 5¼
4¾ 4½ 4
8° 37′ 2¾
5 6¼
27° 43′ 6¼

Line 3.—Ship ϕ north end of land. N. 15° E.
24° 00′ 5½
5½
10° 20′ 3¾
1¼
7° 43′ 4
4 × ×
4° 20′ 4
3¼ 3 4½
3° 15′ 5½

Line 4.—From ship S. 27° W.

3° 02′ . . . . . . $5\frac{1}{2}$

5 × 4 ×

S. 28° W. 4° 49′ . . . . $3\frac{1}{2}$

4 × ×

S. 25° W. 8° 37′ . . . . $3\frac{1}{4}$

$4\frac{1}{4}$ $5\frac{1}{2}$ 6

Ship 50 feet . . . . . $5\frac{3}{4}$

9.0 A.M.

Line 5.—For Mac Point, from ship, S. 45° W.

S. 45° W. 21° 43′ . . . . $5\frac{1}{2}$

5 $4\frac{1}{2}$ 4 4 $3\frac{3}{4}$

S. 46° W. 5° 41′ . . . . $3\frac{1}{2}$

$3\frac{1}{4}$ $2\frac{3}{4}$ $2\frac{1}{2}$

S. 45° W. 3° 55′ . . . . $1\frac{1}{2}$

Dry reef 30 feet, and drying into Point: reef dries off south-east end of Point about 100 yards.

Position of a dry patch of rocks.

From ship S. 39° W. 4° 28′.

Pulled along edge of reef, extending 100 yards from shore.

Line 6.

S. 66° W. 3° 55′ . . . . . $\frac{1}{2}$

$1\frac{1}{4}$ $1\frac{1}{2}$ 2

S. 66° W. 6° 15′ . . . . . $\frac{1}{4}$ small patch

$2\frac{3}{4}$ 3 $3\frac{1}{2}$

S. 62° W. 10° 30′ 4

$4\frac{1}{4}$ $4\frac{1}{2}$ 5

S. 66° W. 33° . . . . $5\frac{1}{4}$

Line 7.—From ship.

S. 85° W. 60 feet. . . . . . $5\frac{3}{4}$

$5\frac{1}{2}$ $5\frac{1}{2}$ $4\frac{3}{4}$

S. 83° W. 9° 29 . . . . $3\frac{3}{4}$

$3\frac{1}{4}$ $3\frac{3}{4}$ × $2\frac{1}{4}$ ×

S. 86° W. 5° 17′ . . . . . $1\frac{3}{4}$

$1\frac{1}{2}$ $1\frac{1}{4}$

S. 85° W. 4° 16′

H.W. line, 30 yards.

Line 8.

From ship, N. 77° W. 4° 30′ 1

Reef dry 20 feet nearer the shore.

Between it and H.W. line is dry sand.

H.W. line is about 130 yards.

$1\frac{1}{4}$ $1\frac{3}{4}$ $2\frac{3}{4}$ 3 $3\frac{1}{4}$

N. 77° W. 8° 37′ . . . . . $3\frac{3}{4}$

4 $5\frac{1}{4}$ $5\frac{1}{2}$

N. 77° W. Ship 150 feet. . . . $5\frac{1}{2}$

10 A.M.

Line 9.—From ship.

N. 56° W. 33° 0′ . . . . . $5\frac{1}{4}$

$5 \quad 4\frac{3}{4} \quad \times$

N. 56° W. 8° 37′ . . . . . $3\frac{1}{2}$

$3\frac{1}{4} \quad 2\frac{3}{4} \quad 2$

N. 58° W. 5° 17′ . . . . . $1\frac{3}{4}$

1

N. 60° W. 4° 25′ . . . . . $\frac{3}{4}$

Shore, 90 yards.

A patch of reef dry between this fix and shore.

Line 10.—Reef dry a boat's length inside fix.

N. 34° W. 5° 17′ 1

$1\frac{3}{4} \quad 2\frac{3}{4} \quad 3\frac{1}{4}$

N. 32° W. 9° 16′ . . . . . $3\frac{3}{4}$

$4 \quad 4\frac{3}{4}$

N. 34° W. 27° 45′ 5

Line 11.

N. 14° W. 27° 45′ 5

$4\frac{1}{4} \quad 4$

N. 14° W. 9° 16′ . . . . . $3\frac{3}{4}$

$3\frac{1}{4} \quad 3\frac{1}{4} \quad 2\frac{1}{4}$

N. 12° W. 4° 58′ . . . . . $1\frac{3}{4}$

$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{2}$

Close to shore, 10 yards.

N. 13° W. 3° 06′

The coast forms a small cove about 120 yards across and 100 yards deep.

Line 12.

At south extreme of Smerby Point H.W. line 10 yards—reef dries out half-way from shore.

N. 1° E. 4° 19′ . . . . . $2\frac{1}{4}$

$2\frac{1}{4} \quad 3\frac{1}{4} \quad 3\frac{3}{4} \quad 4$

North 9° 41′ . . . . . $4\frac{1}{4}$

$4\frac{1}{2} \quad 5\frac{1}{4} \quad 5\frac{3}{4}$

$6\frac{1}{4}$ at ship.

11.0 A.M.

Line 13.—From ship.

N. 16° E. 30° . . . . . $5\frac{3}{4}$

$5\frac{1}{2} \quad 4\frac{3}{4}$

N. 14° E. 8° 01′ . . . . . $3\frac{3}{4}$

$4\frac{1}{4} \quad 3\frac{3}{4} \quad 3\frac{1}{4}$

N. 15° E. 3° 55′ . . . . . $2\frac{1}{2}$

Shore reef N. 70° W.—20 yards—dry half-way to shore.

Smerby Point is a narrow peninsula about 60 feet wide and 100 yards long, with a rounded extremity, on which stands a small mound.

Line 14.—From ship.

N. 29° E. 3° 43′ 3

$3\frac{1}{2}$ 4

N. 27° E. 5° 17′ . . . . . $4\frac{1}{2}$

$4\frac{3}{4}$ 5 6

N. 29° E. 21° 50′ 6

Line 15.—From ship.

N. 46° E. 21° 10′ 6

$5\frac{3}{4}$ 6 $6\frac{1}{4}$ $6\frac{1}{2}$ ×

N. 46° E. 3° 50′ . . . . . $7\frac{1}{2}$

Noon.—Dinner. Lead line O.K.

This concludes the M.H. angle survey of the roadstead: after its fashion a completed piece of work.

PART II.—For true bearing.—Landed at Smerby Point.

Sunrise, 28th March 1904—lat. 55° 26′ N., long. 5° 31′ W. At 5.45 A.M., ship's head S. 27° W., corrected dec. 2° 50′ N.

O| 104° 32′. ← Macringan Point.
Ship 25° 50′.

For distance from ship.

M.H. elevation on arc 4° 15′ 0″.

„ off „ 4° 12′ 0″. M.H. 119·5.

The above by T.B. and distance connects the position of the ship with the triangulated survey of the coast-line which follows.

By landing compass, bearing of ← Mac Point S. 30° 20′ W.

For coast-line.

At Smerby Point.

Mac Point	9°	50′	Inner Mac.
	27	55	Bottle.
	41	55	Kirk.
	58	10	Cross.
	78	02	Knock.
	98	00	Cove.
	112	00	Ox.
Knock	58	00	Inner Smerby.
Mac Point	76	50	Summit (*b*).
	41	45	„ (*a*).
	46	16	Houses.

At Ox.

Knock	49	00	Cove.
Inner Smerby	45	00	Smerby.

At Kirk.			
Mac.	4°	50′	Inner Mac.
	27	00	Bottle.
At Bottle.			
Kirk	25	20	Smerby Point.
Knock	19	20	
Ox	18	00	
Inner Smerby	12	30	
At Mac Point.			
Inner Smerby	5	50	Smerby Point.
Ox	10	00	Smerby Point 19° 26′ Ship. M.H. angle on 3° 11′ 30″.
Knock	13	20	M.H. angle off 3° 09′ 00″.
Cross	23	30	
Kirk	24	50	
B Summit	57	30	
House	38	45	
A Summit	67	10	

Coast is sketched in between the stations. From Smerby Point to Ox is a hard rocky coast. From Ox round the small bight to Knock is sand beach. Round Knock is rocky. Thence through Cove to 50 yards off Kirk is sand. Round Kirk is rocky, and all the way to Inner Mac is rocky. From Inner Mac to Mac Point is a gradually rising cliff, height at Mac about 60 feet.

Position of rocks off coast:—

1. Rock dry at L.W.
   Mac, 79° 40′; Inner Mac, 29° 30′, Kirk.
2. Rock dries 2 ft., L.W. springs.
   „ 55° 50′; „ 86° 30′.
3. Rock awash, L.W., off Smerby Point.
   Mac, 69° 00′; Knock ϕ Smerby Point.

For elevation and topography.

1. Fix of boat at noon:—

   Mac, 54° 50′, Bottle; 90° 10′, Smerby Point.

   ϕ
   A Summit.

Angle of elevation of A Summit to Bottle:—

3° 38′ 30″; eye, 5 feet.

2. Knock. 25° 50′; Smerby Point.

   In. Mac, 61° 10′. Kirk 63° 50′; Smerby Point.
   Mac, 84° 50′.

Elevation of B Summit to Cross, 6° 34′. Eye, 5 feet.

Ship at anchor:—

Knock 23° 30′; Smerby Point.
Mac, 78° 05′ Kirk 57° 05′.
Inner Mac 110° 40′. 8.0 A.M.
A Peak 84° 00′. 29 March
B Peak 57° 30′. Weighed
House 64° 30′. L.W.

Ship sounding:—

Steered to the east, keeping Kirk mark ϕ B Summit.

6 8¼ 10¼
Mac Point, 55° 20′ Kirk. 35° 40′, Smerby . 10¼
ϕ
B.

Altered to southward:—

8¼ 5½
Mac, 44° 00′ House. 36° 36′, Smerby . 6¼
ϕ
B.

9¼ 10¼
Mac, 23° 14′ Bot. 35° 20′, Smerby . 16
ϕ
B.

9.0 A.M. Turned to starboard, ½ helm:—

12
Mac, 14° 30′ In. Mac. 40° 43′, Smerby . 7¼
ϕ
B.

Heading to northward:—

8¼ 7¼ 6¼ 5½ 8
60° 40′ Kirk ϕ. 39° 20′ . . . 11¼
B.

11¾ 11¼ 10½ Cross ϕ B.
8¼ 7¼
Mac, 53° 20′ Smerby Point. . . . 8¼
ϕ
B.

9.30 A.M. Finished. Lead line O.K.

M.H. Angle Table.

Height M.H., 119½ feet; dist. by ¼-inch scale $= \dfrac{119{\cdot}5 \,.\, \cot a \,.\, 6{\cdot}6}{6072} \times 4$

$$= \frac{119{\cdot}5 \,.\, \cot a \,.\, {\cdot}1 \times 4}{92}$$

$$= \frac{47{\cdot}8}{92} \,.\, \cot a = {\cdot}5 \cot a \text{ nearly.}$$

Scale, 6·6 inches = 6072 feet.

Observed Angle.	Distance on ¼-inch Scale.	Observed Angle.	Distance on ¼-inch Scale.
3° 00′	9·54	9° 00′	3·15
20	8·58	10 00	2·83
40	7·8	11 00	2·57
4 00	7·15	12 00	2·35
20	6·6	13 00	2·16
40	6·12	14 00	2·00
5 00	5·71	16 00	1·74
30	5·19	18 00	1·53
6 00	4·75	20 00	1·37
30	4·38	24 00	1·12
7 00	4·07	28 00	·94
30	3·79		
8 00	3·55		
30	3·34		

CHAPTER VIII.

SOUNDING.

The latitude and longitude of a port or of a river is principally for the purpose of locating it; and its longitude may be used for the determination of the errors of chronometer.

The coast-line and coast details are for the purposes of recognition, fixing positions, and indicating what sea-room there is.

530. The soundings are to impart safety and confidence as to the depth of water; and since the bottom is not seen, those using a chart must blindly trust the depths and contours given: this part of hydrographic surveying is probably the most important matter connected with it.

The ultimate object of sounding is to procure the information required for delineating the contour of the bottom, as seen from skyward: the only known method of satisfying this is to measure the depth of the water, by means of a lead line or a long pole, from a standard level of the tide, this standard being L.W.O.S.

To do this for every spot in a harbour or river would be a human impossibility. The next best thing is to measure this depth from as many spots as are desirable, and to infer the remainder. An accurate idea of the contour of the bottom, where the depth approaches to the normal draught of water of a ship, is essential for the avoidance of danger to shipping; at this depth there must be less room for inference, and more measurements will be required than in deeper water.

The first question that arises is, What is the best method of covering the most space by the same labour?

No better illustration can be given than that of mowing a lawn. Evidently parallel lines do the most work.

531. **Sounding in Parallel Lines.**—For sounding, then, parallel lines are the most effective.

532. **Direction of Lines of Sounding.**—The next questions are, What shall be their direction? and, How far shall these lines be apart? And to these there are almost endless answers.

As regards direction, keeping the fact in mind that we want to delineate the bottom, what would be the best direction on which a line of soundings will quickest show the change of depths?

It certainly is one at right angles to the irregular contours: a hill can best be contoured by walking up and over it, and the most direct way is at right angles to the base. Our base for the contours of soundings is the level of the coast-line at L.W.O.S.

533. Probable Shape and Size of Shoals.—The irregularities or rises on the bottom of the sea are of various shapes, and are sometimes a continuance of the natural formation of the land, below the surface of the water.

For example, land whose characteristic is hills of bare rocks, which is fronted by a coast studded with detached boulders, lying off this again there being detached rocks, or islets, ranging from 1 to 100 feet high, such land will probably be associated with a *submerged* continuation of that characteristic.

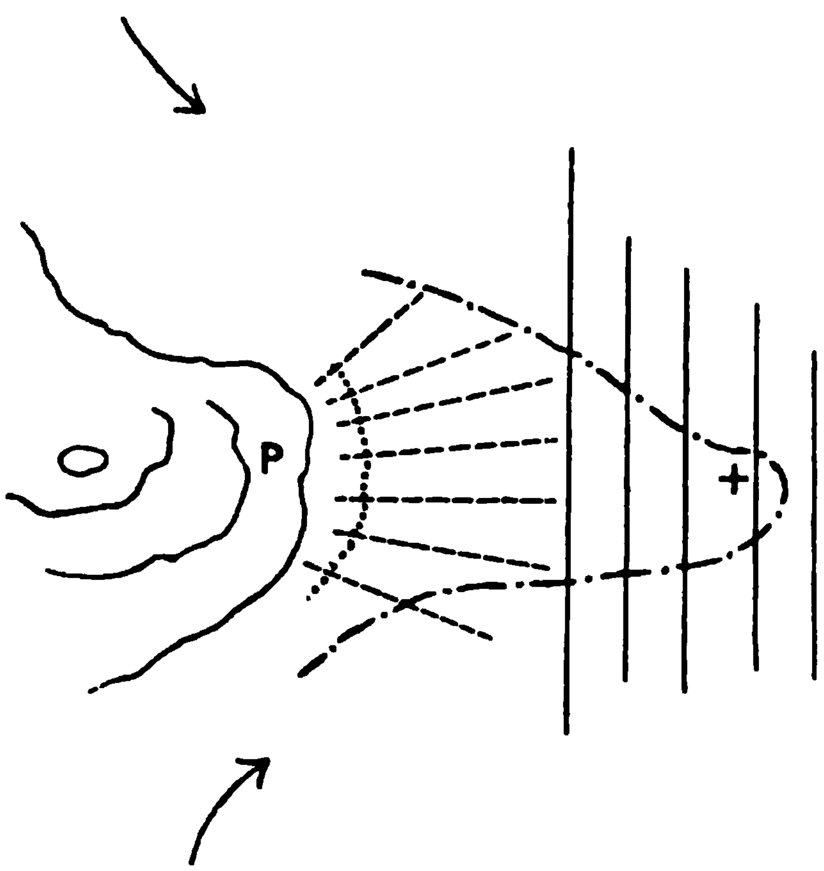


FIG. 196.

In such a locality, only a 'sweep,' such as fishermen's 'drogues' or 'doors' connected together by a light line, and towed at a regulated depth, could discover all the rocks.

Beyond the foreshore, where the coast is tide-swept, the offlying irregularities will collect round themselves, and arrest, the silt which is carried by the tide; this silt forming a shelf, or a bank or shoal lying parallel to the direction of the tidal stream, or with the general trend of the coast.

There could be no better illustration of this than is shown in fig. 196.

P is a point of land, and owing to its nature—huge boulders in wide disorder—there was a possibility of outlying rocks existing. One did exist where shown; aided by the direction of the tide, it had collected sufficient silt to form a 10-fathom bank with a rock at the end of it; outside of it the soundings were 12 fathoms.

The arrows show how the tides assisted in the silting-up process; only by that very silting was the existence of the rock first hinted at; the first indication is shown by the abnormal curve of the 10-fathom line; though the rock itself was missed by

the first sounding lines—the 'hard' lines in the illustration—by running intermediate lines between them the rock was eventually discovered.

Where the silt is carried down by a river current, it is arrested near the open sea by the sea-wall, and where the matter is deposited, there forms a 'bar' parallel with the sea-wall or coast-line, and at right angles to the stream; or, if the tidal stream or current impacts at an angle with the sea-wall, the silt will be deposited alongside of it, partly parallel with the stream.

Then, generally speaking, the direction of the line of sounding should be at right angles to the general trend of the coast; but, as shown in fig. 196, points of land should be treated with lines at right angles to the coast-line round them; or, in technical language, the lines should star out from the point, as shown by the dotted lines.

534. Distance Apart of Lines and Soundings.—As to the distance the lines should be apart, one authority says $\frac{1}{4}$ inch irrespective of scales; that is, $\frac{1}{4}$ inch might represent $\frac{1}{16}$ mile

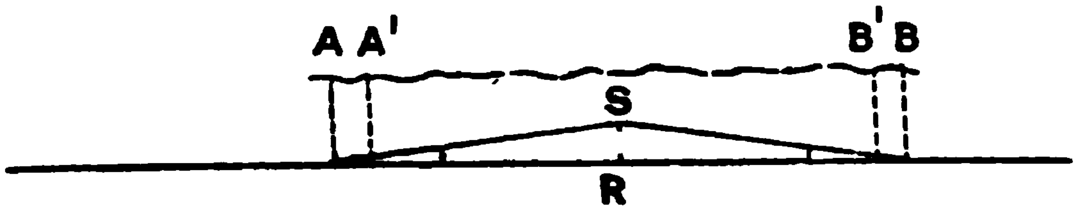


FIG. 197.

on a 4-inch scale, or $\frac{1}{4}$ mile on an inch scale: one is 100 yards, and the other 400 yards.

Now when these first lines are run, they must be looked upon as a sort of reconnoitring line, and they reconnoitre best and quickest when parallel; if, as the line is run out from the shore, the soundings are increasing regularly, it would seem at first sight that since nothing had been caught nothing existed to catch.

But, taking the first distance of the lines apart as, say, 100 yards $= 300$ feet, let fig. 197 represent a section of the bottom. A and B are two depths, obtained on each line A, B.

Suppose these depths are 5 fathoms or 30 feet, half these would be $2\frac{1}{2}$ fathoms—a dangerous depth.

R represents a shoal. Could such a shoal exist in the particular place that is being sounded, where A and B are 300 feet apart and $RS = 15$ feet?

Of course, it could exist somewhere, but will it at that particular locality? A to R is 50 yards $= 150$ feet, R to S is 15 feet ($2\frac{1}{2}$ fathoms), therefore the angle and slope of that shoal is $\frac{15}{150} = \tan a = 6°$.

Such a slope could probably exist almost anywhere; in coral water, sometimes the side may be perpendicular, and in abnormal

places the slope might be 45°, and the shoal be correspondingly less in diameter. But supposing it to have a normal inclination, such as 10°, and be 100 yards in greatest diameter, then, as shown in fig. 197, the lines A and B, 100 yards apart, would have in their sounding missed the side of the shoal. If their distance is 20 yards less, as at A′ and B′, they would have obtained a shallower sounding when on the edges of the 'bank' than those both before and after, on one or both the lines.

This is called obtaining an indication; and it is reconnoitring for these indications that the first lines are run: at what distance apart will depend on the part of the world, the character of the land and bottom, the part of the chart, etc.; for evidently a blind passage is not important, a greater margin of speculation being admissible; whereas the main channel or depth is. Evidently a depth of 1 fathom and under is not navigable water: at a depth of 50 feet (7 fathoms) the slope of the shoal would be abnormal, so that its shallowest part shall be within $2\frac{1}{2}$ fathoms of the surface. Therefore, from 7 upwards the lines can gradually be extended, and probably then, in main channels up to 5 fathoms, the lines should be about 100 yards apart; at $1\frac{1}{2}$ and under, about 200 yards; and after 7 gradually increase the distance.

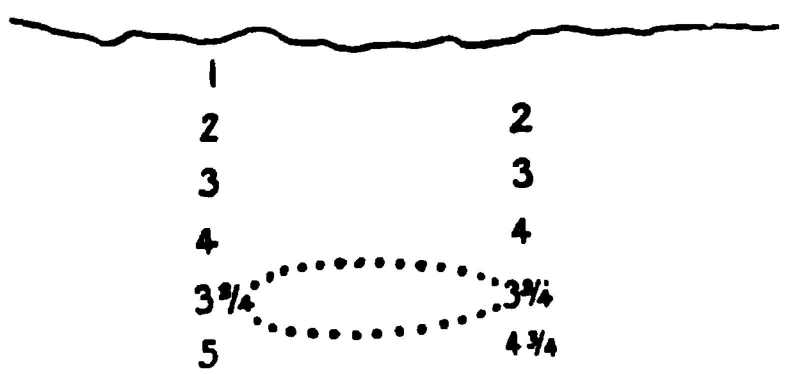


Fig. 198.

Fig. 198 shows a bird's-eye view of the indication; for on running out from the shore the depth suddenly shallows on both lines, and then increases again.

In whatever way the lines on fig. 193 are drawn, one or other will strike an indication—possibly only a small one.

The pith of this part of the work is, that every sounding must be taken with the line in the same position. It must be 'up and down'; if not, the relative soundings are of no use.

535. Lead Line must be 'Up and Down.'—For suppose in the above case one line of soundings only had passed over the edge of the shoal, that at 4 the line was up and down, and that at the next sounding, which should be $3\frac{3}{4}$, the line says and registers 4 or over 4 because the position of the 'line' was not true, then the shoal is lost, or is never found.

536. Care in Reducing the Soundings.—Again, each sounding must be reduced to the same datum, or the shoal will not be discovered; for if one sounding is 4 and the reduction is $\frac{1}{2}$ fathom, this reduces it to $3\frac{1}{2}$; if the next sounding to it is $4\frac{1}{2}$, and only a reduction of $\frac{1}{2}$ fathom is applied when it should be $\frac{3}{4}$, then both will appear as 4; whereas one is 4 and the other $3\frac{3}{4}$.

It is not likely to happen in such a case as the above, but may well occur under many conditions, when dealing in reductions by ¼ fathoms.

537. Effect of Bad Fixes.—There is still one more way of missing the shoal, and that is by bad fixing.

It has been explained, under the section on Tides, that over shoals the tide sets stronger than in the surrounding deeper water.

In the case given above, it may well happen, and does happen, that the boat will be swept a little to one side, and instead of the sounder being on the line he intended, he may unwittingly be slightly off it. Now a bad fix is one in which the centre of the station pointer may be moved some appreciable distance without disturbing the legs off the objects. Well, the fix may put him either *on* his line, or *off* it, in which case he naturally, for choice, accepts it on; but the position might equally be fixed to one side of his line, and this is where he probably is; and so the shoal is missed.

Then, having decided that the lines are to be ¼ inch of paper apart (representing 100 yards), and generally at right angles to the line of the coast, the sounder must endeavour by all possible means to keep to these lines, to sound with the lead 'up and down,' and to be very careful about his relative reductions.

538. Necessary Conditions as to Effective Sounding.—No place can be said to be adequately reconnoitred unless—

(1) The 1, 3, and 5 fathom contour lines off the coast can be distinctly contoured; that is, are reasonably free from ambiguity;

(2) A shoal cast among deeper soundings is sounded in all directions until the normal depth is reached;

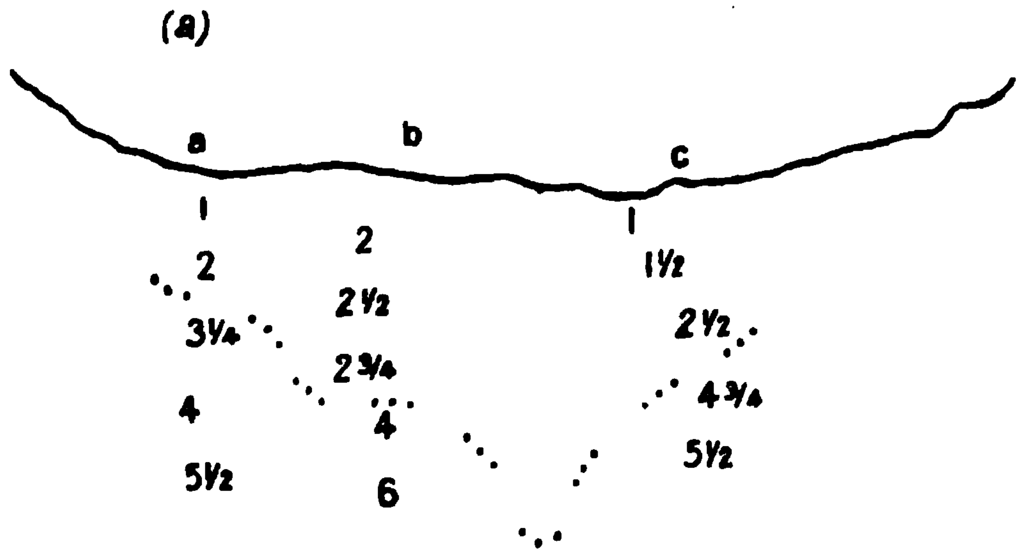


Fig. 199 (*a*).

(3) And, combining the two, the contour between two soundings of any depth can be reliably drawn; the distance between the soundings depending upon the depth, and, in the case of a shoal, depending upon the relative depth of the shoal to the normal depth.

For instance, in Case 1, between lines *b* and *c*, fig. 199 (*a*), the 3-fathom contour cannot be inserted for that depth, as the

distance between the lines is too great. The 3-fathom line might take the shape shown by the dotted lines, there being no contra-indication; as a matter of fact, such crooked lines have existed, and are very liable to exist again.

Case 2, fig. 199 (*b*), illustrates the case of a shoal cast among deeper soundings. Left as it is, the 5-fathom contour line cannot be completed; or it may be drawn in a number of directions: nor can the 6-fathom line be drawn nor the 7; 8 being the normal depth.

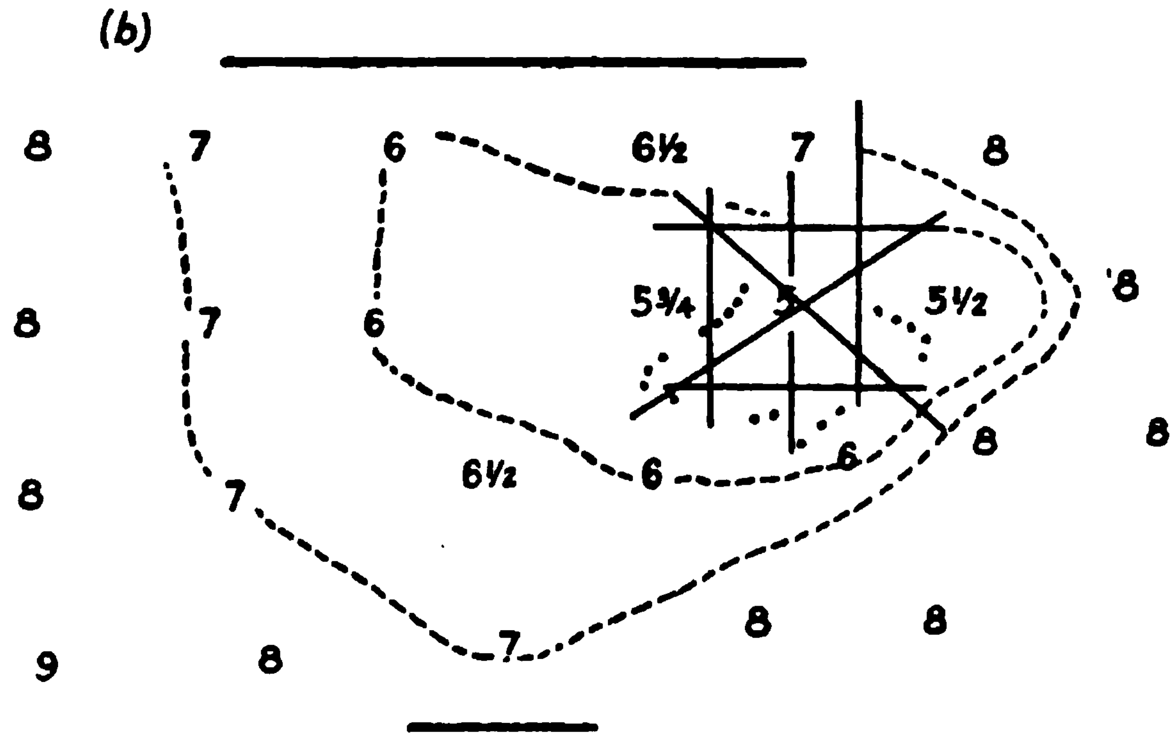


FIG. 199 (*b*).

539. When Necessary to Run More Lines.—It will be necessary here to run more lines, as shown by the vertical hard lines; and after that, it may still be found necessary to 'star' it in search of the shallowest possible sounding; or again, it may have to be done on a larger scale, to make it plainer. (See *Enlarging for Sounding*, p. 346, par. 580; p. 340, par. 578.)

After the intermediate lines are run, there will be more information as to the possible shape of the shoal, and if it should happen to show a shape such as indicated in fig. 199 (*b*), then more horizontal lines would have to be run, but this time across the first; as well as others as shown.

540. Promiscuous Soundings.—Promiscuous sounding, one here and one there, conveys no useful information.

Fig. 200 represents a chart. According to the scale, each sounding is 200 yards apart; from their irregularity, there might be as many rocks lying about as a battalion in skirmishing order.

No doubt the soundings that are shown are there, and somewhere near perhaps where they are placed, and a ship would be safe enough over the depths mentioned; but then the trouble arises in getting from one sounding to the other.

Notice the anchorage carefully marked.

The general deduction, looking at these soundings, is that at no cost should a ship go into such a place. It would be better if there were no soundings at all, as a vessel would then send a boat sounding ahead of her.

541. The Ineffectiveness of 'Negative' Soundings.—The same remark applies to a negative sounding: $\dot{\overline{10}}$ means that bottom was not reached with 10 fathoms of line out; but there is no information as to what is the condition between the soundings—anything, in fact, might exist: the only time when such a form of sounding would be justifiable in a port, is when an area has been 'swept' and found to be free from rocks or shoals or patches within 10 fathoms of the surface, or of the negative sounding stated; or if an area is dredged out.

But this excellent, and seemingly perfect idea, may not yet come to pass, apparently for two reasons; one being that it would be a strenuous and expensive job, and the other, that the insurance premiums would be lessened; and it is hard to say which is the more important consideration for the end attained.

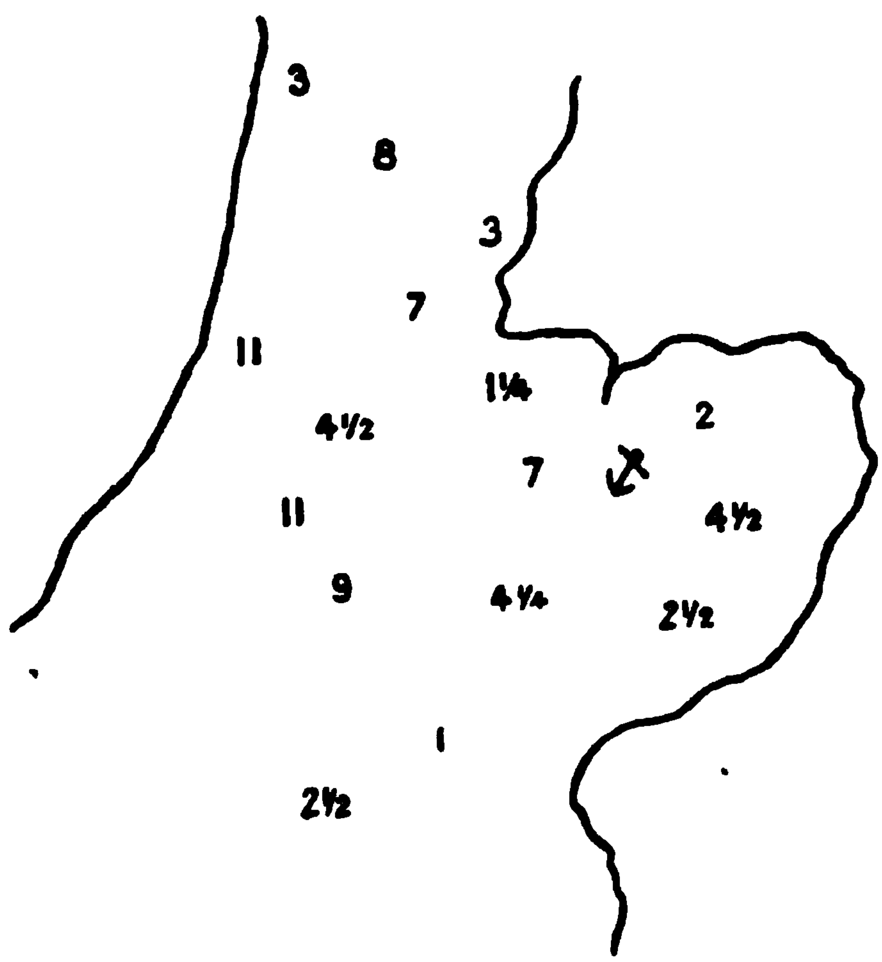


FIG. 200.

To recapitulate: to sound most effectively, or, in other words, to equally cover any space with the least chance of missing a shoal, it is better at first to run all lines of soundings parallel and equidistant from each other; and the soundings on these lines should, up to 5 fathoms, be not less than 100 feet apart.

542. Excessive Soundings Thinned Out.—On a scale of less than 6 inches there would in this case be more soundings than could be plotted without utter confusion; under such conditions, only those that can be plotted are filled in, though the intermediate ones must be watched carefully for any indications they might suggest. For instance, 3, 3½, 3¾, 4, 4½, 4¼, 4, 4½, 5 (reduced); supposing there is no room for all these, the natural act would be to leave out the odd ones, and to start from the second sounding; in such case they might be written on the board

$3\frac{1}{2}$, 4, $4\frac{1}{4}$, $4\frac{1}{2}$, and in consequence of such a slip the shoal is lost.

It is noticeable that the second sounding of 4, which is missed out, is an indication of the shoal.

After 5 fathoms, up to 7 or 8, the distance apart of both the lines and of the soundings might be increased to, say, 200 yards for the lines, and the soundings be 150 feet apart; and it must be pointed out that even this leaves room for many small dangers between the lines, for, no matter what the system may be, sounding is never infallible.

But distance between lines must depend upon the nature of the coast and the probable shape of the bottom, and also the nature of the bottom: a rocky bottom is more likely to produce rocky and steep-sided shoals than a muddy and sandy bottom.

In coral water, and where the coast is bold, reefs are almost 'steep to'; and where pinnacle-shaped islets and rocks lie off the coast above water, the probability is that the bottom follows suit; and here, sight is as effective as, or perhaps more so than, the lead, since whirls and eddies, under perfect conditions of wind and weather, would show themselves; but as many are found by accident as by regular sounding. By 'accident' is meant that a tide stream may drift a boat over a patch which is off the line of soundings, as also other 'unavoidable' accidents, when the ship herself bumps a shoal.

Around bays and 'bights,' sound at right angles to the general trend, and here also the lines may be further apart; but off salient points of land, at the entrance to a harbour, etc., the soundings should be closer together.

543. Fig. 201 illustrates a general idea of the direction lines of soundings might take in a harbour. First, rule lightly the lines intended to be run, and closely adhere to them for the reasons already explained.

Having made clear the purpose of sounding, and the why and wherefore of a system, the details only will be explained.

First, there is the boat; it must be large enough to allow room for one 'hand' to throw the lead without hitting the man behind him on the head; and since his footing is necessarily precarious in any small boat, the leadsman is usually given a stanchion or pole, erected somewhere in the bows, for him to hold by with one hand.

When he pulls in the lead and line, he will, if the boat is not under control by steam or sufficient oars, pull the boat's head all over the place. Besides him, there is the other operator in the stern of the boat to be considered; he must have room for a board on his knees, or resting somewhere, so that he can plot each fix as he goes along, otherwise his lines will, to say the least of it, be erratic and confusing.

For suppose the fix is only plotted at the beginning and end of the line, then the result shown in fig. 202 may occur, viz. a blank

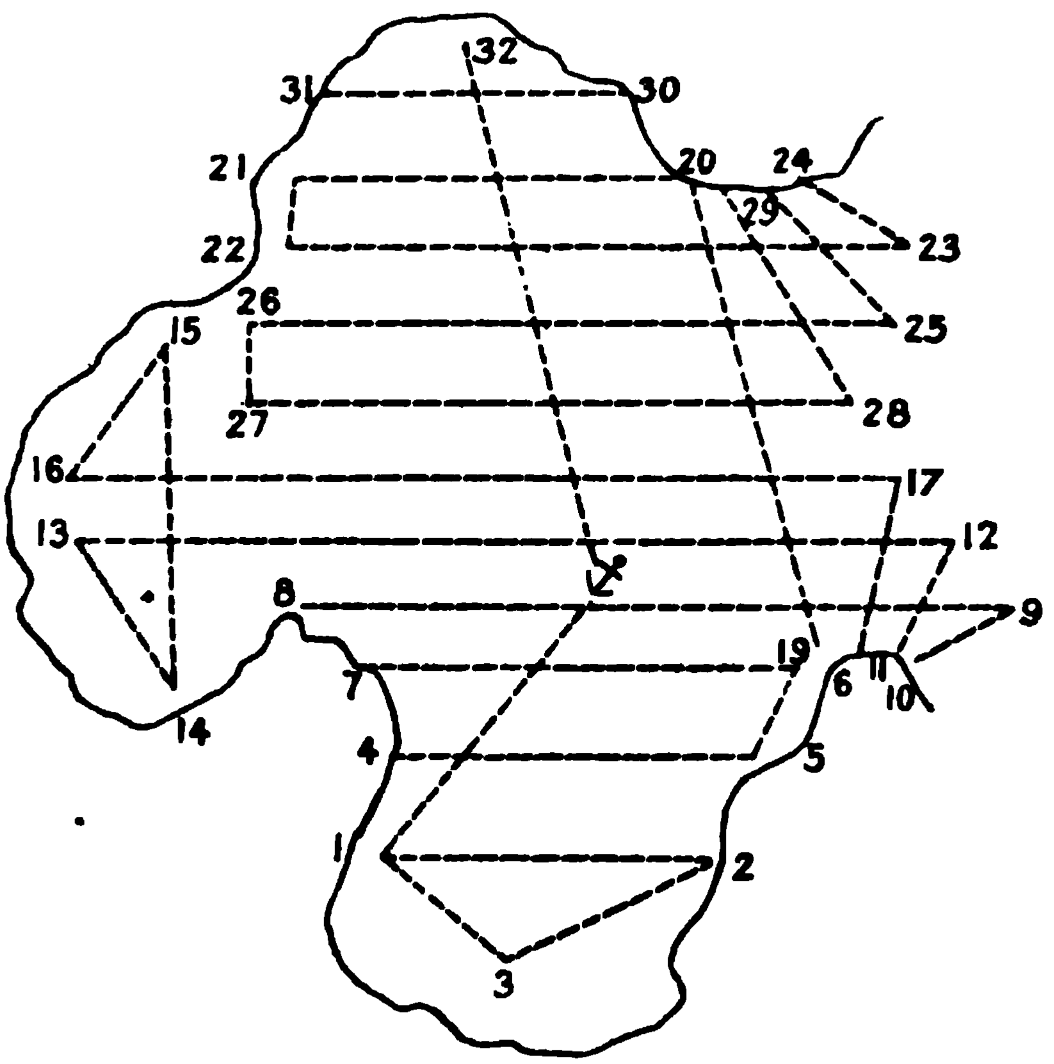


FIG. 201.

between lines *a* and *b*, where two more lines are required to fill up. The dotted lines are those that should have been, and the wavy lines are those actually, run. They may not even be so straight as shown in figure; but, anyway, they are only an encumbrance.

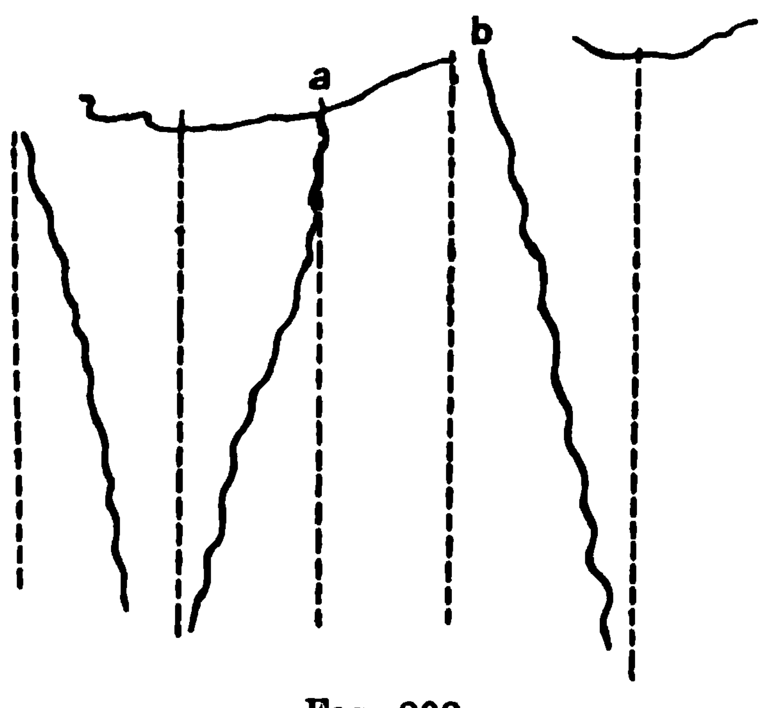


FIG. 202.

But had the fixes been plotted as they were taken, then the error in the direction could have been rectified.

So that for the operation of running systematic lines of soundings, two men in a dinghy are useless; though for the purpose of verifying an occasional sounding on the chart, they may suffice. The beau-ideal of a boat for sounding is a half-decked steamboat of shallow draught, about 40 feet or less long, twin-screwed, with room for a table or a stern cabin,

on the roof of which the board and instruments can be placed and worked.

The leadsman can either throw the lead or use a 30-foot pole, and walk aft with the line in his hand. The sounding is obtained 'aft,' the fix is at the same spot, and the line being hauled in from aft there is less possibility of the lead line fouling the screw.

544. Distance Covered by a Boat Sounding.—In such a boat, in soundings ranging from 2 to 10 fathoms, it is possible to run 30 or more miles of lines of soundings in a day of 10 hours; in a rowing boat, the distance covered will be about 12 miles. The time required to sound out an area can be estimated accordingly.

545. Marking a Lead Line.—A lead line should be marked in feet up to 5 fathoms. Wharton recommends the following plan.

At 1, 2, 3 fathoms, pieces of leather with respectively 1, 2, 3 tails; 4 fathoms, red bunting; 5 fathoms, white; 6, blue; 7, red, and so on. At the intermediate ½ fathoms up to 4½, blue bunting is used; and each foot is marked by a knot, the numbering starting from each complete and ½ fathom. Thus 3 fathoms 1 foot is indicated by 1 knot; 3 fathoms 2 feet by 2 knots; 3 fathoms 4 feet by 1 knot again; 3 fathoms 5 feet by 2 knots, and so on.

The size and thickness of the lead are regulated by the supply, and individual taste, remembering that a small line with a small lead reaches bottom as quickly as a thicker line with a heavier lead; and bearing in mind the labour entailed in heaving a heavy lead, and also the awkwardness of hauling in a thin line, for many hours, as also the depth of water to be sounded. A sounding pole is a most efficient instrument for harbour or river work where the depth is less than 5 fathoms, as, with practice, it is as easy to handle as a lead and line.

546. Sounding Pole.—Obtain a light and strong pole, or a baton about 2 inches square, or a bamboo pole, about 32 feet long. At the heel fix an iron shoe sufficiently heavy to sink the pole nearly to its head in an upright position, but not so heavy as to drag the pole right under; mark it in feet and inches.

The 'leadsman' launches it, leaning forward as he does this, holding on to the end with one hand. The weight of the shoe pulls it downwards, and the speed of the boat is so regulated that the pole shall be exactly upright by the time the boat reaches the spot where the shoe touched the water. The forward movement of the boat assists the sinking of the shoe end, and the final 'coup' is given by the leadsman pushing it 'home.'

There can be no doubt that soundings obtained by such means are truer than those made with the lead and line.

In the half-decked boat suggested, the operation is simplicity itself, because the 'leadsman' walks aft with it while it is gaining the upright.

547.—To fix one's position on a piece of paper there must of course exist on the paper the correct position of a number of objects that are required for fixing; and there should be such a sufficiency that the boat's position at any part of the work can be fixed. (See *Triangulation and Marks*, p. 212, par. 425, and p. 213, par. 427.)

These 'points' will be ringed, so as to catch the eye, with a small circle in Indian ink or with ink that will not run; and each labelled with the name abbreviated, or as short as possible. Then, armed with this paper, ruled with the lines to be 'run' and mounted on the board, a sextant, station pointer, notebook, pencil, protractor, watch, and rough table of reductions, the boat shoves off.

When clear of the ship, or when nearing the locality where

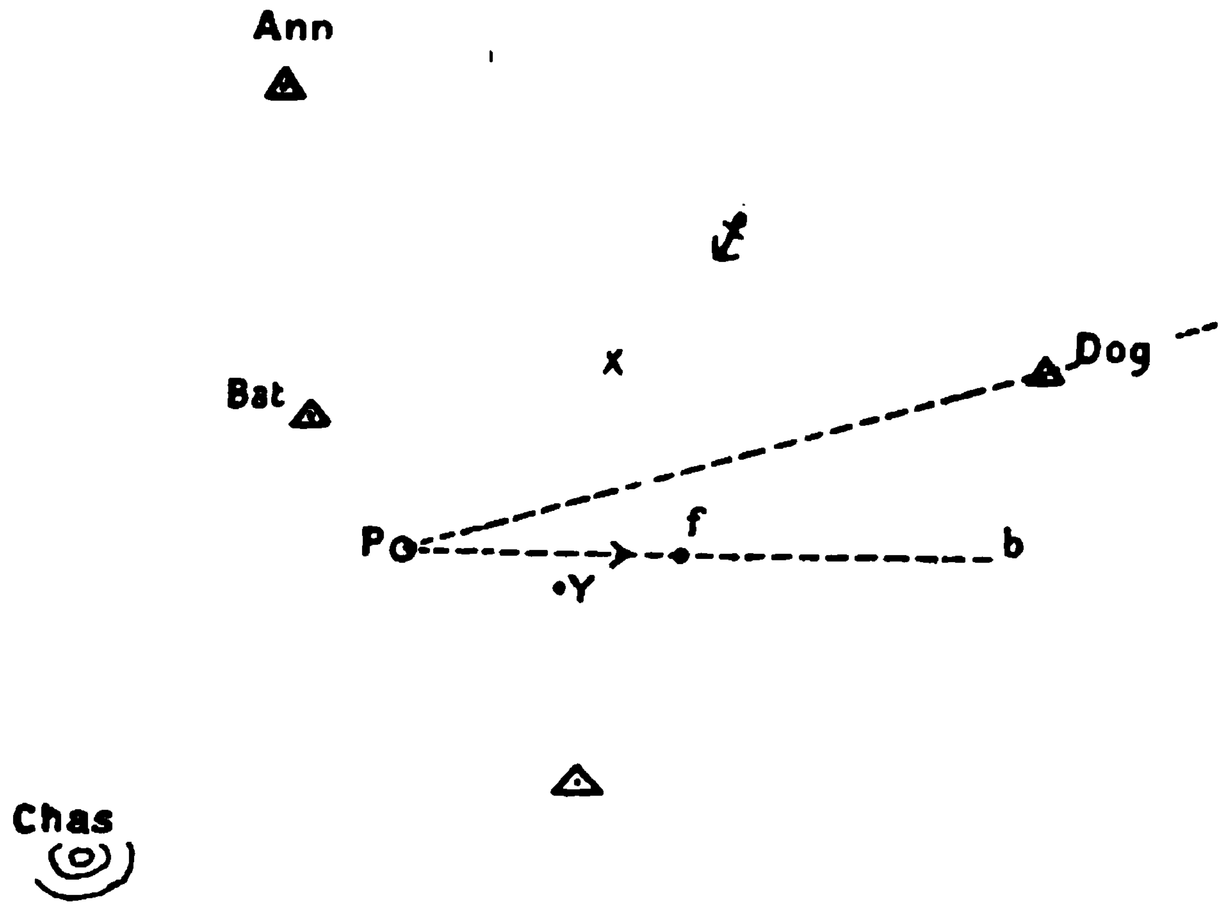


FIG. 203.

operations are to commence (see fig. 203), stop, and fix according to the following directions.

548. First Steps in Sounding.—Noting quite roughly where the boat appears to be, look on the paper for suitable objects to fix with. Roughly speaking, take a suitable near object as the middle one; look for another on the left of it, making an angle of about 90° with the first object, and nearly the same distance off; then another object to the right of the middle one, but much further away; or adopt one of the many other good fixes available.

No one undertaking responsible work in a survey, should be permitted to take angles for fixing who has not been instructed in the matter by good teachers, and passed through a preliminary examination (see chapters on *Fixing*, pp. 64–96). The angles are

placed on the station pointer, and, as explained on p. 71, par. 181, the boat's position found, which is marked with a pencil dot.

In fig. 203, let X be the position of the boat at this fix. The surveyor writes down date, hour, sounding, in the following way:—

Chas (short for Charles) 78° 10′ Bat 45° 40′ Ann . . $6\frac{1}{2}$

Looking at the lines as drawn, it is decided to commence at P, where the ring is shown, and to continue on the line indicated by the direction of the arrow.

So the boat is steered towards Charles, and judging P to be about half-way, the boat is stopped near the mid distance, and "fixed" again:

Chas 42° 50′ Bat 115° 00′ Ann.

This fix places the observer at Y. Evidently he must steer a little towards Bat; in doing so, the angle between Charles and Bat will shift very rapidly, and that between Bat and Ann not at all, or very slightly.

549. Picking up a Position by an Angle.—If the centre of the station pointer is placed at P, the middle leg over Bat, and the left over Chas, the angle indicated by them, if placed on the sextant, will be the left one at the spot P: that particular angle was chosen because it was the one that would be changing the most rapidly in going from Y to P.

Had the direction been directly towards Bat, then the whole angle (the sum of the two), that is from Chas to Ann, would have changed the most, and it would have been taken off the pointer and set on the sextant.

Having set the angle C – B on the sextant, now steer in a direction about mid-way between A and C (any object selected on the coast-line will give this direction), and looking at Chas through the clear part of the horizon-glass, move on till Bat appears in the reflected part; then fix again, and the boat will be at P ⊙ or somewhere near it. It is not absolutely necessary to be exactly on the dot, but as close to it as is reasonable.

This, then, is the position to begin from, and it is advisable to 'hang on' here, either by anchoring, or keeping the lead on the bottom and the boat over the lead; the object being to maintain your position until the future line of direction is ascertained.

550. Laying Off and Setting Angle to Direction of Line.—It is now required to steer in the direction P to *b* (see fig. 203).

Place the centre of the station pointer at P, the middle leg over Dog, and the right (if possible) along the line P*b*; if not possible, on account of the small angle, use the left over Dog and the middle over P*b*; read off the angle D P*b* as indicated on the pointer, and set this angle on the sextant.

Point the sextant to Dog, and look at it through the clear part of the horizon-glass; some shore object will be seen through the silvered portion of the glass superposed on Dog. Take special note of this object, which may be the branch of a tree, or even a leaf, or one stone darker than another; this object will be on the line P*b* produced to the shore; then put the sextant down, and select another object in line with the first, but much further off; by keeping these two objects in transit, the boat will remain on the line P*b*, though, owing to wind or tide, she may be heading in another direction.

Then up lead or anchor, and begin sounding, always keeping the marks in transit.

551. When to 'Fix' on the Lines.—The last 'fix' at P reads VII. 30, C 73° 00′ B, 45° 00′ A . . . . 1 fathom. The directions given were that soundings were to be obtained at every 100 feet, so an attempt will be made at first to guess where they should be taken. If the boat is being pulled against the tide, try every 10 strokes of the oars; or if a steamer, say every 5 seconds of time. At the end of every 10 strokes or 5 seconds, sound. The soundings are, say, $1\frac{3}{4}$, 3, $4\frac{1}{2}$, 5, $5\frac{1}{4}$.

If a rough table of reduction supplied directs that at VII. 30 the reduction to be made is about $\frac{1}{4}$ fathom, then the last sounding taken will reduce to about 5.

The 5-fathom line is important because it is a warning danger-line for shipping; therefore it must be fixed as accurately as possible. The only way to do this will be to 'fix' when the last 5 (reduced) is found when pulling away from the shore, and where the first 5 (reduced) is picked up going towards the shore.

In this case it is decided to fix at the next sounding, which is C 75° 30′ B 47° 10′ A . . . . $5\frac{1}{2}$.

552. How to Space the Soundings.—This 'fix' is plotted by station pointer and marked with a pencil dot as shown in fig. 203; P being the starting-place and *f* the last 'fix.' This is

FIG. 204.

shown more distinctly in fig. 204, an enlargement of part of fig. 203. At P the sounding reduced was $\frac{3}{4}$ and at *f* it is $5\frac{1}{4}$; between P and *f* there are 5 soundings.

Make a dot half-way between P and *f*, and between that dot and P and *f* place two more dots at equal distances apart; there will then be five dots shown by lines in fig. 204. Starting from P, plot the soundings, and they will appear as in that figure.

553. Regulating the Distance between Soundings.—Now on measuring the distance between P and *f* in fig. 203 it is

found to be ·8 of an inch; and this on a 6 inches to the mile scale, supposing the plan to be on that scale, is $\frac{8}{10} \times \frac{1}{6} . 6000$, or roughly $= 800$ feet; and since there are 6 spaces between P and *f*, then each space will $=$ 135 feet. That is longer than was ordered. This indicates that 10 strokes of the oars, or 5 seconds of time, gave too long an interval, and another guess is made in reducing it, which will probably not be far out. This will constantly have to be altered to allow for wind and tide.

If it happens that 5 soundings cannot be legibly plotted, 3 soundings only are entered. But when it comes to a shallow patch, every sounding taken must be plotted, and be weeded out later in the quietude and seclusion of a room; or the whole shoal be re-plotted on a scale enlarged from the original, and then, if necessary, re-sounded, as demonstrated in *Example*, p. 324.

Thus the soundings continue; but by the time these soundings have been plotted the boat may be off her line, unless she was anchored. Up again on the line of the transits she must go, and the soundings continue—5¼, 6, 6, 5, 3¾, 3½; and it is decided to 'fix' the next.

Then VIII. 00, C 88° 30′ B 87° 00′ A . . . . 3¼. Sometimes the transits may be 'lost,' that is, the back one may be shut out; in that case another and nearer one must be taken. Sometimes, while the necessary directing angle is being 'taken off' with the station pointer, the boat has drifted; then, of course, the line of direction will be in error; and that would produce such lines as are shown in fig. 202.

It is entirely a question of quickness and accuracy in fixing, and in the use of instruments, and practice in 'picking up' objects to run on; and see Appendix II.

At first, then, it is far better for a beginner to anchor at both ends of the line, so as to make quite sure where he really is, and about the directing objects to run on.

Having then arrived at the end of the line, the boat is again 'fixed,' and supposing she is anchored, it may be found easier to plot the soundings between the fixes, than when the attention is drawn off by the steering and plotting while moving from 'fix' to 'fix'; but each 'fix' must be plotted when it is taken.

Then the sounder moves on to the next line, 'picking up' his position at the end of the line as he did before.

554. Steering by Transits Astern.—In fig. 201 it will be noticed that when following line 8–9 the boat is heading seawards, and there is nothing ahead for fixing the direction by; in such case the observer takes 'transits astern,' by simply projecting the line to be run through the land astern of him, laying off an angle, as before, from the fixed object making the smallest angle with

his line (see Appendix II., *Errors in Projecting Lines from a Position in Error*), and then the process continues as before.

On this line he may have a fair wind, and the strokes of the oars, or the time, will have to be reconsidered from what it was when he was going head to wind.

Cases might arise where there are no possible transits astern which can be picked up. If too near the shore, pull out a little way.

If the coast be a barren desert of sand, or devoid of prominent marks, the observer must erect his own transits with anything handy: one set for running out on, and another for running back on; and these will be shifted at the end of each pair of lines.

The distance between the outer pair can be measured by lead line, and the inner pair adjusted by angles, as in fig. 205 ($a = 180 - b$), ($a' = 180 - b'$); or they can be adjusted by prismatic compass bearings; but when far enough from the coast, the boat must be fixed by angles as soon as possible, and as often as possible. The transit line in going in must not be accepted as a fixing line if this can possibly be avoided.

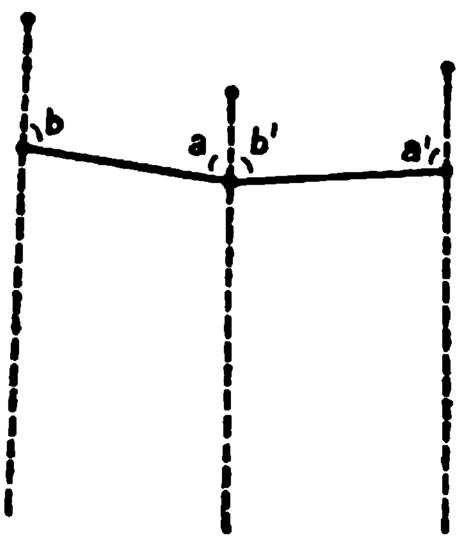


FIG. 205.

In the case of a steep cliff faced by an open sea there may be no means of obtaining transits either in or out. The only possible way out of the difficulty is to steer a course by boat's compass, and find what the compass does show for the direction you want to go in; or, conversely, steer any course near the supposed one by the compass, and let that course take you where it will. The next line parallel to this in that direction will be the same course; but of course, fixing by angles must be resorted to as soon as possible, as until this is done the boat's course by compass and the distance by patent log must suffice.

When close in, the fix will be 'shore 100 yards' (see example, par. 565); and, judging distance by the shore, the next line will be run; usually there should have been arrangements made for such a contingency, and here and there along the cliffs whitewashes fixed for the very purpose of sounding. (See example, 'Extending the Marks for Coast Sounding,' p. 262, par. 453.)

555. Necessity to 'Keep Over' the Lead.—Let it be repeated that the lead line must be vertical 'up and down,' as when working with the wind the boat is very liable to overrun the line.

In a strong tide, where it is impossible to have the lead 'up and down,' and soundings at such times are all but useless, wait for slack water. In a lumpy sea, harbour, or river, soundings are also very doubtful readings. In deeper water it is possible to

make out a table of corrections for the angle of the line from the perpendicular; here a small error of a foot or two is not of so much consequence.

556. Sounding should be done Deliberately.—Sounding under the best conditions is necessarily strenuous work; some are better adapted to this style of work than others. Sounding for continuous hours and days is trying to the nervous system; and, as in every other branch of labour, long hours do not necessarily mean a corresponding increase of good work. Under no conditions should sounding be done hurriedly. Probably the slower and more deliberately it is carried out the better will the result be. (See par. 544.)

557. Final Reductions made Later.—Later in the evening the tide registration will have to be produced, and, if necessary, all the soundings again reduced, plotted, and inked in; while later still the work can be analysed to see if, and where, intermediate lines should be run. (See example, P.M. Bay, Plate XIV.)

In this way the whole harbour is gradually sounded out; the buoys, piles, etc., can be subsequently 'fixed' by going to each one and taking angles.

558. Sounding over Mud- and Sand-flats Necessary.—It is immaterial at what period of the tide soundings are obtained, though to obtain the necessary data to enable a truthful statement to be made about the drying of mud- or sand-flats they will have to be sounded while there is water on them; and supposing the sounding is 2 feet, and the reduction is 6, then evidently that particular spot will dry 4 feet at L.W.S., and is marked $\underline{4}$.

If one boat can go over these flats at a certain time of tide, another of the some draught can do so also; and a chart is for the use of boats as well as for ships.

There are short cuts for barges across most of the flats at the estuary of the Thames. And these short cuts will save hours, and may save days and perhaps many pounds to the owners. It serves no purpose whatever to write across a mud or sand-flat that it dries at L.W.S.; but it would seem to serve the purpose best of all if the depth at half-tide were stated on such places.

559. Soundings in a River.—In a river the soundings would be carried out in exactly the same manner, from shore to shore in parallel lines.

In a creek, a line up the middle must suffice; here a line of negative soundings would serve the purpose, and this introduces the 'submarine sentry.'

560. Description and Use of the 'Submarine Sentry.'—This ingenious contrivance is evolved from the following idea. Suppose a stupid small boy is flying a kite and there is no wind.

If he runs along at a steady pace, with a certain length of line, the kite will remain aloft at exactly the same vertical height from the ground, the vertical height depending upon the length of the string 'paid out.' Reverse the process, and tow a wooden kite in the water. Going at a steady speed, with the same length of line out, the kite will tow at a certain vertical depth; and the vertical depth will vary with the speed and the length of line. The arrangement is not kite-shaped, but it is slung on the same system as a kite. A fisherman's 'drogue,' or 'door,' acts exactly on the same principle, and is made use of by them, to sink their nets below the surface. The ingenious part of the sentry is that it has an iron tripper on its stem, which projects below the base of the arrangement.

Now the kite can be set to a particular depth; and is towed over an area, in parallel lines, at any speed; like mowing a lawn. If it should strike a less depth, the iron tripper takes the shock, releases a trigger, and shifts the line from the kite-slung fashion, to a position at the end of the kite, and it bobs up on the water. Where it has been towed and has not struck bottom, it registers an area of 'no bottom' at the depth set—a record supplementary to the soundings already shown.

In shifting this position of the 'sling' a great deal of the pressure is taken off, and a contrivance can be rigged up supplementary to that already supplied, by connecting it with the ship's or boat's steam whistle or syren, whereby that sounds an unmistakable alarm, and no watching is therefore required.

561. Re-measuring Lead Line.—On returning from the day's labour the lead line must be re-measured.

It may have been in use eight or ten hours, and it was probably stretched. For example, at the standard 3-fathom measurement, the line may show $2\frac{3}{4}$ fathoms; this is the equivalent to using a wrong scale.

If the scale is too long, distances or lengths measured on it will be too small; therefore the correction will have to be + to all the soundings, proportioned to the hours the line was in use; and as every division on the scale is an error, each scale division will have to be proportioned. For instance, supposing during 10 hours the 2-fathom mark has stretched 3 feet—that is, for a correct length of 2 fathoms it shows $1\frac{1}{2}$—it has to be accepted as having stretched $\frac{3}{10}$ foot per hour.

Starting at VII. 00 A.M.; 3 hours after, viz. X. 00 A.M., it will give a sounding of 1 foot too little *at 2 fathoms*; at 4 fathoms, it has stretched 6 feet, *i.e.* $\frac{6}{10}$ foot per hour, therefore at X. 00 A.M. it will give a sounding of nearly 2 feet ($3 \times \frac{6}{10}$ foot) too small; at 4 fathoms at noon the error will be half a fathom.

These little adjustments may quite alter the complexion of

the results of the soundings. If working in fractions of fathoms:

From 6 inches up to 2 feet	the reduction is		$\frac{1}{4}$	fathom.
From over 2 feet to 3 feet 9 inches	,,	,,	$\frac{1}{2}$	,,
Over 3 feet 9 inches to 5 feet	,,	,,	$\frac{3}{4}$	,,
Over 5 feet to 6 feet	,,	,,	1	,,

562. Quarter Fathoms not shown above 6 Fathoms.—Soundings up to 6 fathoms are inserted in $\frac{1}{4}$ fathoms. No $\frac{1}{4}$'s are registered between 6 and 7; so that $6\frac{1}{4}$ is entered as 6, and $6\frac{3}{4}$ as $6\frac{1}{2}$. At greater depths than 7 fathoms, no fractions are shown; thus $7\frac{1}{4}$ and $7\frac{1}{2}$ appear as 7, while $7\frac{3}{4}$ is 8.

563. Demonstration of Reductions, and the Relative Alteration in the Depths.—Fig. 206 illustrates graphically what is meant by tide reductions. Here the soundings are in

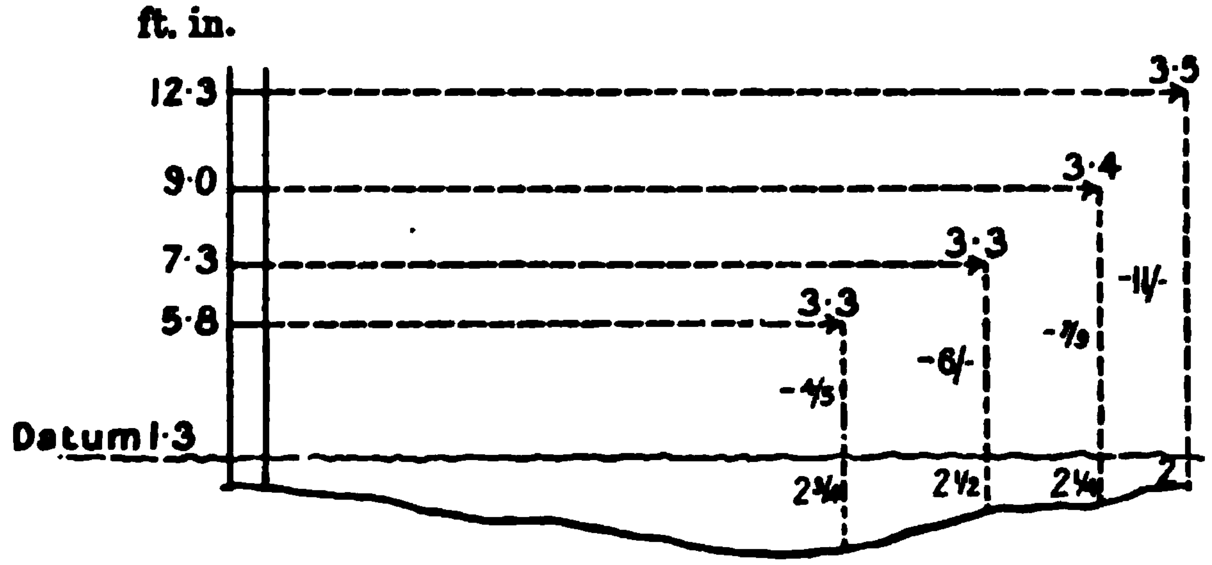


Fig. 206.

fathoms and feet, and the reduction is made in feet; the depth is shown in fractions of fathoms. The vertical dotted line is the amount of the reduction, and is the difference in the reading of the tide-pole at L.W.S., and at the instant of sounding, as shown on the left of the figure; the soundings actually taken are 3/3, 3/3, 3/4, 3/5, and the figure illustrates the case of two similar soundings (viz. $3\frac{1}{2}$, $3\frac{1}{2}$) obtained near the same spot at different hours of the tide.

But when the reductions are applied they produce a different depth at L.W.S. and show a rise in the bottom; and moreover as the soundings gradually increase the depths are shown to be correspondingly shallower. The unevenness of the bottom is not evident to the sounder, unless he applies the reduction. In the absence of a table of reduction, any empirical scale near the correct one (see p. 282, par. 515*a*) must serve the purpose at first, if the work is to be intelligently carried out.

The soundings shown in fig. 206 are not necessarily taken all on

the same line; they may have been obtained partly by one line, and partly on another crossing the first, but at a different hour.

Three examples are given below:—

(1) *Sounding on and off a coast-line, showing the lines of soundings as well as the subsidiary lines, on account of the shallower casts that were obtained in the first going off.*

(2) *Sounding of a point.*

(3) *Starring, or crossing and re-crossing a shoal.*

564. Sounding off a Coast in Feet and Inches, and finding a Shoal, showing the Subsidiary Lines run.—*Example* 1 (Plate V.).—Plot the following positions (fig. 207),

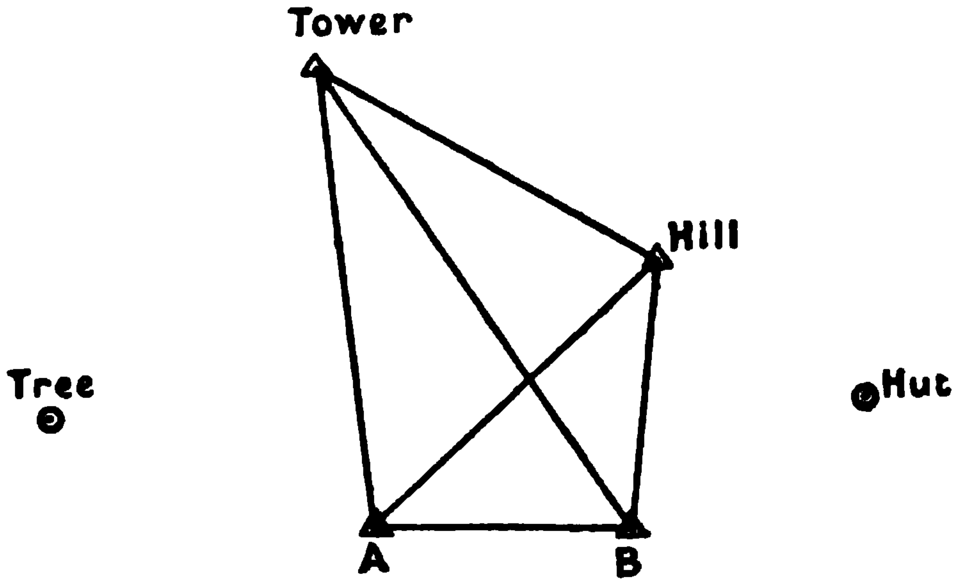


FIG. 207.

which are necessary for 'fixing' purposes. A B is a measured length = 1520 feet. The following angles were observed:—

At A,	Hill	55°	20′	B
	Tower	94	15	
	Hut	20	30	
	Tree	67	10	Tower
At B,	Tower	24	55	Hill (H)
		86	10	Hut
	Tree	62	20	Tower
	A	62	05	

At Tower (T), A is not visible from T, though the staff at Tower is visible from A.

B	61°	50′	Tree
Hill	23	45	B
Hut	37	30	

To plot these positions, B to Tower (T) is the adopted plotting side; and its distance must be calculated 'through' triangle T A B and 'through' triangle T B H.

In Δ T A B, A 94° 15′
B 62 05
T 23 40 (calculated)

180 00

T B : A B : : sin A : sin T

$$TB = AB \cdot \frac{\sin A}{\sin T} = AB \cdot \sin A \cdot \operatorname{cosec} T.$$

log A B 3·161844
log sin 94° 15′ 9·998804
log cosec 23 40 10·396406

log 3·577054 = 3776·2 feet = T.B.

In Δ A B H, A 55° 20′
B 87 00 (24° 55′ + 62° 45′)
H 37 40 (calculated)

180 00

B H : A B : : sin A : sin H.

$$BH = AB \cdot \frac{\sin A}{\sin H} = AB \cdot \sin A \cdot \operatorname{cosec} H.$$

log A B 3·181844
log sin 55° 20′ 9·915123
log cosec 37 40 10·213911

log 3·310878 = 2045·8 feet.

In Δ T B H, T 23° 45′
B 24 55
H 131 20 (calculated)

180 00

T B : B H : : sin H : sin T.

$$TB = BH \cdot \frac{\sin H}{\sin T} = BH \cdot \sin H \cdot \operatorname{cosec} T.$$

log B H 3·310878
log sin 131° 20′ 9·875571
log cosec 23 45 10·394968

log 3·581417 = 3814·3 feet = T.B.

The first result of T B = 3776·2
The second „ „ = 3814·3

2 | 7590·5

mean = 3795·25 feet.

The scale adopted was 4 inches = 1 mile = 6081 feet.
Therefore 6081 : 3795·2 :: 4 : X.

$$\text{X, in inches of paper,} = \frac{4 \,.\, 3795{\cdot}25}{6081} = \frac{15181}{6081}$$

log 15181	4·181301
log 6081	3·783975
	0·397326 = 2·496 inches.

The true bearing deduced of T from B was N. 30° 30′ W.

Now draw a true meridian through the centre of the paper.

Assume B on the meridian, to be in the middle of the line.

Lay off by chords (see p. 56) the T. B. of T (30° 30′), and along the line projected measure 2·496 inches. This will place T on the paper.

Then from B and from T project all the angles taken; and so plot the positions of Tree, A, Hill, and Hut. When A is plotted from B and T, project the angles taken at A.

There is bound to be a small 'cocked hat' at Hill, because the line T B is a compromise between the length obtained 'through' triangle A B T and 'through' triangle T B H; but probably the 'cocked hat' is smaller than if only one of the results had been accepted instead of the mean.

A tide-pole was set up on the beach, and read—

At X. 00 A.M. 6 feet . . L.W.
IV. 15 P.M. 22 feet . . H.W.
Springs rise, 22 feet.

From the above data the 'range' of the tide for the day is 16 feet; half 'range' = 8 feet; the springs rise is 22 feet; therefore mean water-level = 11 feet.

(Refer to Appendix XI., Plate XIX., or to table in *Inman's Tables.*)

The reductions to the soundings will be :—

	ft.	ft. in.	ft. in.		ft.	ft. in.	ft. in.
At X. 00	11 − 8	0 =	3 0	At IV. 00	11 + 8	0 =	19 0
30	11 − 7	9 =	3 3	III. 30	11 + 7	9 =	18 9
XI. 00	11 − 6	11 =	4 1	III. 00	11 + 6	11 =	17 11
30	11 − 5	8 =	5 4	II. 30	11 + 5	8 =	16 8
XII. 00	11 − 4	0 =	7 0	II. 00	11 + 4	0 =	15 0
30	11 − 2	1 =	8 11	I. 30	11 + 2	1 =	13 1
I. 00	11 − 0	0 =	11 0				

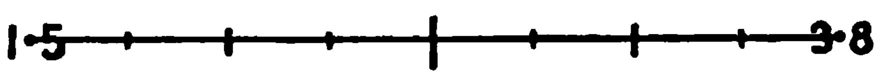
X. 00 A.M. Commenced sounding. Soundings are in feet, and taken to 3 inches in the shallow parts.

21

Time.	Fix.	Sounding at Fix.	Reduction.
X. 00 A.M.	Tree 84° 00′ A 14° 10′ B . . .	1	3
	3 9 12 15 18		
X. 15 A.M.	Tree 39° 00′ A 20° 30′ . . .	21	3/2
	23 24 27 29 33		
X. 30 A.M.	Tree 22° 30′ A 25° 40′ B . . .	37	3/3
	36 35 34	37/6	
	36/5 35/5 34/5		
	Tower ϕ A 34° 00′ Hut . . .	34	3/8
	31 28 25 22 20	34/8	
	31/6 28/8 25/8 22/4 20/6		
XI. 00 A.M.	Tower ϕ A 46° 10′ B . . .	19	4/1
	16/1 13 10/2 7/1 3/1	19/6	
	16/11 13/2 10/4 7/2 3/1		
	Tower ϕ A, and B ϕ Hut . .	2/2	4/8
	6/4 9/3 14/5		
	6/6 9/6 14/8		
XI. 30 A.M.	Tree 42° 30′ A 36° 15′ Hut . .	18/4	5/4
	20/4 23/3 26 27 29	18/10	
	20/10 24 26/10 27/11 30		
	Tree 25° 10′ A 32° 10′ . . .	31	7
	29 29 27	32	
	30 30 28		
Noon.	Tree 19° 00′ 43° 15′ Hut . .	30	8
	24 22 20 18 16/4	31/4	
	25 22/11 20/11 18/10 16/11		
	Tree 23° 00′ A 67° 40′ Hut . .	14	
	Tree 58° 30′ Hill ϕ B . . .	8/7	9
	12/7 14/5 17/2 14/5 12/7		
	13 14/11 18 14/11 13		
XII. 30 P.M.	Tree 41° 30′ Hill ϕ B . . .	21	10
	27 29 32 35 39		
	29 30/8 33/10 37/1 41/4		
I. 00 P.M.	Tree 32° 00′ Hill ϕ B . . .	40	11
	41 41 41	44	
	44 44 44		
	Tree 25° 00′ B 26° 30′ Hut . .	43	12
	39 38 37 35 32	47	
	41/9 40/9 39/9 37/9 34/6		
I. 30 P.M.	Tower ϕ B 38° 40′ Hut . . .	34	13
	31 29 24 21/3 17/10	36	
	33/4 31/4 26/3 22/8 19		

Time.	Fix.							Sounding at Fix.	Reduction.
	Tree 16° 50′ B 76° 30′ Hut . .							12/11	14
								13/6	
II. 00 P.M.	Tower ϕ B 58° 00′ Hut							15/8	15
	19	24	26	29	32	34	35	16/8	
	20/4	26	28/4	31/8	35	37	38		
	Tree 31° 30′ B 29° 00′ Hut . .							38	15/10
	38	37	40					41/4	
	41/7	41/5	44/5						
II. 30 P.M.	Tree 21° 10′ B 33° 00′ Hut . .							42	16/8
								46	
III. 00 P.M.	Tree 35° 00′ B 28° 00′ Hut . .							36	17/3
	34	32	21/6	20	21/6			40/7	
	37/9	35/9	23/7	21/9	23/11				
	Tree 44° 15′ B 41° 00′ Hut . .							25	
								27/9	
	Tree 56° 40′ B 30° 10′ Hut . .							27/8	18/9
	24	21/3	21/3	21	33			31/3	
	25/9	23/9	23/9	23/6	37/2				
IV. 00 P.M.	Tree 40° 20′ B 23° 40′ Hut . .							44	19
								48/6	

NOTE.—To divide a space to fit in seven soundings. By eye bisect the line, make a dot, bisect each of the two spaces, then bisect the remaining four spaces; thus



The amount for the reduction for the tide is written boldly right across the soundings in red pencil, but is here printed at the side, under 'reduction.'

And the sounding, corrected for the error of the line, is written under each.

The labour incurred in making this last correction suggests that much of it might have been avoided by the use of a sounding pole instead of the line, when such an occasion as this arises for all possible accuracy.

On returning on board, the lead line was re-measured.

At 7	fathoms	it was 6	feet	too long.
„ 6	„	„ 5	„	
„ 5	„	„ 4	„	
„ 3	„	„ 2	„	
„ 2	„	„ 1	„	

TABULATED LEAD LINE CORRECTIONS: + TO SOUNDINGS.

Depth.	Error.	X. 30	XI.	XI. 30	XII.	XII. 30	I.	I. 30	II.	II. 30	III.	III. 30	IV.	Rate.
42	6	/6	/11	1/6	2/	2/6	3/	3/6	4/	4/6	5/	5/6	6/	1 ft. p. hr
39	6	/6	/11	1/4	1/10	2/4	2/6	3/3	3/8	4/2	4/7	5/	5/6	
36	5	/5	/10	1/3	1/8	1/8	2/6	2/11	3/4	3/9	4/2	4/7	5/	/10 in. ,,
33	4	/5	/9	1/2	1/6	2/	2/3	2/8	3/	3/6	3/9	4/1	4/6	/10 ,, ,,
30	3	/4	/8	1/	1/4	1/8	2/	2/4	2/8	3/	3/4	3/8	4/	/8 ,, ,,
24	2	/3	/6	/9	1/	1/3	1/6	1/9	2/	2/3	2/6	2/9	3/	/6 ,, ,,
18	2	/2	/4	/6	/8	/10	1/	1/2	1/4	1/6	1/9	1/10	2/	/4 ,, ,,
12	1	/1	/2	/3	/4	/5	/6	/7	/8	/9	/10	/11	1/	/2 ,, ,,
6	/6	...	/1	/2	/2	/3	/3	/4	/4	/5	/5	/6	/6	/1 ,, ,,

565. *Example* 2.—This example is intended to show how the soundings off a point of land may be examined, and fixing marks set up, without reference to the marks on the published chart; or when the published chart is on too small a scale.

A small triangulation will have to be made; the plotting side for this will be two accessible points on the coast, visible from each other, such as Cape and A. (See Plate VI.)

A length is obtained by a series of measurements on the beach, between A and B. (See *Measurement of Base*, p. 160, par. 337 and 338.)

The following angles having been taken, to plot A, B, Church, and Cape. Cape to A was the side plotted from.

At A.	At B.	At Cape.
B 32° 28′ Cape.	Cape 127° 20′ A.	A 20° 22′ B.
Cape 44 10 Church.	Church 72 55 A.	Church 53 45

So as not to wait for any calculations, it was decided to assume the length of Cape to A as 5·5 inches of paper; this was three times the length in inches of that shown on the chart.

The bearing of A from Cape is derived from the chart, and is N. 20° 30′ E. Variation, also from the chart, is 7° 10′ E.

To plot. Draw a true meridian through the middle of the paper from one end of it to the other; and, since Cape is the most southern point, prick its position on the meridian line below the centre of the paper.

From Cape, project the true bearing of A, and measure along the line so projected 5·5 inches; this will give the position of A.

With Cape to A as a plotting side, project the angles to B, and to Church, from each of them. This will plot these two objects by two lines.

Finally, from B, from the line B–Cape, project the angle to Church; it should intersect at the same spot as the other two

lines from A and from Cape; if it does not, exactly, then either the angles taken at B or at A are wrong, or the plotting is in error, or B is plotted wrong.

With sextant angles there is sure to be a 'cocked hat,' and the only way out of it is a compromise; in this particular case the 'receiving angle' at Church is better than that at B, and an error in plotting will throw out B more than Church, as also would errors in taking the angles.

But in Δ A B C all three angles of the triangle were observed, and each angle took its share of the total error; each angle in it is therefore more correct probably than the two unchecked angles observed in Δ A-Cape-Church; and in this particular instance the centre of the 'cocked hat' is probably nearer correct than at any other two intersections.

When four points—A, B, Cape, and Church—have been plotted, they can be pricked through on to the paper, which is mounted on a board ready for taking away in a boat; and the sounding can be proceeded with.

A tide-pole has been erected somewhere near the point, or inside the harbour round the corner.

Accept the reductions as 9 feet during the whole period of sounding. The soundings are in fathoms.

Line 1.	Cape ϕ B 77° A[1]	$11\frac{1}{2}$
	Steered in on the transit	
	$9\frac{1}{2}$ $7\frac{3}{4}$ $6\frac{1}{2}$ $5\frac{1}{4}$ $4\frac{1}{2}$	
	B 100 yards	$2\frac{1}{2}$
Line 2.	Church ϕ B 100 yards from B . .	$2\frac{1}{2}$
	Steered out on the transit	
	$4\frac{3}{4}$ $3\frac{3}{4}$ $6\frac{1}{4}$ $8\frac{1}{2}$ $10\frac{1}{2}$	
	Church ϕ B 40° A[1]	$12\frac{1}{2}$
Line 3.	Cape 20° B 60° A	$12\frac{1}{2}$
	Steered for B	
	$10\frac{1}{2}$ 7 5	
	B about 200 yards.	
	Bearing by prismatic compass, S. 63° W. .	$3\frac{1}{2}$
Line 4.	B bearing N. 58° W. about 200 yards .	$3\frac{1}{2}$
	Steered outwards	
	$5\frac{1}{4}$ $6\frac{1}{2}$ $10\frac{1}{2}$	
	Cape 66° B 24° A	$13\frac{1}{2}$
Line 5.	Cape 100° A ϕ B[1]	$14\frac{1}{2}$
	Steered in on the transit	
	$12\frac{1}{2}$ $10\frac{1}{2}$ $8\frac{1}{2}$ $6\frac{1}{4}$ 4	
	A Ȯ B 100 yards from B	$2\frac{1}{2}$

It was found that as the boat approached the shore the Cape mark was obscured, and the boat was 'fixed' by a bearing and

[1] These 'fixes' should be plotted with the protractor.

approximate distance; but since there was no scale these fixes could not be plotted.

To find the scale, we require to know the length of Cape to A in feet, and the calculation of this took place after the sounding was completed.

To find the length of A to Cape.

		Observed Angles.	Corrected Angles.
In Δ CAB,	C	$20° \; 22' - 2'$	$20° \; 20'$
	A	$32 \quad 28 - 3$	$32 \quad 25$
	B	$127 \quad 20 - 5$	$127 \quad 15$
		$180 \quad 10$	$180 \quad 00$

$$AC : AB :: \sin B : \sin C.$$

$$AC = \frac{AB \,.\, \sin B}{\sin C} = AB \,.\, \sin B \,.\, \operatorname{cosec} C.$$

The length of AB is found to be 4455 feet.

log AB	3·648848
log sin B	9·900914
log cosec C	10·459059
	4·008821 = 10205·04 feet.

Now since Cape to A was drawn 5·5 inches long, and corresponds to 10205·04 feet, how many feet will 1 inch of paper represent?

$$1 \text{ inch} = \frac{10205{\cdot}04}{5{\cdot}5}$$

log 10205·04	4·008831
log 5·5	·740363
	3·268468 = 1855·5 feet.

And since it is more convenient to have the units of the scale in thousands of feet rather than in inches, then how many inches = 1000 feet?

$$1855{\cdot}5 : 1000 :: 1 : x$$

$$x = \frac{1000}{1855{\cdot}5}$$

log 1000	3·000000
log 1855·5	3·268468
	log 1·731532 = ·539 inch nearly.

If the $\frac{1}{4}$-inch scale on the base of the protractor is used, then ·539 is 2·156, and that is the measurement on that scale for 1000 feet; 2000 feet will be 4·312, and 3000 feet 6·466; next, without moving the protractor from its initial position, those

measurements of the $\frac{1}{4}$-inch scale can be ticked off along the line drawn for the scale.

It is advisable to subdivide one of the 1000 feet into ten hundreds. Then the first mark of 100 feet, on the $\frac{1}{4}$-inch scale of the protractor is ·21; the second, ·43; the third, ·64; the next, ·86, and so on; and these can all be ticked off on to the scale line without moving the protractor from its first initial position.

566. *Example* 3. *On Sounding.*—This is to illustrate the *system of 'starring' lines through a shoal.* (See Plate VII.)

Place the position of the following points, which were plotted as part of the general triangulation in a harbour, and would in the ordinary course of events have been pricked through on to the 'field' board.

Assume the distance from A to Turret to be 2·9 inches, and the bearing of turret N. 8° 53′ W.

At A, Turret 71° 50′ B.		At B, A 68° 10′ Turret.
58 20 Hut.		Turret 82 20 Hut.
F.S. 77 35 Turret.		Battery 54 55 Turret.
Battery 80 20		F.S. 47 00

At Turret, B 40° 00′ A.
Hut 71 20
A 35 00 Battery.
57 05 F.S.

The tide-pole readings for the day were as follows:—

		ft.	in.		ft.	in.	
H.W.	X. 00	15	0	I. 30	6	0	
	XI. 00	14	0	II. 00	4	6	
	30	13	0	30	3	0	
	XII. 00	11	6	III. 00	2	0	
	30	10	0	30	1	7	
	I. 00	8	0	IV. 00	1	5	
				30	1	4	L.W.

Springs rise, 20 feet.

From this it will be found that the datum for L.W. springs is 1 foot 10 inches below the 0 of the tide-pole.

[Soundings are in feet and to be so plotted.]

No.	Time. H. M.		
(1)	12 30	F.S. 40° 30′ Turret ɸ A 34° 30′ Hut . .	32
		31 30 28 27 26 24 22	
(2)	12 40	Turret ɸ A 54° 0′ B	16
(3)	12 50	F.S. ɸ Bat. 60° 30′ Turret	15
		17 20 22 21 23 27 29	
(4)	1 0	F.S. 35° 40′ A 39° 15′ Hut	30

No.	Time. H. M.		
(5)	1 15	F.S. 29° 55′ A 44° 5′ Hut	30
		27 25 19 14 15 14 14	
(6)	1 20	F.S. ⏀ Bat. 56° 5′ Turret	14
(7)	1 30	Bat. 17° A 94° Hut	12
		13 (14 Bat. ⏀ F.S.) 15 14 12 15 20 26 27	
(8)	1 45	F.S. 24° 15′ A 48° 45′ Hut	28
(9)	1 55	F.S. 19° 0′ A 52° 0′ Hut	29
		27 25 24 23 20 (20 Bat. ⏀ F.S.) 19 19 17	
(10)	2 10	Bat. 7° 20′ A 48° 20′ Turret	14
		11 11 11 14 18 20 20	
(11)	2 20	F.S. 55° 30′ A 34° Hut	22
		21 18 16 11 11 14 18	
(12)	2 30	T 36° 10′ B 15° 50′ Hut	19
		17 15 13 10 11 12 15	
(13)	2 45	F.S. 44° 30′ A 41° 10′ Hut	18
		16 13 10 8 9 15 18	
(14)	3 0	T 27° 30′ B 17° 20′ Hut	20
		17 16 11 12 14 16 18	
(15)	3 15	F.S. 39° 50′ A 40° 10′ Hut	22
		20 19 18 16 18	
(16)	3 30	A 47° 30′ B 13° 10′ Hut	20
		16 13 11 10 14 16 18	
(17)	3 45	F.S. 61° A 33° 50′ Hut	20
		19 16 14 9 5 8 16	
(18)	4 0	A 52° 50′ B 15° 30′ Hut	18
		13 8 8 8 11 12 13	
(19)	4 15	Bat. 67° 40′ A 36° 30′ B	11

Within 1 fathom contour, bottom is all rock; outside, it is coarse gravel.

CHAPTER IX.

REPORTING THE EXISTENCE OF A SHOAL, AND LOCATING IT.

567. Always Sound on what Appears to be a Shoal.—Never trust to the eye only for the existence of a shoal: the E.D. (existence doubtful) sometimes seen on charts is often enough due to an optical illusion. Many things resemble a shoal, and in certain lights strikingly so: fish spawn for instance, tide rips, and even shoals of fish, seen at certain angles with the sun, have been mistaken for shallow water. When sighted, a sounding must be obtained over it.

568. Tentative 'Fix' of a Shoal, and Sketch.—The vessel should 'fix' her position there and then. If the shore is near enough, and there are sufficient objects visible, then take angles for a station pointer 'fix,' and take a compass bearing of the nearest objects; there being only two distant objects in sight, take a true bearing of one and the angle between them (see p. 108). In either of these two cases make a rapid sketch (see sketch, p. 50). The vessel is supposed to have 'stopped' for the purpose of 'fixing.'

569. T.B. of a Point on Shore most Important.—Failing the sight of the shore, or if only one distant object is visible, then take a true bearing of that, and take sights with the sea horizon to obtain a line of position to intersect the true bearing line (see *T.B. Lines and Fixing Position*, example, p. 108, par. 235). Probably the sun will not be on a suitable bearing to take so that the position lines will cut at a well-conditioned angle. It may be suggested that an angle of less than 30° is ill conditioned. For instance, it may be near noon, and 'sights' by the sun would give a position line nearly east and west (see *Example*, p. 331); this would be prohibitive, so far as a true bearing of the shore object is concerned, if it also bears anywhere near east or west. In fact, there may be a number of conditions which make it practically impossible to 'fix' dependably the absolute position at that moment

by these means, though by waiting at the place till after the sun's bearing has changed at least 30° a fairly accurate position can be obtained (see *Example*, p. 331, par. 572). But the true-bearing line is the most important, and that should be obtained at any cost: it is not the true or compass bearing of the end of the land that is wanted, but that of a definite object, and it would save future time, labour, and expense, if one waits to get the bearing, or the sight that is wanted, to fix the position.

570. Position of Shoal Relative to Vessel.—From a bearing line the position of the shoal can always be recovered. By a sketch it is easier still to find it again; but a combination of the two is the best means for re-locating it at a future time. When the vessel's position is fixed, take a compass bearing of the shoal; a mast-head angle from the boat that has obtained a sounding on it will give its distance off the ship.

If the above steps have been taken, and the shoal is not sounded out there and then, it will be easily found again, and sounded out at some other convenient time, either by the ship, or by a boat if the land is sufficiently near, or by mast-head angle and bearing from the ship moored on the shoal. In this last case the shoal is sounded out relative to the ship's position (see example, p. 288); which method is adopted will depend upon the distance from the shore, and whether there are suitable objects to fix by either from the boat or from the ship; or the ship's position may be fixed astronomically.

An example of 'fixing' with the aid of astronomical lines of position is given further on, p. 331.

571. Occasions that Call for Amending the Soundings.—According to regulations, a leadsman must always be in the 'chains' when a vessel is in the proximity of land; but the soundings he often supposes he gets should not be looked upon as infallible. Still, if he does call a shallower cast than the soundings recorded on the chart at that particular spot, some notice must be taken of it, because there is an off-chance of his being correct.

In navigating in or out of a harbour, it sometimes occurs that a leadsman's anxiety or zeal or other influence, leads him to imagine he has touched bottom with the lead at a greater depth than the conditions would seem to warrant. For instance, at a speed of 6 knots, with a throw-out of say 13 fathoms, the lead is 'up and down' when there are 8 fathoms of line out; the leadsman having 'gathered in' 5 fathoms. At a speed of 10 knots the ship is moving at the rate of 1000 feet a minute, and will take 5 seconds to reach the spot where the lead touched the water, the leadsman having gathered in the slack in that time. During that 5 seconds the lead, dragging down so much line, will have sunk 6 fathoms; 3 seconds after that the line is 50 feet

abaft the leadsman; by that time the lead has dragged the line through his hands about 2 more fathoms; the lead therefore reaches 'bottom' at 8 fathoms about 50 feet abaft the leadsman, and remains there a small fraction of a second. He is called upon, there and then, to make a rapid calculation of what the depth would have been with the lead 'up and down'; and, as often as not, the calculation begins from the nearest mark to his hand. Under such conditions it is not improbable that he will make a mistake in arithmetic or in trigonometry!

Sometimes the soundings called cannot be accounted for. For instance, on one occasion *two* leadsmen, one in each of the chains, called a sounding of 7¼ fathoms where it was ascertained, after some anxious moments, that there was no bottom at 100 fathoms. When a sounding has been called considerably less than that charted, as it was in this case, someone must check it, and even sound out the whole area, so as to leave no doubt as to the actual state of things.

This shows the necessity of the ability required to take angles and 'fix' rapidly by station pointer while the vessel is navigating a channel, so as to locate the area with sufficient accuracy to allow of its being sounded out later on (see p. 339).

Example 1.—Example of a combination of astronomical lines of position (by day) with a true bearing of an object (fixed) on shore.

572. The problem, from a surveying point of view, is to 'fix' the position of a buoy or shoal, the shore being so distant that only one known object is visible.

First case is *when the ship anchors for the purpose.*

At about $8^h\ 30^m$ A.M. on the 9th of June 1908.

Approximate position, lat. 51° 30′ N., long. 4° 10′ E.

Watch showed $6^h\ 02^m\ 59^s{\cdot}5$. Obs. alt., $\underline{\odot}$ 41° 00′ 10″.

I.E., + 2′ 28″; height, 30 feet.

Simultaneously, O| 76° 47′ Lt. Ho.

1. Given the H.A., which will be deduced from the watch time, and the assumed latitude and longitude, the sun's altitude will be calculated and also the true bearing.

[EXAMPLE.

	h.	m.	s.	
Watch time	6	02	59·5	
comparison	2	05	16·2	fast
chron. time	3	57	43·3	
error chron.	4	17	05·8	slow
G.M.T.	20	14	49·1	(8th)
long.		16	40	E
mean time	20	31	29·1	(8th)
eq. time		1	05·4	
app. time	20	32	34·5	
	24	00	00	
H.A.	3	27	25·5	

	m.	s.		
Eq. time	1	14·9	(8th)	·47
–		9 ·5		20·24
	1	05·4		9·5128

Dec. (8th)	22°	50′	25″	N.
		+4	36	
	22	55	00	
	90	00	00	
P.D.	67	05	00	

ch. 13·66 +
20·24
60|276·4784
4′ 36″

log hav. H.A.	$3^h\ 27^m\ 25^s{\cdot}5$	9·281446
log cos lat.	51° 30′	9·794150
log cos dec.	22 55	9·964294
diff.	28 35	9·039890 hav 38° 40′ 11

log vers.	38° 40′ 11″	0·219239
log vers.	28 35 00	0·121878
vers.		0·341117

48° 47′ 07″ zenith distance
90 00 00

41 12 53 calculated altitude.

To find T.B.

log sec lat.	51° 30′	10·205850
log sec alt.	41 12 53″	10·123640
	10 17 07	
P. dist.	67 05 00	
log ½ hav.	77 22 07	4·794318
log ½ hav.	56 47 53	4·677250

9·801058 hav. = N. 105° 22′ E. Sun's T.B.

2. Find the true altitude.

Obs. alt.	41°	00′	10″
I.E. +		2	28
	41	02	38
dip −		5	24
	40	57	14
ref. par. −		1	00
	40	56	14
S.D. +		15	46
true alt.	41	12	00
cal. alt.	41	12	53
diff.	00	00	53 = ·883 mile.

By 'sights' the observer is therefore ·883 mile further from the sun than was calculated, because the true altitude is less than the calculated.

3. To find the horizontal angle between the sun and the Lt. Ho., and from this to deduce the true bearing of the Lt. Ho. (see fig. 207A).

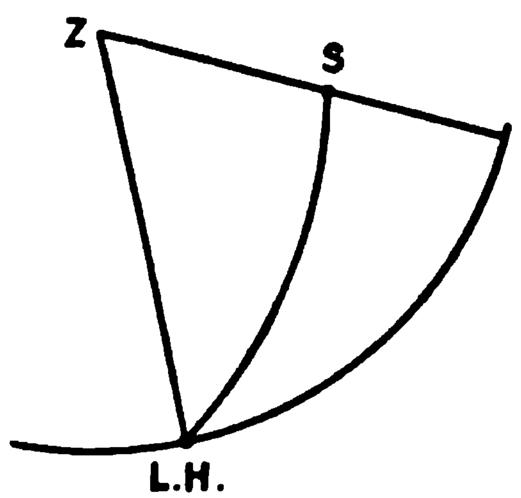


Fig. 207A.

SL = obs. dist. =	76°	47′	00″ to near limb.
I.E.		2	28
S.D.		15	46
corrected dist.	77	05	00

S Z L is the hor. angle.

$$\text{Cos SZL} = \frac{\cos \text{SL}}{\sin \text{ZS}} = \frac{\cos \text{LS}}{\cos \text{app. alt.}}$$

log cos LS = 9·349343
log cos app. alt. = 9·876347

cos 9·472996 = 72° 43′, the hor. angle between ☉ and L

The T.B. of ☉ is N. 105° 22′ E.
hor. angle to L.H. 72 43 to the right of sun.

T.B. of Lt. Ho. N. 178 05 E., or S. 1° 55′ E.;

and, conversely, T.B. of observer from L.H. is N. 1° 55′ W.

To plot these particulars.

Let L M (fig. 208) be the T.B. line of the observer, projected from the Lt. Ho.

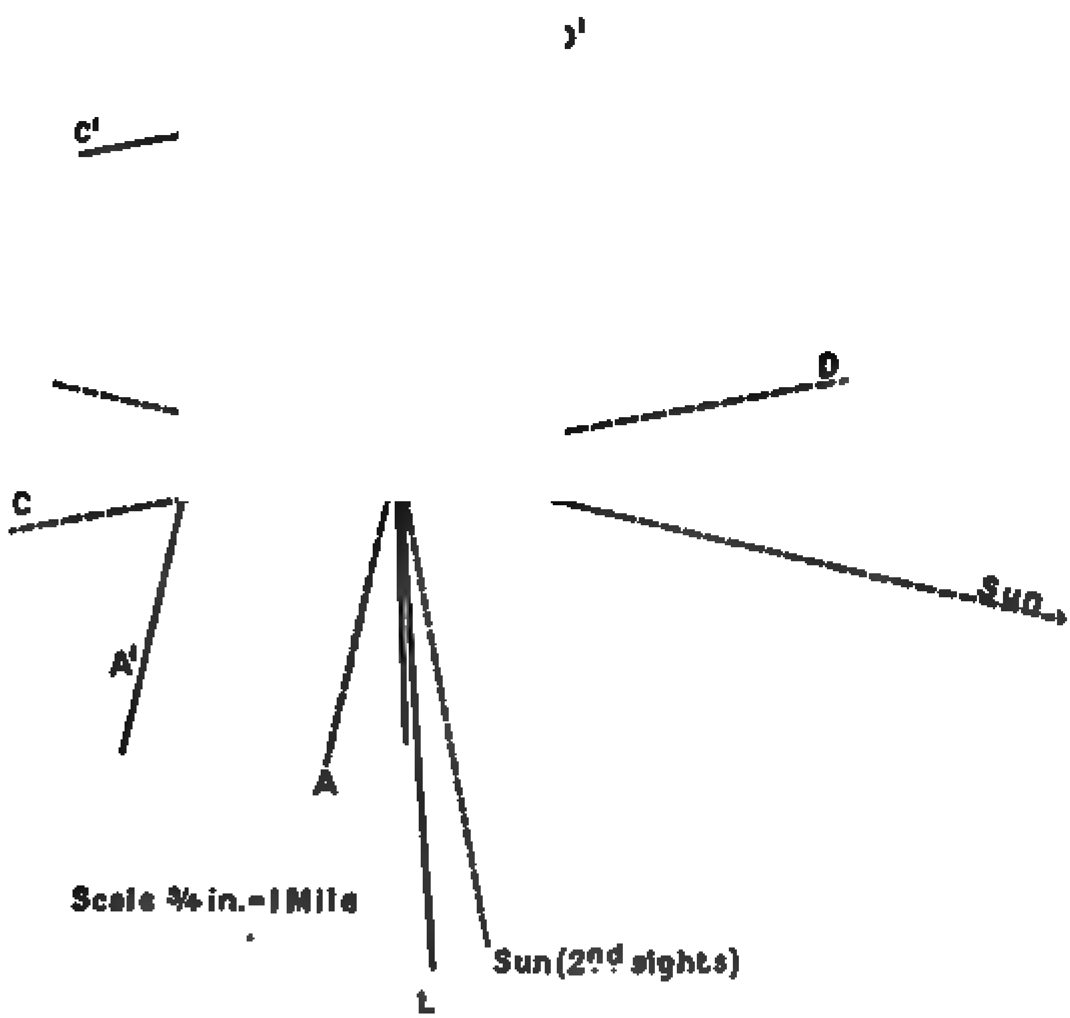


Fig. 208.

Since he is on that line, let O^0, anywhere along the line, be his assumed position, viz. lat. 51° 30′ N., long. 4° 10′ E.

Now at O^0, at the time of 'sights,' the sun's T.B. is N. 105° 22′ E, or 72° 43′ to the left of the L.H. line; then at O^0 project O^0S

72° 43′ to the left of L.M. A line A B at right angles to this will be a part of a circle of 'equal altitudes,' along which, at any part of the line for a limited distance, the sun's altitude will be as calculated (41° 12′ 53″), and the bearing N. 105° 22′ E.

But the corrected observed altitude (41° 12′ 00″) indicates that the observer is further from the sun along the line S O^0 than he assumed he was; and the effect of this will be to move A B further back, parallel to itself, ·883 mile (41° 12′ 53″ – 41° 12′ 00″), *i.e.*, ·883 inch (since in this case we are plotting it on 1 inch = 1 mile); this length is measured from O^0 along the line S O^0 produced, and A^1B^1 is drawn parallel to A B.

The intersection of L M with A^1B^1 at O gives *a* position of the observer.

Now the line O^0O^1 is in error ± owing to 'errors of observation' (see p. 42), and the 'cut' at the position O is only 17°. It is an 'ill-conditioned' 'cut'; for a small error in the distance O^0O^1 will make an *increased* error along the line O O^0; and, conversely, had A^1B^1 cut L M at right angles, an error in O^0O^1 would only show an amount equal to the error along O^0O^1; hence the position of O is not satisfactorily 'fixed.'

What is now required is a line of position cutting L M at right angles or nearly so; and such a line will be obtained when the sun bears about south, or, in fact, when it is on the meridian. The next 'sight' should be taken at or about noon, and be calculated exactly in the same way as the above.

Without showing the calculation, suppose C D is the 'equal alt.' line obtained with the same latitude and longitude as before, and C^1D^1 the line of position obtained from the difference between the true and calculated altitudes; O^0O^2 being the 'intercept.' Then L M is intersected by A^1B^1 at O, and at O^2 by C^1D^1. The intersection of A^1B^1 with C^1D^1 is at X, the two *astronomical* 'lines of position,' being here disregarded, as the line L M is more accurate than A^1B^1. But the question is, which position is nearer right in latitude, O^2 or O? An error of observation is common to both: but if it is a consistent one and about the same in 'time,' then the correct position must be, if it is – in both cases, from O^2 towards O^0; if it is a + error in both, the corrected position would be to the north of O; but the size of the errors would not be consistent with the movement in altitude when the sun is on these bearings. Therefore, in this case, both errors of observation were doubtless – ; but there is no possible means of telling what the amount of the error is, except by another 'sight' taken with a celestial body which bears about N., bearing in mind that the position is more doubtful in latitude than in longitude. If this is obtained, some position, such as Y, would probably be found; and midway between O^2 and Y would be the correct position. Notice that the first line of position

found was of little use in determining the position in latitude, and that L M gave the longitude very nearly. A line at right angles to L M was the one required, and this could only be obtained after an interval of about $3\frac{1}{2}$ hours.

573. Effect of Position Lines in Latitude and Longitude.—This happens with all bearing lines: each one will give a better position in latitude or in longitude, unless the body bears exactly towards one of the quadrantal points of the compass; and the intersection of any two lines will give a better latitude or longitude according to the mean angle of intersection.

574. Best Direction for Line of Position.—If the mean bearing is E. or W. there will be no error in the latitude, but all the error will be in longitude; if the mean bearing is N. or S. the longitude will be correct, and all the error will be in latitude; and therefore, if the sights can be taken with a *mean* bearing both E. and W. and N. and S., it is the most accurate possible way of 'fixing astronomically.' As a *surveying* problem, this must be carried out.

575. Fixing Astronomically without Stopping for Second Line of Position.—The second case is, if the vessel *does not anchor*, or stays for too short a time for obtaining all the requisite sights. For instance, supposing in the case above, if, after taking the first 'sights,' the vessel proceeds N. 20° E. for 30 miles, or roughly for three hours, and at about noon the second set of 'sights' is taken. She may then proceed to a third position, and take 'sights' for, say, the Pole Star.

Exactly the same calculations follow as were done before, but allowing for the difference of latitude and longitude, which depends upon the course and distance run by 'dead reckoning.'

If neither the true course nor distance are correct, as in all probability they are not, then the second latitude and longitude is not, nor would the third be relatively correct to the first; and the second position line obtained will not be correct relatively to the first; therefore the value of the 'fix' of the first position obtained through the second set of sights cannot be so good as that obtained if the ship remained at (1).

In this case, N. 20° E. 30 miles $= 28'\cdot2$ of diff. lat., and $10'\cdot2$ dep. by natural log tables.

$$\begin{aligned}\text{diff. long.} &= \text{dep. sec lat. } (51^\circ\ 44') \\ &= 10\cdot2 \,.\, 1\cdot6 \\ &= 16\cdot32 \text{ miles E.} = 1^m\ 08^s = 1^m\ 04^s\cdot8\end{aligned}$$

lat. at (1)	51° 30′ N.,		long. at (1)	$0^h\ 16^m\ 40^s$
diff. lat.	28′ 12″		diff. long.	1 04·8 E.
lat. at (2)	51 58 12		long. at (2)	0 17 45 E.

With this new derived latitude and longitude calculate, as before, the sun's altitude and bearing, and find the difference between the true and calculated altitude. The line of bearing obtained can be laid through the position (1), just the same as if it had been found there; *and always provided the course and distance is correct* the intersection of the lines give as accurate a 'fix' as if the vessel had remained at (1).

CHAPTER X.

AMENDING THE DETAILS ON A CHART.

576. Amending, implies making small corrections, or additions to; not testing the triangulation, because this could not be effected without reference to the original plotting-sheet.

It has already been explained that, owing to the process of engraving, the positions of 'objects' on a published chart are not infallibly correct. But changes in the depth of water do occur from time to time, especially over bars at the entrance of harbours or of rivers, and over sand and mud flats elsewhere; also dredging takes place sometimes; and, in consequence of these changes, the 'buoyage' may have to be altered; perhaps, also, the nature of the buoys undergoes a change, though this should not be a radical one; or there may be additional marks, buoys, or beacons necessary; pile lights may be introduced, and many other changes of condition may arise from time to time. Prominent objects may be built, and the conspicuous objects of earlier times may have ceased to be prominent. For example, in a prominent harbour not far from London, the whole buoyage has been shifted; pile lights, both afloat and on the shore, have also been bodily shifted; objects that were conspicuous fifty years ago have ceased to exist; the conspicuousness of an isolated red house has been wiped out by several streets of them; the leading marks of a 'church in line, etc.,' has been obscured by a forest of trees; and, conversely, a clearing has revealed a conspicuous factory chimney; a conspicuous barn has vanished; in short, but for a few remaining details, the place is hardly recognisable by what remained on the original chart.

It may or may not have been the duty of the port-master or port authorities to supply reliable information as to these changes, but certainly they ought to have been put on record by some authority.

The method of verifying the soundings, and of sounding out an area or a channel has been explained and exemplified in

Chapter IX.; how, also, the rise of springs and of neaps, and the direction, duration, and strength of the tidal streams can be verified is described in Chapter VI. on *Tides*, pp. 265–283.

If, on sounding out the channel or channels, it is found necessary afterwards to shift the buoys, their new positions should be 'fixed' by station pointer angles (see *Station Pointer Fixing*, Chapter XI., p. 64), and the same remark applies in the case of having to shift the position of 'leading marks,' or pile lights, whether on the edge of the channel or on shore. If a conspicuous mill has ceased to exist, notice of such a fact should be given; if a conspicuous factory has come into existence, its position should be placed on the chart, by 'shooting it up' from three or more positions (see p. 343); and all the latest information relating to the harbour should be submitted to the Hydrographic Department of the Admiralty, for the general use of all seafaring folk who may have occasion to enter the port, to give them more confidence in Admiralty charts, and it is in this manner that charts are corrected up to date.

If it is desired to make a fresh and independent survey of the harbour on a larger scale than that published, see, and read, *Survey of a Harbour*, p. 218, or *River*, p. 250.

577. Re-triangulating a Portion of a Chart in order to Plot Amendments.—Sometimes, if only a portion of the harbour requires verifying, for instance that portion leading to the anchorage, it is almost better to make a fresh triangulation of the whole; this may be done on a larger scale, and on that foundation put in what work is required. Later, when time and opportunity arises, other parts of the harbour can be verified in detail, and added. These details will be re-plotted on to the original chart.

The newly triangulated marks will be dependent on those already fixed on either the Ordnance Survey, or in the previous Hydrographic triangulation, and one at least of these points must be included in the new work, as a means of connecting it up with the original (see *Triangulation*, p. 211).

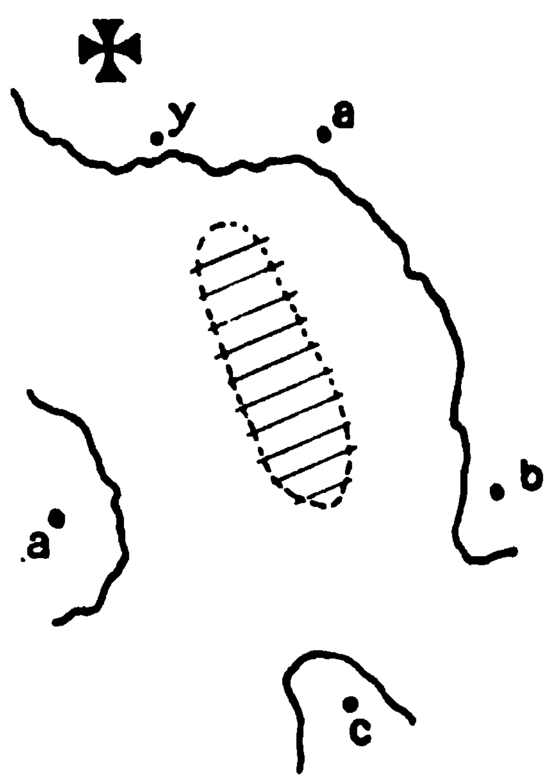


FIG. 209.

For example: take such a plan as shown in fig. 209. The part that is dotted is a shoal on one side of the channel; it is intended to verify its extent, but working on a larger scale.

✠ is a church tower: this appears in the Ordnance Survey,

and also in the H.O. triangulation: *a*, *b*, *c*, *d*, etc., are points selected in the new triangulation; ✠ must be connected up with one of them by a True Bearing and distance; for preference take *c*, because it is the most distant from it.

By this means the plan retains its location, or a relative position to some point already known, which otherwise it will not have.

Or, suppose *c* also is a point on the Ordnance or H.O. triangulation, then it will be an additional connecting link; and, from *c* and ✠ the position of every detail will be in relation to these two. Again, if these two places exist on an Ordnance chart of a much larger scale than is now intended to work upon; *c* being (without any doubt about it) the same spot on both plans, then, the bearing and distance of ✠ from *c* can be adopted from the Ordnance survey (see *Scales*, p. 176), and the remainder of the new triangulation built up on that side.

It would be better, of course, if ✠ is accessible for taking angles from; but if it is not, then, take angles at *c*, between ✠ and *a*; and at *a*, between ✠ and *c*; and with the length of ✠ to *c*, calculate the length from *c* to *a*; and use *a* to *c* for a plotting side (see *Plotting and Triangulation*, p. 198).

Then follow, stage by stage, coast-lining, (no true bearing is here required), sounding, fixing, and tides, as explained in the various chapters on the subject, so that the whole port can be done on any scale required, just when time and opportunity admits of it.

It is as well to remember that to sound out a shoal, or to thoroughly do a channel, there must be room on the paper to plot distinctly the greater part of the soundings obtained; otherwise there will be a hopeless tangle of figures, and it is not easy to describe the various contours of the bottom, nor to follow and fully realise the shape of the shallowest part. The scale commensurate with this cannot be less than 6 inches to a mile.

If a still smaller portion of the harbour is required to be re-sounded out, such as a small shoal, or a small part of the channel on a larger scale than the present one, exactly the same process as above will have to be carried out; of course, no details of the coast will be required, but a sufficient number of marks, 'points,' or objects must be plotted, for the purpose of fixing the boat's position, though the number of marks will probably not exceed four or five. For example, referring to fig. 209, *a*, *b*, *c*, and *d* are almost sufficient in themselves for the area shown in dotted line; but perhaps *y*, another mark on the water-line, might be required as a substitute for ✠ for fixing purposes.

578. Enlarging a Published Chart for Purposes of Amendment.—Should, however, there be no accurate chart on a larger scale than that intended to be presently used for sounding the shoal, or for the channel or harbour, then the scale

of the published chart will have to be enlarged; not in all its details, but only so far as it applies to the plotting side.

Suppose, for example, ✠ to *c* (fig. 209) is adopted as the plotting side; that the published chart is on a 4-inch scale; and that you want to sound the channel or a shoal on a 10-inch scale, *i.e.* two and a half times the scale of the published chart.

Measure from the published chart the distance in inches from ✠ to *c*; suppose it to be 10·2 inches.

For the convenience of this example accept the true bearing from ✠ to *c* to be the same as that taken from the chart.

Draw on the paper the line representing the relative bearing from ✠ to *c*, assuming the sides of the paper to be roughly north and south; prick a hole on this line to represent either, in about the position it occupies relatively to the remainder of the work, and measure, along the line, 10·2 . $2\frac{1}{2}$ inches = 25·5 inches.

Then at *c*, observe the angle between ✠ and *a*; and at *a*, observe the angle between ✠ and *c*; 180° − the sum of these angles will give the '*calculated angle*' at ✠ between *a* and *c* (that is, supposing ✠ is inaccessible).

Call ✠ H. In Δ *a*H*c*, at H project angle H from the line H*c*; and at *c*, *c* from the line H*c*: the intersection of these lines will give the presumed position of *a*.

Follow out the same thing in Δ *b*H*c*, and presume a position of *b*; also find *d*, in the same way.

Then in Δ *acb*, at *a*, project the angle *cab* from the line *ac*; it should cut through the position of *b* as previously plotted. Do the same for *d*, and also do it again from *b* to *c*, that is, lay off angle *b* of Δ *cbd* from the line *bc*. The line so projected should go through *d*; if it does, then all the points are correctly plotted. There are, however, sure to be some 'cocked hats'; for the discussion of this see p. 62, par. 160.

When all the 'points' are satisfactorily plotted, then sounding can be carried out.

In the foregoing example it has been assumed that one end of the adopted plotting side is accessible. This may not always be the case. In the example which follows on p. 346, it is presumed that neither end of the adopted plotting side is accessible; but, since the two objects at the end of it are assumed or known for certain to be the same objects as in the original survey, and are the most distant apart, they were therefore adopted.

579. Plotting Amendments Directly from Points in the Published Chart.—The first example of 'amending' a chart will be the simplest; it does not need the enlargement of the published chart, and the details of the coast were easily 'shot up' from a boat or from a vessel while entering and leaving the harbour.

Practically it amounts to this. If one or more objects are

'shot up' from two ends of a line, the plotting side of the triangle will depend for its length and its bearing upon two positions deduced from sextant angles, and plotted with a station pointer, between objects fixed on the chart, and assumed to be correct.

A number of errors necessarily creep in (see *Errors of Observation and of Positions on the Chart*, p. 62); thus neither the length nor the bearing of the plotting side is really correct; but still, the lines projected from both ends of this side will meet somewhere, and, if the lines of reference to the object 'shot up' are correct, both in angles and in drawing, the position of the object so 'shot up' will be relatively correct to the length and bearing of the plotting side, whatever the scale of that side may be, and whatever be the direction of the true meridian.

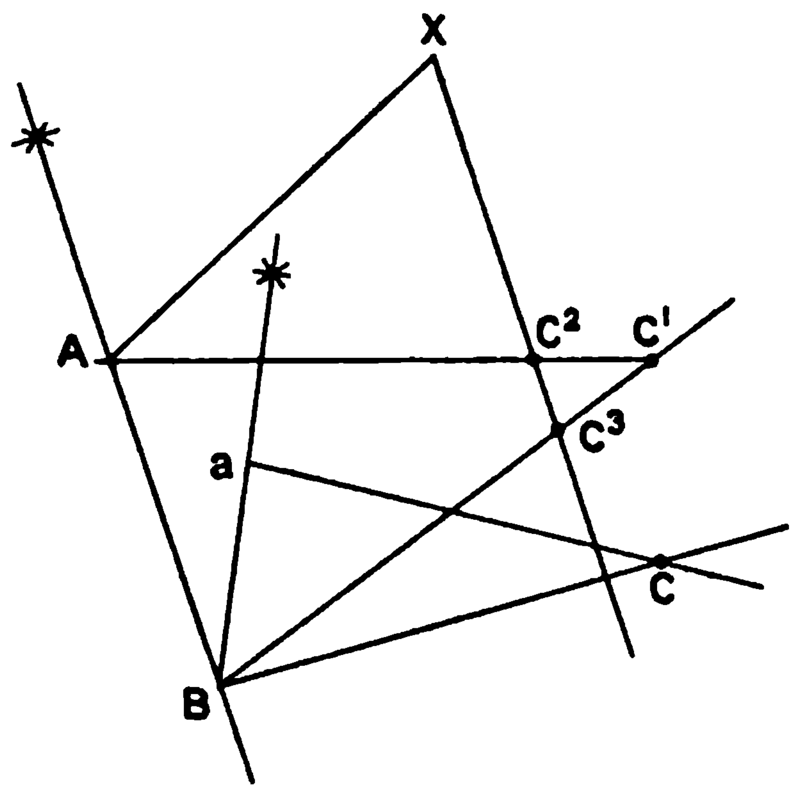


FIG. 210.

Thus in fig. 210, if the angles A B C^1 and B A C^1 are correctly taken and plotted, then C^1 is quite correct relatively to A and B. If B A is B*a*, and the angles at B and at *a* are taken to be the same as before, then *c* is quite correct in bearing and distance relatively to B*a*; but the scale of the two triangles is different, and the true meridian is not in the same place for both.

So that if A and B represent two positions fixed by sextant angles from three objects on shore, and they are both in error, then the scale for the distance between them is not the same exactly as the scale of the chart, from which, of course, all the other distances are measured; but C^1 is correct relatively to the new scale derived from A and B.

Now suppose another position X is 'fixed' by three objects on the published chart (see fig. 210); then it too is in error, and there is not necessarily the same error in the distance A X as there was in B A, though the amount of the difference will be minimised when one has acquired a thorough knowledge of 'fixing' by means of the station pointer; and, consequently, the scale from which A X can be measured is not the same as either that of the chart or that from which B A can be measured.

The new C^2, plotted from A and X, is correct relatively to A and X; but the scale of distance and the position of the true meridian belonging to A X and C^2 is not the same as that of

A, B, and C^1; the natural result is that C may be in three places, *i.e.* at C^1 relatively to A and B, at C^2 relatively to A and X, and at C^3 relatively to B and X. In these cases of 'fixing,' and especially when the boat or vessel does not anchor for the purpose, the 'cocked hat' will necessarily be large—much larger than in the previous example—and when, as in the following example, the objects 'shot up' are the approximated centre of buildings, that is, no definite mark, the same spot will probably not be observed from the three fixes; but the illustration is not intended to show more than, that the position of the buildings would be approximately located only. By 'without attracting undue notice' is meant, in the case of a foreign harbour, that a deliberate survey would be resented by the authorities.

If, however, their susceptibilities can be overcome, and permission be obtained, so that the work can be deliberately carried out, the system is the same; but the boat might anchor at each 'fix,' as when doing coast-line (see previous example, p. 233); and where there are buildings, a particular spot on each, such as a chimney, might be localised; or an angle may be taken to both sides of them, that is → or ←.

Whenever possible, 'fix' by the same three objects; under the best conditions, the error of position should not exceed $\frac{1}{20}$ inch.

For points of reference to the objects 'shot up,' take an equidistant point, and that which makes the smallest angle with the objects (see Appendix II., p. 438).

Adding to a Published Chart. (Plate VIII.)

(Plotted in 1 hour.)

Example 1.—Entering a port, on chart No. 212. It is intended, without attracting undue notice, to 'shoot up' a number of important and conspicuous objects, and place them on the published chart. The chart is published on a 6-inch scale.

The following marks, selected as the most suitable for 'fixing,' are pricked through from the chart on to a piece of clean paper:—

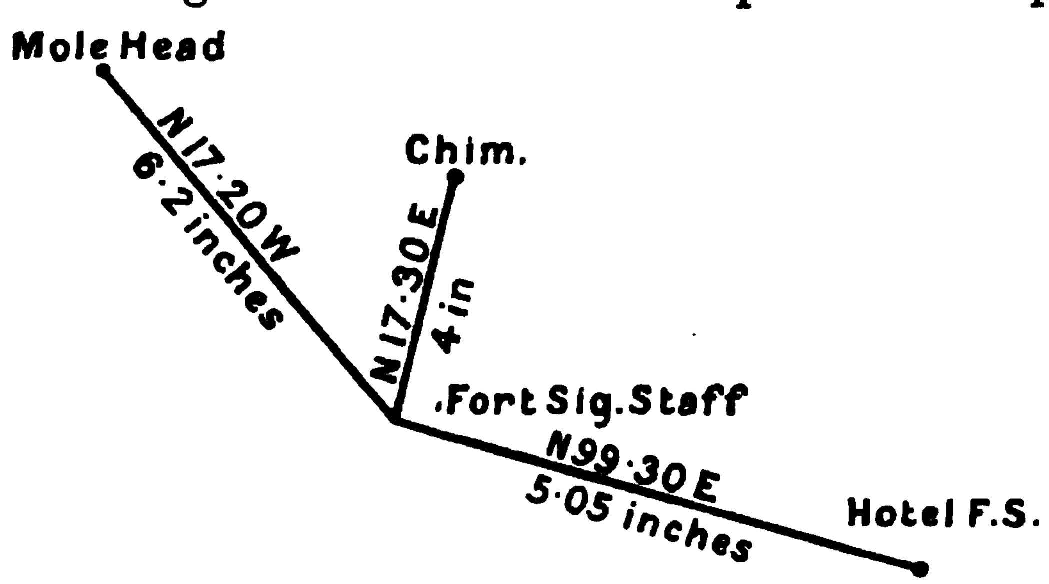


Fig. 211.

The bearings given are true; the distances are in inches.

At positions (1) and (2) the ship stopped, and the angles were taken simultaneously by three observers.

At position (1)—

Mole . .	29° 40′	S.S.	54° 05′	Hotel.
,, . .	39 00	Naval Store		
,, . .	42 40	Military Store		
,, . .	46 35	Bush Hill		
,, . .	55 20	Barracks		
,, . .	63 22	Magazine		
,, . .	79 15	Hospital		

In going from position (1) to position (2)—

When Bush Hill ϕ S.S.	S.S.	21° 15′	Magazine.
		30 45	Hospital.

At position (2), just outside and close alongside a red conical buoy with globe—

Mole . .	99° 43′	S.S.	ϕ Hotel	
Magazine .	2 05		1° 00′	Hospital.
Military Store .	16 30			
Bush . .	23 40			
Naval Store .	26 25			

At position (3), ship is at anchor—

Mole . .	73° 15′	Chimney	89° 44′	S.S.
		Military Store	24 50	
		Naval Store	34 50	
		Bush	40 40	
		Barracks	53 10	

Soundings taken on the way in—

IV. 30 A.M. At fix (1) . . . . . . $4\frac{1}{2}$

$4\frac{1}{4}$ ×

Bush Hill ϕ S.S. 4

$3\frac{1}{2}$ 3

At fix (2) 3

3 $3\frac{1}{2}$ × 4 ×

V. 00 A.M. At anchorage . . . . . $4\frac{1}{2}$

Weighed the next day, and sounded on the way out—

Mole 70° 20′ Chimney 76° 25′ S.S. 4½
IX. 00 A.M. 3¾ × 4½ 4¾
Mole 86° 45′ S.S.
93 36 Hotel . . . 3¼
3¼ 3 3½ × 3¾
Mole 97° 23′ Hotel . . . 4
9 37 S.S.
4¼ 4½ 5 × 5¼
Bush 75° 20′ Hotel
41 30 Hospital . . . 5¼
S.S. 113 55 Hotel
etc.

On the next occasion of visiting the port, more soundings are taken going in and out.

Measured lead line, 1 ft. 6 in. short at 3 fathoms.
" 2 0 " 4 "
" 3 0 " 5 "

All the soundings are reduced.

The following sketches were made to 'fix' the summits :—

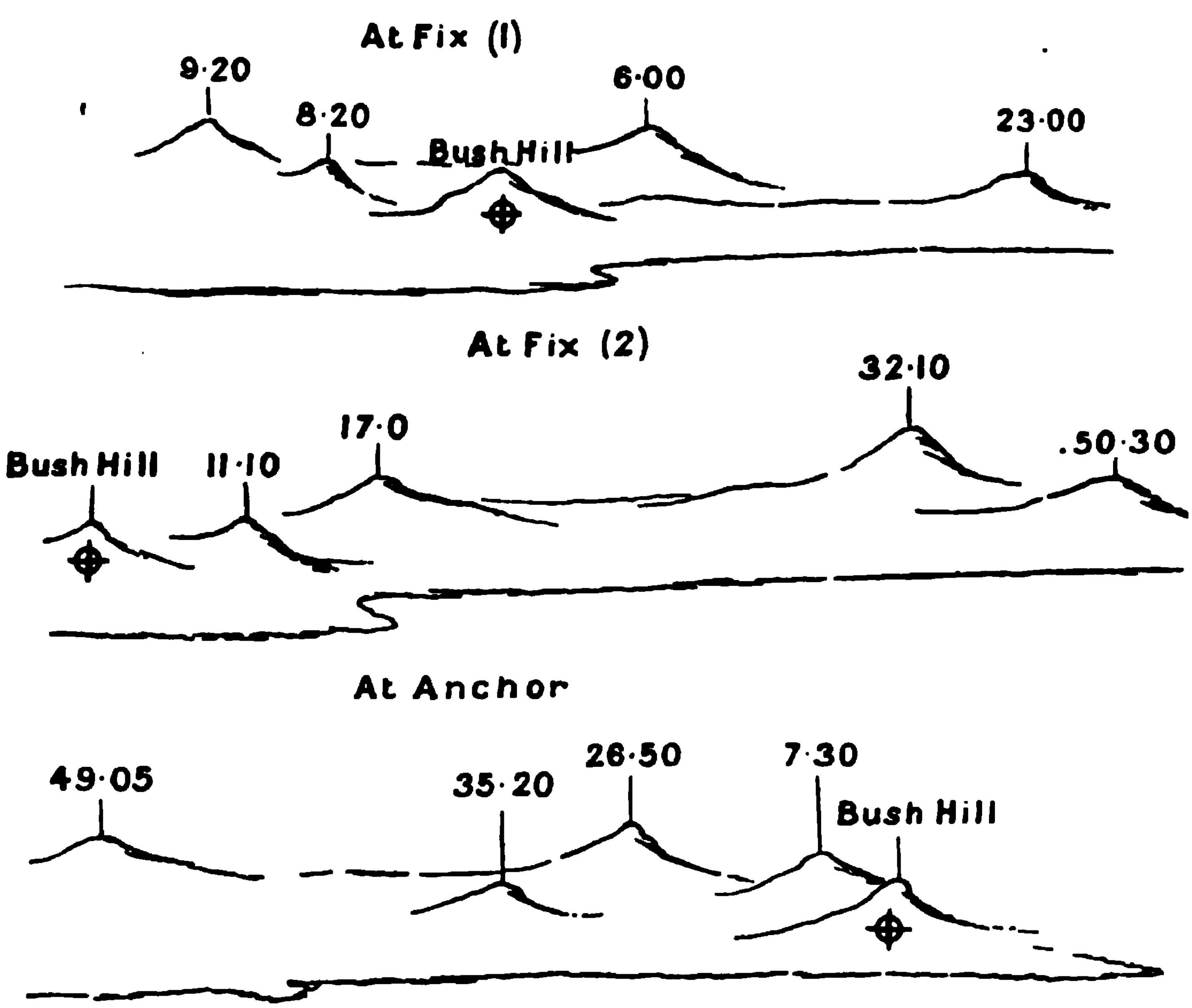


FIG. 212.

580. Sounding a Shoal on a Scale Enlarged from the Published Chart, when Neither End of the Plotting Side is Accessible.—*Example* 2 (Plate IX.).—It is required to sound out a shoal lying about a mile to the north-east of Ex. Pier.

The scale of the published chart is 1 inch = 1 mile, and it is proposed to work on a 5-inch scale.

From Abbey to Church, measured on the chart, = 3·38 inches (see Plate IX.).

Rough true bearing, $\frac{\text{N.}}{\text{S.}}\ 78^{\circ}\ \frac{\text{W.}}{\text{E.}}$

(*a*) For purposes of 'fixing,' it is necessary to plot the positions of Staff and Club for middle objects.

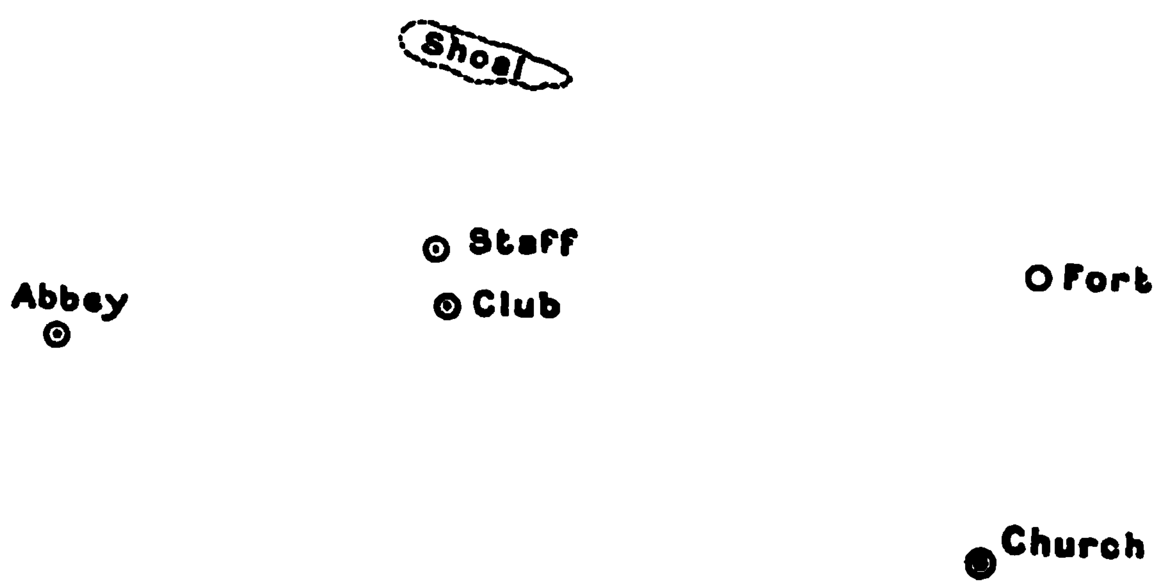


Fig. 218.

(*b*) Angles are taken only at Staff and Club, Abbey and Church being inaccessible.

(*c*) It is therefore necessary to find the *calculated* angles at Abbey and Church to Staff, Club, and Fort, and to plot these three ojects.

In Δ S.C.Ch., assuming S.C. = 1, find S.Ch. }
In Δ S.C.A., „ „ „ S.A. } This gives the relative values only
In Δ S.C.F., „ „ „ S.F. } of three sides.

In Δ S.F.A., given S.F. and S.A., find ∠ A. and project it from the line A.Ch., giving a line of reference through Fort.

In Δ S.A.Ch., given S.Ch. and S.A. and ∠ A.S.Ch., find ∠s A. and Ch.

1. And plot S. from A. and Ch.

2. Plot C. by direct angle from S., and calculated angles Ch. and A.

In Δ F.S.Ch. find ∠ Ch.

3. Plot Fort by the intersection of lines from A. and Ch. and a check from S.

The following angles were observed :—

At Staff—

Fort 32° 18′ Church 129° 46′ Abbey.
82 56 Club.

At Club—
Fort 32° 38′ Church 150° 22′ Abbey.
F.S. 121 10.
At S., Observations for T.B. of Abbey :—
Lat. 50° 45′ N., long. 1° 7′ W.
Corrected declination, 20° 24′ N. Eqn. T. 6^m 11^s—to M.T. S.D., 16′.
About 7.0 P.M. Watch, 6^h 57^m 20^s. Abbey (alt. 5°), 45° 42′ |⊙. I.E., + 2′.
Comparison, 0^h 12^m 25^s slow. Chron., 0^h 01^m 26^s slow on G.M.T.
Pole is erected at L.W. at pier end, and registers—
VII. (low water), 3 feet 9 inches. Springs rise given on chart, 18 feet 6 inches.
I.00 P.M. (high water), 15 feet 9 inches.

IX. A.M. (commenced sounding).

(1) Fort 93° 50′ F.S. 27° 30′ Abbey . . 7¼
4 5¾
(2) 98 05 27 20 . . . 6¼
(3) 101 02 25 20 . . . 6¼
5¾ 4 ×
(4) • 95 42 26 00 . . . 9¾
(5) 97 26 24 27 . . . 6¾
3½ × 4¾
(6) 103 06 23 32 . . . 8¾
(7) 105 30 21 40 . . . 7¾
4¼ 4
(8) 100 31 22 35 . . . 7¾
(9) 102 18 21 10 . . . 6¾
4 4¾
(10) 107 25 20 14 . . . 7¼

X. 00.

(11) 109° 47′ 18° 25′ . . . 9¼
5¼ 4¼ 5¼
(12) 104 08 19 26 . . . 8¼
(13) 105 57 18 18 . . . 9¼
4¼ 3¾ 4¾
(14) 111 37 17 02 . . . 8¼
(15) 113 28 15 33 . . . 8¼
6¼ 3¾ 4
(16) Fort 107 16 F.S. 17 02 Abbey . . 8¼
(17) Fort 97 16 Club 27 21 Abbey . . 8¼
4¼ 4 6¼
(18) 102 28 27 10 Abbey . . 5¾
(19) 103 39 26 13 . . . 7¼
5¼ 4
(20) 99 07 26 25 . . . 9¼

XI. 00.

(21)		100° 29′	23° 20′ . . .	$8\frac{1}{4}$
	6 7			
(22)		105 16	25 14 . . .	$8\frac{1}{4}$
(23)		107 22	23 47 . . .	$6\frac{1}{2}$
	$5\frac{3}{4}$			
(24)		102 00	24 35 . . .	$8\frac{3}{4}$
(25)	Church	61 40	23 12 . . .	$10\frac{3}{4}$
	5 $5\frac{1}{4}$			
(26)		69 09	22 55 . . .	$7\frac{1}{4}$
(27)		70 21	21 10 . . .	8
	5 ×			
(28)		66 41	21 32 . . .	$7\frac{3}{4}$
(29)		67 32	20 43 . . .	$7\frac{3}{4}$
	5 $4\frac{3}{4}$			
(30)		71 40	19 56 . . .	$8\frac{1}{4}$
(31)		70 43	19 11 . . .	$9\frac{3}{4}$

Ship sounding, I. 30 P.M.

(*a*)	Fort	94° 54′	F.S. 29° 13′ Abbey . . .	9
	9			
(*b*)		88 54	29 30 . . .	15
			turning to Port . . .	15
(*c*)		78 58	Club 41° 35″ . . .	9
	Perch	12 46		
	Beacon	27 00		
	Post (1)	43 47		
	Post (2)	46 16		
		9 10	turning to Port	
(*d*)	Fort	87 35′	31° 53′ . . .	10
	10			
(*e*)		97 13	30 30 . . .	11
	11 × 9			
(*f*)		118 40	18 45 . . .	12
	Perch	4 07		
	Beacon	25 43		
	Post (1)	60 21		
	Post (2)	65 52		

II. 00 P.M.

	10 13 13			
(*g*)	Church	71° 20′	6° 23′ . . .	14
			10 37 F.S.	
	Beacon	12 08	1 20 Perch	

	Post (1) 44 54			
	Post (2) 33 34			
	turning to Port			
(*h*)	Church 67 32	8° 22′	Abbey . .	16
		10 40	F.S.	
	18 14 16			
(*j*)	Fort 115 00	16 23	. . .	$13\frac{3}{4}$
	$11\frac{3}{4}$ $11\frac{3}{4}$ $13\frac{3}{4}$			
(*k*)	105° 20′	17 07	. . .	$14\frac{3}{4}$
	$13\frac{3}{4}$ ×			
(*l*)	88 34	29 30	. . .	$14\frac{3}{4}$

II. 45 P.M., measured lead line. O.K.

Contour the shoal by the usual contours.

Scale, 5 inches = 1 mile = 6085 feet. Draw scales of latitude and longitude and natural scale. Var., 17° W. Give the bearings and distances of east and west extremities of shoal from F.S.

581.—*Explanation of Example* 2.—In the example above, the purpose is to sound out a shoal, on account of a ship having grounded on it, and reported that it extended further than was shown on the chart.

The scale of the published chart is too small for the purpose of plotting the soundings distinctly.

It is therefore necessary to make a fresh triangulation so as to include only just sufficient marks to fix with at any part of the shoal; this necessitates a suitable right, left, and middle object.

Take the longest possible side from the chart, between two objects whose identity are the same as those 'fixed' in the original survey; meaning that there is no doubt about those being the objects marked on the chart.

The objects chosen were an old abbey, and a conspicuous church; and there was the extra advantage that they would serve the purpose also of right and left objects to 'fix' by as well as points to triangulate from.

A near object for a middle one is also required: a staff at the end of a pier, also the flag-staff in front of a club-house would suit admirably; especially as from them, Abbey and Church are visible: this is an absolute necessity in this particular case.

Angles were taken at Staff and at Club, to Abbey, Church, and also to another supplementary left object, Fort.

Since Abbey to Church is the plotting side, it is required to find the angles *at* those places to plot Staff, Club, and Fort; and, since both places are inaccessible, then the required angles will have to be 'calculated.'

Let H stand for Church.

In Δ S C H (fig. 214), S 50° 38′ (observed),
C 121 10 (observed)
H 8 12 (calculated)

180 00

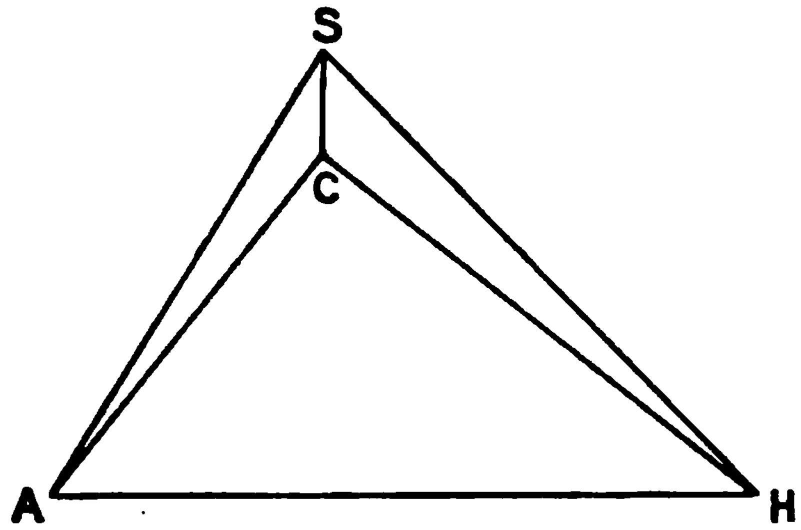


FIG. 214.

Let the side S C be assumed to = 1.

Then the length S H can be calculated; it will only be relative to S C being 1.

$$S H = S C . \sin C . \operatorname{cosec} H.$$

This gives the value of S H = 5·999 (times S C).

Similarly, in Δ S C A, S 79° 08′ (observed),
C 88 28 (observed),
A 12 24 (calculated).

Again, assuming the length of S C to be 1,

$$S A = S C . \sin C . \operatorname{cosec} A.$$

This gives the length of S A = 4·655 (times S C).

Then in the triangle S A H, the proportion (not absolute) of the length of the sides is S A = 4·655,

S H = 5·999,

and angle A S H 129° 46′ (measured at S).

Given two sides and the included angle, find S A H and S H A.

The calculation gives A 28° 30′
H 25 44

These angles are projected from A, and from H, and plot S.

To plot the work from the beginning.

At first presume the rough true bearing of A from H, taken off the chart, as N. 78° W.

Draw a line across the bottom of the paper on the bearing of $\frac{\text{N.}}{\text{S.}}$ 78° $\frac{\text{W.}}{\text{E.}}$, assuming the bottom of the paper to be roughly east and west.

At near the left end of the line, prick the position of A. Now, from A to H, on the published chart, is 3·38 inches; and it is intended to work on a scale 5 times that of the chart. (This is an exaggerated example; such an enlargement is an excessive one, and would lead to a considerable error in the scale. But if, as probably will be the case, the shoalest part of the shoal will eventually be 'reduced' back on to the original chart on the same scale as that chart, and from the same side, A to H, whatever error there is in the magnifying in the first case, will be cancelled when the same part is reduced again; but if the shoalest part only of the re-sounded part is transferred by its bearing and distance from C or F, then the error in distance remains.)

It must not be forgotten that the triangulation is really being worked 'outwards' on a large scale from a short side, S C, though the calculated angles are projected at A and H, which depend upon those taken at S and C; and the utmost possible accuracy is required so that S and C may be plotted relatively correct to A and H.

So, from A along the line of the direction to H, a distance of $5 . 3{\cdot}38 = 16{\cdot}9$ inches, is measured: this will give the position of H.

At A, project the angle as found above 28° 30′ from line A H.
At H, „ „ „ „ 25 44 „

The intersection of these lines gives the position of S.

By the same form of calculation the angles A H C and H A C can be calculated, as also A H F, and H A F, and when projected from H and A will fix C and F; and if C and F are 'checked' by the angles from S to them, their positions can be accepted.

From here onwards it is only a matter of plotting the boat's 'fixes' and the soundings when reduced.

When the soundings are completed, the true bearing of H from A can be calculated from the observations taken at S; and the true meridian drawn on the paper.

Draw a scale, and give the bearing and distance of the shoalest part found, which, on the reduced scale to the original 1 inch, will in this case appear probably as one sounding (see note on error in scale due to enlargement); and report to the proper authorities, so that the necessary amendment will be promulgated and the chart corrected. (See Plate IX.)

582. **Fixing the Position of a Line of Range and Target Buoys.**—*Example* 3 (Plate X.).—Range and target buoys, and a guide buoy, are to be laid out for gun practice at the south-east end of Long Sands.

The range buoys will be numbered 1, 2, 3 from left to right. The target buoys are called *a* and *b*.

The Commander-in-Chief requires that the position of these buoys be plotted on a scale twice that of the published chart;

and their bearings and distances shown from a permanent mark, for the purpose of giving notice of their exact position.

The scale of the published chart is 10 miles = 6·82 inches.

The only serviceable objects for 'fixing' with, that are visible on shore, are Reculvers Church and the North Foreland Lighthouse.

Their distance apart, measured on the chart, is 6·33 inches; and they bear roughly east and west (true) of each other.

A boat, with her mast up, flying a large flag, is moored about a couple of miles to the southward of where it is proposed to lay out the buoys.

The ship anchors at the intended position of the central buoy, No. 2; drops that buoy, and moors it.

A boat will then take out No. 3 buoy to the eastward, directed in bearing from the ship, and her distance from the ship found by rough M.H. angle.

Simultaneously another buoy, No. 1, will be moored in the opposite direction, roughly by M.H. angle, and ship ϕ No. 3.

The target buoys will be moored approximately at right angles to line joining No. 1 and No. 3, by bearing from ship; and by M.H. angle for distance.

Angles taken at flag-boat—

		N. Fore	59° 16′	Reculvers	68° 30′	Lt. Vessel.
		Reculvers	139 45	Ship.		
No. 1	20° 14′	Ship	15 16	No. 3.		

Angles taken at ship (No. 2 Buoy)—

N. Fore	15° 12′	F.B.
	49 31	Reculvers.
No. 3	119 39	N. Fore.
N. Fore	60 21	No. 1.
a	66 30	No. 3.
b	58 50	No. 3.
Reculvers	2 45	Light Vessel.

Angles taken at No. 1—

b	30° 25′	No. 3
a	33 54	

Angles taken at No. 3—

No. 1	44° 25′	*a*.
	47 45	*b*.

The ship then weighed, and anchored for the night at the position intended for the guide buoy.

To fix: N. Fore 28° 05′ Tongue Lightship 121° 36′ Pole Star.

Pole Star was exactly on the meridian. Corrected altitude,

52° 46′. Declination, 88° 49′ N. The position of the ship should be fixed by the intersection of three lines.

For true bearing:

At ship at No. 2 buoy.

24th March, 1908, at about 5.0 P.M.

Declination, 1° 22″ N. Eqn. T., 6^m 26^s—to M.T. S.D., 16′.

Time by watch, 4^h 43^m 55^s. N. Fore (altitude 1°), 82° 36′ |☉.

Watch fast on chronometer, 7^m 09^s.

Chronometer slow on G.M.T., 16^m 10^s.

Latitude, N. Fore, 51° 22′ 27″. Longitude, 1° 26′ 45″ E. Variation, 14° 40′ W.

Deduce the true bearing of Reculvers from North Foreland. Draw the T.M. and Mag. M. through N. Fore.

Give the magnetic bearing, and distance in cables, of each buoy from Tongue Lightship.

NOTE.—Take the position of North Foreland 1 inch from the bottom, and 1 inch from the right edge of the paper.

The fair chart must be accompanied by a legible copy of the computations, either as a separate sheet, or on the fair chart itself: as well as the other answers required.

The utmost accuracy is called for in calculating and projecting the long lines.

Soundings taken at each position:—

At flag-boat . . .	IX. 0 A.M. . .	6½ fms.
At ship (No. 2 buoy) .	IX. 0 „ . .	14 „
At No. 1 buoy . .	X. 0 „ . .	13¼ „
At No. 3 buoy . .	X. 30 „ . .	13 „
Midway between *a* and *b*	XI. 00 „ . .	6¼ „
At guide buoy . .	IV. 30 P.M. . .	8½ „

Springs rise, 20 feet. Neaps rise, 13 feet. Age of tide, 2 days. Age of moon, 2 days. Moon's mer. pass., 1^h 36^m. H.W.F. and C. XI. 45.

Explanation of Example 3.—The foregoing example resolves itself into 'fixing' the position of some buoys, by the aid of two distant objects visible on the land. A new element in 'fixing' is introduced by the aid of the Pole Star.

And, as stated, the scale is to be twice that of the published chart.

Very much the same system will be carried out as in the preceding example; but in this case, there being no handy shore objects for middle objects to 'fix' by, a boat with a flag flying is brought into requisition.

The Tongue Light-vessel would have suited for a middle object, but as it is not always that a light-vessel may be at hand, her existence is partly disregarded, and the first part of the under-

taking will be carried out as if the Tongue Light-vessel had no existence.

Later, however, she is made use of merely as a light on a ship or a boat, which otherwise might have to be placed thereabouts.

R is Reculvers Church; and N is North Foreland. From R to N will be the plotting side.

S and B are respectively the ship and the boat at anchor; and angles are taken between each other and R and N.

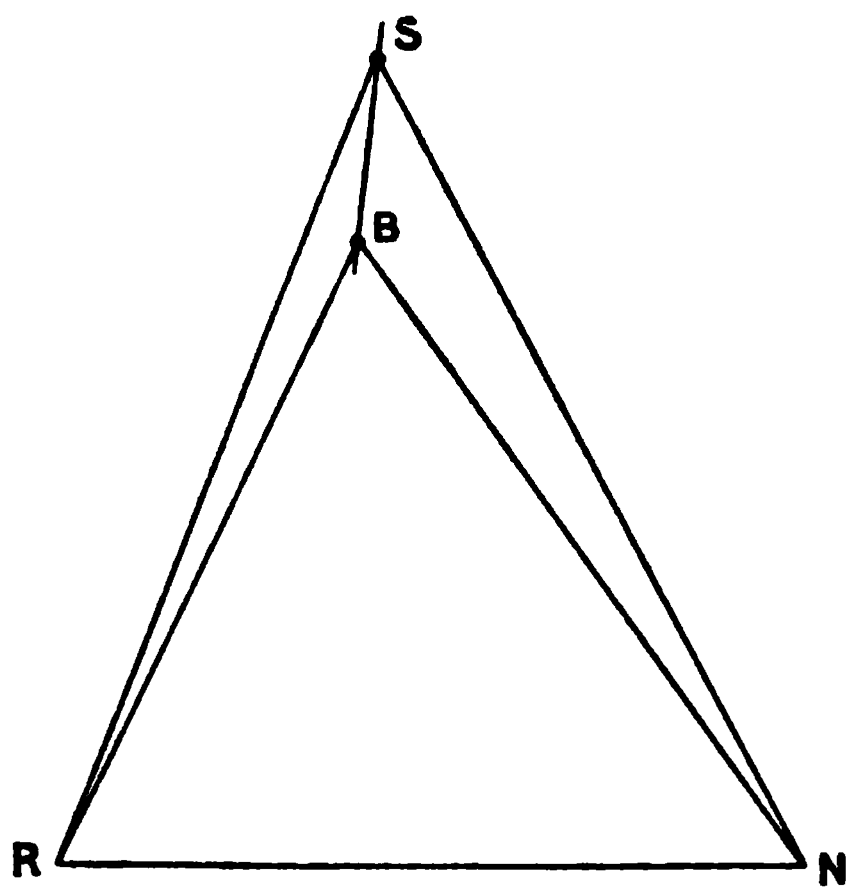


Fig. 215.

Assuming the length of side R N, it is required to find the angle at them so as to plot B and S.

The distance from B to S will be assumed to be 1.

1. In Δ RBS, S 34° 19′ (49° 31′ − 15° 12′)
B 139 45
R 5 56 (calculated angle)

180 00

$$\begin{aligned} RB &= BS \,.\, \sin S \,.\, \operatorname{cosec} R \\ &= 1 \,.\, \sin 34^\circ\, 19' \,.\, \operatorname{cosec} 5^\circ\, 56' \\ &= 5{\cdot}4538. \\ RS &= BS \,.\, \sin B \,.\, \operatorname{cosec} R \\ &= 1 \,.\, \sin 139^\circ\, 45' \,.\, \operatorname{cosec} 5^\circ\, 56' \\ &= 6{\cdot}2504. \end{aligned}$$

This only gives the proportion of the two sides of Δ R B S as 1 to 5·4538 to 6·2504.

2. In Δ SBN S 15° 12′,
B 161 59 (360° − (139° 45′ + 59° 16′),
N 2 49 (calculated angle).

And assuming $BS = 1$, then $BN = 3{\cdot}9389$,
$SN = 4{\cdot}8952$.

3. In Δ RSN, $RS = 6{\cdot}2504$,
$SN = 4{\cdot}8952$ (in those proportions only),
and angle RSN = 49° 31′.

Find angles SRN and SNR.

SRN = 50° 28′ 26″.
SNR = 80 00 34.

4. In Δ RBN, $RB = 5{\cdot}4538$,
$BN = 3{\cdot}9389$,
and angle RBN = 59° 16′.

Find angles BRN and BNR.

BNR = 76° 11′ 43″.
BRN = 44 02 17.

5. Then, since R N is given on the chart as 6·33 inches, and it is desired to plot on twice that scale, the length of our plotting side will be 12·66 inches.

The approximate bearing taken from the chart is east and west. Draw a line east and west approximately parallel to the bottom of the paper, and on it prick two points 12·66 inches apart: one being R, on the west; the other, N.

From R and N project the angles as found above, and plot S and B; and from S as a check, plot the angle R S B; the line projected should go through B.

That gives four positions on the paper. The paper used for this work will have to be 24 inches by 18 nearly.

The angles must be plotted with the most rigid accuracy, to the nearest minute, by the length of their chords; and the radius used in each case must not be less than 12 inches.

6. From the position of the flag-boat, and of the ship, the position of the buoys is 'shot up,' and plotted in the ordinary way.

7. For true bearing—

In fig. 216, X is the sun,

PZ is the cos lat.,
PX is the polar distance.
XPZ is the hour-angle.

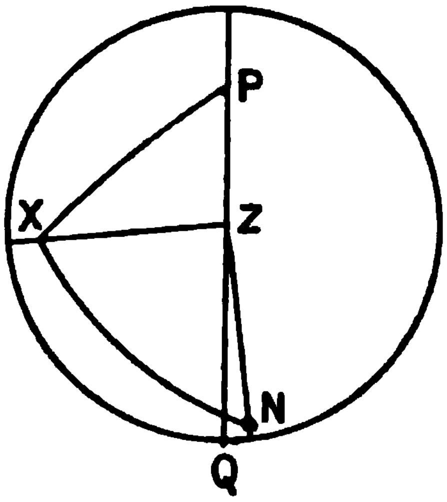


FIG. 216.

For hour-angle—

	h	m	s
watch time	4	43	55
error watch		7	09 fast.
chron. time	4	36	46
error chron.		16	10 slow.
G.M.T.	4	52	56
d. long.		5	44 E.
M.T.	4	58	40
Eq.T.(corrd.) –		6	26
H.A. or A.T.	4	52	14

For latitude—

	°	′		
lat. N.F.	51	22 N.	corrd. decn.	1° 22′ N.
app. d. lat.		10		P.D. 88 38
app. lat.	51	32		

(If, after finding the true d. lat. through the T.B. and distance, this assumed latitude differs widely from the true one, the T.B. will have to be recalculated with a more correct latitude.)

To find Z X (the sun's zenith distance).

log hav. H.A.	$4^h\ 52^m\ 14^s$		9·549372
log cos lat.	51	32	9·793832
log cos dec.	1	22	9·999876
diff.	50	10	9·343080 hav. = 55° 59′ 15″.

log vers. 55° 59′ 15″	0359443
log vers. 50 10 00	0440626
vers.	0800069 = 78° 28′ = Z X
	90
sun's alt.	11 32

To find P Z X (the sun's true bearing).

log sec lat.	51°	32′	10·206158
log sec alt.	11	32	10·008859
	40	00	
P.D.	88	38	
log ½ hav.	128	38	4·954823
log ½ hav.	48	38	4·614675
hav.			9·784515 102° 34′ 30″ W.

In Δ Z X N, Z X = 78° 28′.
Z N = 89 00 (90° 00′ – 1° 00′) (the elevation of N).
X N = angular distance from O = obs. dist. + S.D.
= 82° 36′ + 16′ = 82° 52′.

Find X Z N, which is the horizontal angle between X and N.

log cosec	78° 28′	10·008859
log cosec	89 00	10·000066
	10 32	
X N	82 52	
log ½ hav.	93 24	4·861996
log ½ hav.	72 20	4·770952

hav. 9·641873 = 82° 55′ 30″.

The true bearing of N = Q Z N
= P Z X + X Z N − 180°

102° 34′ 30″	
82 55 30	
185 30	
180	
S. 5 30 E., T.B. of N. from S.	

From this bearing, the true meridian is plotted through N. The distance of S from N, measured from the plotted work, is 9·4 miles, and the T.B. of N is N. 5° 30′ E.;

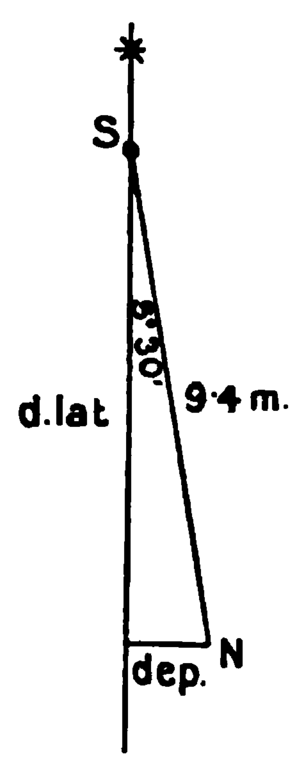


FIG. 217.

therefore, dep. = 9·4 . sin 5° 30′
= 9·4 . ·096 (nat. sine tables)
= 9·02

d. lat. = 9·4 . cos 5° 30′
= 9·4 . ·995
= 9·35 = 9′ 21″

d. long. = dep. . sec lat.
= 9·02 . sec 51° 27′ (mid. lat.)
= 9·02 . 1·6 = 1·44 = 1′ 26″;

and, with this d. long., convergency (see *Formula*, p. 171, par. 356)
= 1·44 . sin 51° 27′
= 1·44 . ·78 = 1′ too small to be taken into account.

Therefore, if the lat. N.F.	51° 22′ 27″ N.		long.	1° 26′ 45″ E.
d. lat.	9 21 N.		d. long.	1 26 W.
lat. of ship	51 31 48		long.	1 25 19″ E.

And, since the latitude assumed was 51° 32′ N. in order to work

out the T.B., it is near enough. The longitude assumed was 1° 26′; this also is correct enough.

It is required to find the position of the guide-buoy.

The meridian altitude of the Pole Star is 52° 46′ (if this is not taken exactly on the meridian, see Table 30, *Inman's Tables*, for its T.B. for any given sidereal time).

Pole Star alt. (corrected)	52°	46′
Z.D.	37	14
dec.	88	49
lat.	51	35 of guide-buoy.

To find the horizontal angle between Pole Star and light-vessel. In Δ P′Z L, Z L = 90° (the light-vessel being on the horizon),

P′Z = Z.D. of Pole Star,

and P′Z L is the horizontal angle.

$$\text{Cos Z} = \cos \text{P}'\text{L} \cdot \text{cosec P}'\text{Z}$$

$$\text{P}'\text{L} = 121°\ 36',$$

log cos P′L	9·719320
log cosec P′Z	10·218200

log cos 9·937520 = 150° 00′, which is the horizontal angle between P′ and Lt. V.

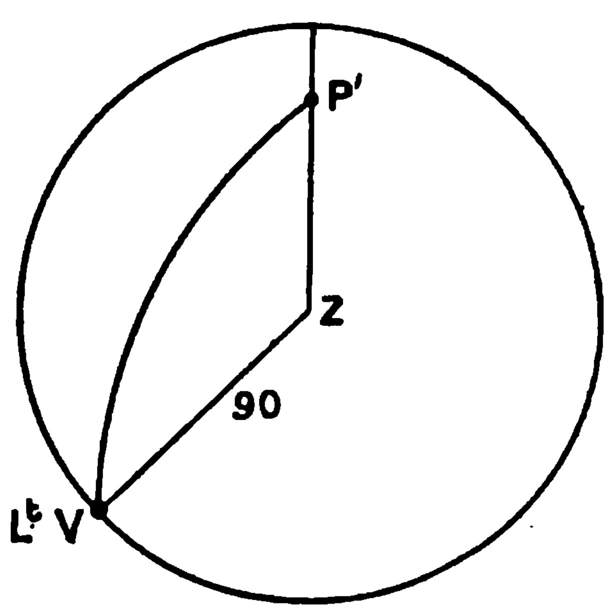


Fig. 218.

Fig. 219.

And, since the Pole Star bears true north (on the meridian), then the light-vessel bears N. 150° 00′ W. from the guide-buoy.

Conversely, the guide-buoy bears from light-vessel S. 150° 00′ E.,

i.e. N. 30° E. at guide-buoy; North Foreland is 28° 05′ to the left of light-vessel; then from the guide-buoy, N.F. bears N. 178° 05′ W.; or S. 1° 55′ E; or the guide-buoy bears from N.F. N. 1° 55′ W.

There are now three lines intersecting at the guide-buoy: one line of T.B. laid off from North Foreland; another line from light-vessel; and the third line is the parallel of latitude; or, since the d. lat. between N.F. and the guide-buoy is 51° 35′ − 51° 22′ 27″ = 12′ 33″ = 12·55 miles; and the bearing is N. 1° 55′ E., then the distance from N.F. to guide-buoy = d. lat. . sec bearing = 12·55 . 1·00056 miles; and in our scale, 6·82 inches = 5 miles.

Then distance in inches of paper

$$= \frac{12{\cdot}55}{5} \, . \, 6{\cdot}82 \, . \, 1{\cdot}00056$$

$$= 2{\cdot}51 \, . \, 6{\cdot}82 \, . \, 1{\cdot}00056$$

log 2·51	·399674
log 6·82	·833784
log 1·00056	241
	1·233699 = 17·13 inches nearly.

This example has been purposely considered from the point of view of a surveying problem, to demonstrate a method of accurately 'fixing' the position of out-lying buoys. (See Plate X.)

CHAPTER XI.

A RUNNING SURVEY.

583. This is probably the roughest possible form of triangulation. It is only adopted when all other available means fail, such as might be the case with a steep-to oceanic island, where there is neither an anchorage, nor even perhaps a landing; or, it may serve as a reconnaissance; but it will be necessary for any weather-bound and everlasting lee shore. It is included here solely as an exercise in possibilities: only a completely equipped surveying vessel would ever be in the position to carry out such a form of survey for an extensive piece of work.

584. Tangenting an Islet.—In its simplest form, fig. 220 shows how a small islet or island can be enclosed or bounded by a number of 'tangents,' obtained from a boat or vessel in several positions.

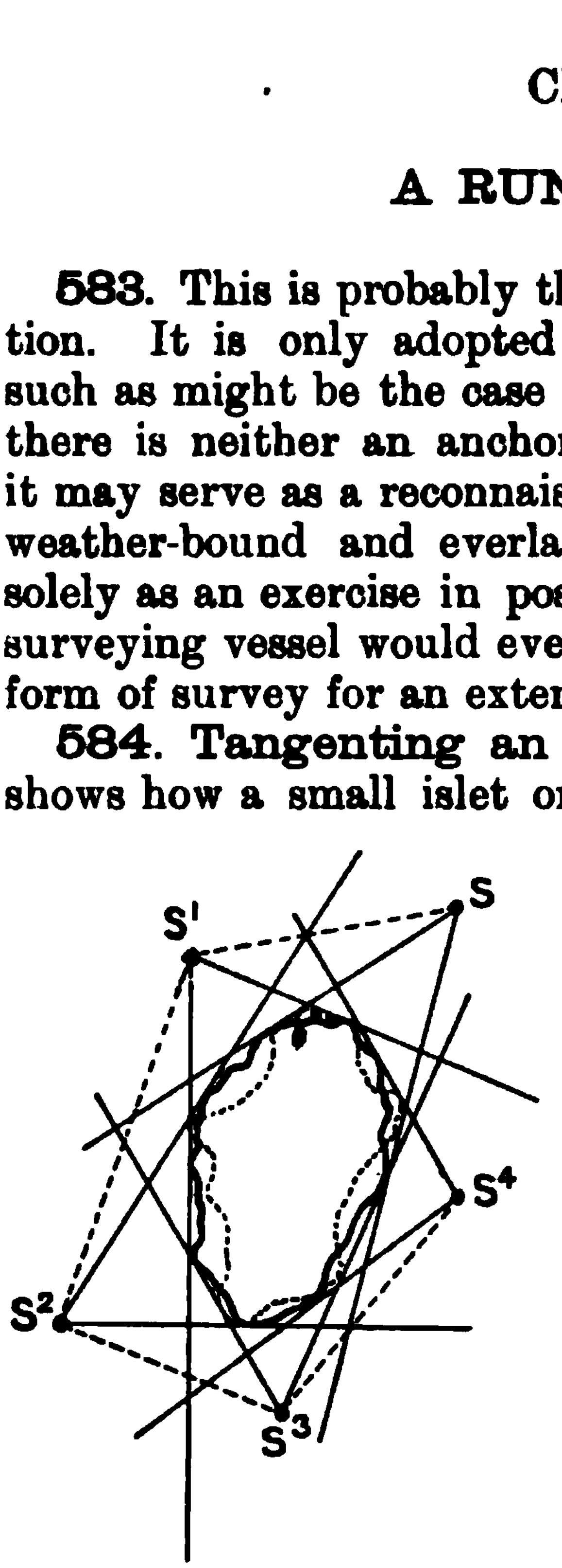


Fig. 220.

Supposing it to be an island distant from any land, its position will be relative to the first position at S.

This position S is either 'fixed' by angles to objects on the contiguous shore, or by true bearings from some distant objects; or a combination of T.B.'s and astronomical lines of position; or entirely astronomically.

Taking the crudest possible case, S is fixed in latitude and longitude (see p. 369, par. 589). A magnetic bearing, *i.e.* a bearing by compass with the deviation applied, of the 'tangent' of the island is taken, and also the angle between this tangent and the other left or right extreme.

The vessel proceeds by course, and distance by patent log, to S^1; then again takes a magnetic bearing of one 'tangent' and the angle between it and the other, and so on, as shown in fig. 220;

and the area enclosed by these tangent lines is filled in, so that no part of the coast-line shall extend beyond them. The work becomes more accurate if the vessel is able to 'fix' by angles to objects elsewhere; then all the 'tangents' become angles measured to some 'fixed' object.

It is needless to point out, that many details are lost, generally more than when 'putting in' the coast-line from a boat; even the shape of the islet may be a matter of taste. The hard line, and the dotted line in fig. 220 show what the possibilities might be; either are limited by the same tangents.

585. Not Applicable unless Coast is Inaccessible for All Time.—From this primitive stage, by enlarging on the system, and by improved means of deducing the bearings and the distances, it merges into the method of a survey of larger detached islands. But it may generally be accepted that coast-lines, unless quite inaccessible at all times and all seasons, are no longer surveyed by this system.

As explained in *Amendments*, p. 341, par. 579, when the ship's position is fixed by angles between three objects on the shore, then the scale of each plotting side, and the position of the true meridians at each 'fix,' differs from each other, owing to the error of observation in the 'fixing' angles, and also to a probable error in the position of the objects used on the chart.

586. Various Methods of Obtaining the Bearing and Distance of Plotting Side.—It will be seen that a running survey is practically carried out in a precisely similar manner; but the distance and bearing of each plotting side is deduced in the one case by taking angles at one 'fix,' then at another, while the running survey 'fixes' are obtained either by patent log or by mast-head angle, and the bearing of one position from the other is obtained with a ship's compass; consequently, the length of each plotting side is dependent on a system of measurement which will never give the same accuracy as is obtained by angles from three objects. The bearing also of each plotting side relative to each other is obtained by methods which will be much greater in error than that given by station pointer fixing.

The step in advance of the tangent system is:

The ship 'fixes' astronomically by sea horizon (see *Example*, p. 369, par. 589), or she may begin from a 'fix' obtained by three objects themselves 'fixed' on shore (see *Example*, p. 343). Where there are no elaborate appliances available, the vessel then steers a course by compass, and a certain distance by patent log, and may, or may not, anchor at one end; the line joining these two positions is the first plotting side.

As regards the distance, it may be measured by patent log; if one way only, *i.e* if the patent log is read at departure, and again at the next stop, the distance may be almost miles in error;

for a five-knot tide is not uncommon, and, whether the distance is being run 'with' or 'against' the tide, may be unknown; it might be possible to allow for a current which is known to exist, but a tidal stream cannot be ascertained without anchoring.

Hence it would be almost absolutely necessary for the boat or ship to anchor, so as to measure the distance by patent log run both back and forward; and for this purpose a boat must be left behind at the starting-point, while another runs the patent log there and back, from the vessel's second position at anchor.

Before using the patent log, its own error in registering should be ascertained (see *Example*, p. 381).

Even when the distance is run to and from the ship's position, the eventual error will probably be about $\frac{1}{20}$th mile for every mile run; this means an error of 100 yards per mile.

Since each position is plotted from the last, the distance in error at the next mile is 200 yards, and so on, gradually increasing. Comparing this with the other method of 'fixing' by angles, it will be seen that there is a considerable difference in the degree of accuracy attained.

The distance may be obtained from the start to the next 'fix' by M.H. angle; but here again an error of 1′ at 1 mile = 100 yards nearly, and, as before, the error in the total distance goes on accumulating.

Of the two methods mentioned, for any distance beyond 1 mile, the patent log will give the more accurate results.

As regards the bearing: if the vessel does not anchor at each 'fix,' entire dependence is placed on a course by compass; and the error is one of degrees. Hence this form of survey might be adopted in the case of oceanic islands, a few hundred yards off which there may be 'no bottom' at hundreds of fathoms; in such case a running survey is the only thing possible. Off weather-bound coasts where there is anchorage, the system is carried out with the aid of beacons anchored, and shifted on; from beacon to beacon, or to the ship, being a plotting side.

Suppose it is possible and safe to anchor, then the bearing of each plotting side can be obtained by angles, taken from one 'fix' to the next.

For, from the first position, which is 'fixed' astronomically (see *Example*, p. 369), a true bearing is obtained of some well-defined point, or object, or summit, on the shore; and, supposing it possible to leave an observer here, in a boat at anchor, in perhaps 150 fathoms of water, the ship proceeds to a second position, as was carried out in *Example* 1, on p. 367.

Both at the ship, and at the boat left behind, angles are taken between each other, as also between them and the object whose T.B. was previously taken. The angle projected from the position of the boat, gives a line of direction or of bearing to the ship.

The distance between the boat and the ship can be found either by another boat running a patent log from one to the other, or by taking a mast-head angle from position 1 if near enough. (In days gone by, such a distance was sometimes obtained by taking the time between the flash and the sound from guns fired at both ends, and such means of measuring a distance not under 4 miles, is still permissible.)

These two positions, *i.e.* of the boat and the ship, constitute the ends of the first plotting side, from whence a number of objects on shore are 'shot up'; or, the shore may only be 'tangented' from them (see fig. 220).

The 'check' to the value of the 'fixes' at 1 and 2 could be obtained by another true bearing from position 2 of the same object that was taken before at position 1. This is, in fact, observing the three angles of the triangle.

If, for instance, Z (fig. 221) is the object whose T.B. was observed at 1 and at 2, then the difference of the two true bearings is the angle 1 Z 2. If this is added to the angles at 1 and 2, the total should equal 180°; but this is not a 'check' on the distance 1 to 2, since at 1′ and at 2′ with the same angle Z the triangle has the same angles as before. The ship or boat next proceeds to 3, and her bearing is found by an angle either from 1 or 2.

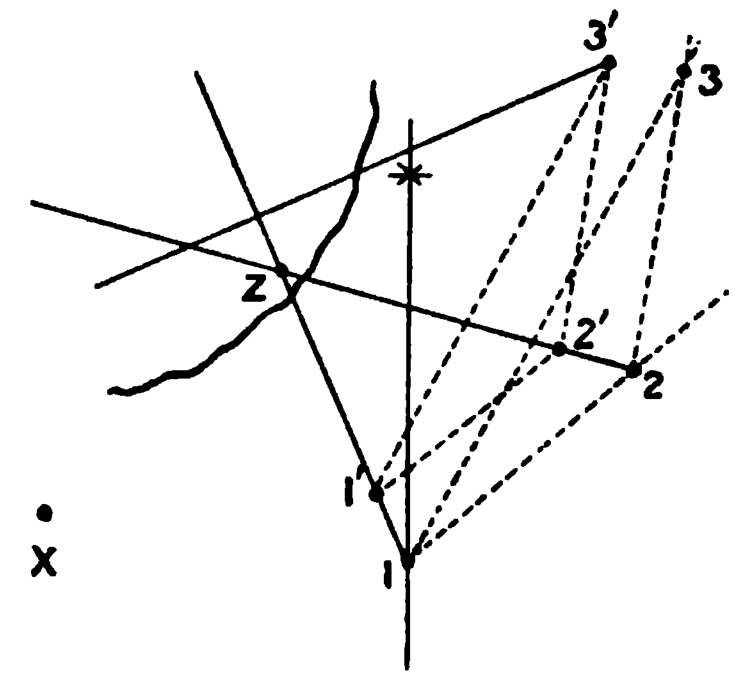


FIG. 221.

In the first case, if the angle is from 1, evidently 1 remains at anchor, in order to take the angle Z 1 3; but if the angle and distance is from 2, then 1 proceeds to 3; and, by a system of true bearings, and distances, and angles 'on' and 'back,' new plotting sides from 2 to 3, 3 to 4, and 4 to 5, etc., are obtained.

The length of each plotting side is on a different scale to the last, owing to varying errors due to the patent log, tides and currents; the result is, that from the first three true bearings of Z, there will be a 'cocked hat' at Z. Accepting the centre of this, the remainder of the work will be a series of compromises between the bearing of Z and the length of the side opposite to Z.

X is the final 'fix'; and if the boat has been left at 1, then the last distance measured from 1 to X should coincide with that on the paper.

We have now much the same sort of error as described in coast-lining round an island with a 10-foot pole, shown on p. 240,

except that in this case a friendly point Z, supposed to have been visible at every 'fix,' has probably kept the bearing, from 'fix' to 'fix' nearly correct; and what probably remains will be an error in distance. This error must be proportioned to each distance run, and, if necessary the whole work be replotted; but do not omit to weigh the importance or otherwise of the undertaking.

587. Method where a Number of Boats are Employed.—There must arise, however, a number of cases favourable for carrying out such form of survey.

For instance, take an ideal case represented in fig. 222.

Suppose two observers A and B land, and find two convenient positions at the ends of the island; each can put up a flag-staff, or erect a substantial mark. Suppose they can see each other's mark; each can find the true bearing of the other by angular measurements from the sun (see *True Bearings*, p. 101; p. 230).

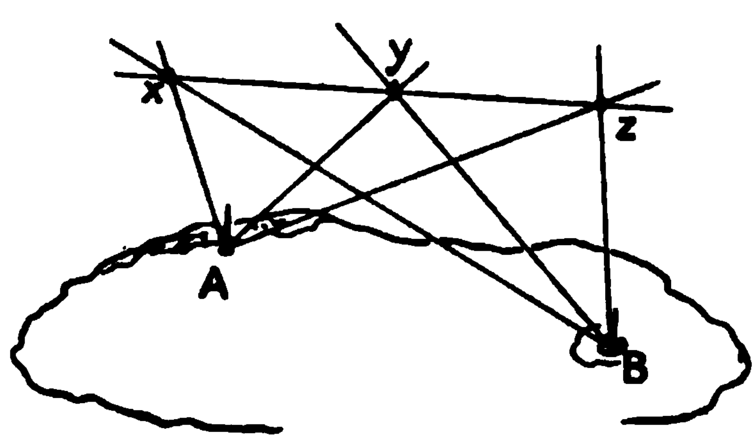


Fig. 222.

Suppose x, y, z, are the positions of three boats.

Then, if simultaneous angles are taken at each position to each other, there is practically a complete triangulation.

From x, y, z a number of objects on shore can be 'shot up,' and the observer at each can make a sketch such as is shown on p. 367, and so 'fix' all the north side of the island.

In the same way the remainder of the coast can be put in, merely by shifting the positions of x, y, z.

The whole work can be plotted from an assumed length of A to B. Let A B be drawn 6 inches long; then x, y, z can be plotted from that side, and everything is relatively quite correct.

For the scale, suppose x, y, z are placed in one line, and that from x and z the mast-head angle of the ship at y is taken. The sum of the distances xy and zy, having been determined, will give the length of xz; and from this length the distance between A and B is calculated. Suppose the length is found to be 1200 yards; then, if A B (6 inches) = 1200 yards, the scale is 6 inches = 1200 yards; *i.e.*, 1 inch = 200 yards = 600 feet.

Or, the distance between x and z may be run by patent log, with probably much the same result as before.

Or again, and better, if xz *is* longer than A B, plot the whole thing from the side xz; and deduce the scale from that side.

But this form of survey is really the connecting link between a triangulation carried out on land, and a triangulation mostly on the water combined with a 'running survey'; and, just as in a running survey pure and simple, requires a number of

observers and boats. Different conditions allow of a great number of potentialities.

One may conceive, however, of a slight modification of the foregoing, by supposing that only one observer is on shore. (See fig. 223.)

He would take a true bearing of the vessel at her first position, and measure the angle between each position arrived at and the previous one, providing there is a mark left there; such as angles S A S^1, S A S^2, or S^1A S^2, while the observer in the boat takes angle A S^1S, or A S^2S.

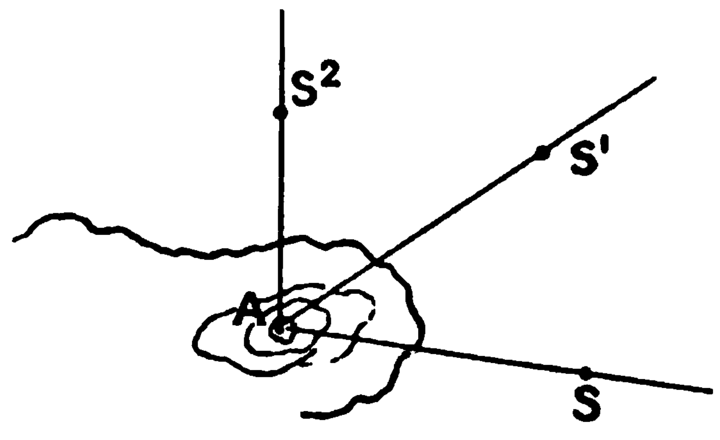


FIG. 223.

Here, as before, the ends of the survey will not meet; but if there is a landing and an anchorage, the island may at some future time serve some purpose, and therefore an attempt should be made to adjust the error.

Where there is an observer on shore, practically all three angles of each triangle are observed, so that each triangle can be adjusted as described on p. 232; and the distance in error will be proportioned out to each side according to its length.

588. Squaring in a Running Survey round an Island.— When no observer is left on shore, but true bearings have been taken to 'fix' the summit from the first three or four positions, and angles to the various objects on the shore have been referred to the summit, the figure described will be a number of sides such as is shown in fig. 224. In this figure, "the sum of all the interior angles equals twice as many right angles as the figure has sides – 4 right angles." Then the angles taken at the ship, and between each other and the summit, can be readjusted so that, as in the above figure, their sum shall equal 14 – 4 right angles, in the same way as described on p. 232. As regards the distance in error, each length must also be readjusted in the proportion to the distance run. After this, the whole work will have to be replotted. Between each of the forms of 'running survey' described, there are innumerable intermediate cases; but generally, in such work as this, one has to study the means of providing the greatest accuracy that is possible under the circumstances: that is, whether it is better to measure the distance by patent log or

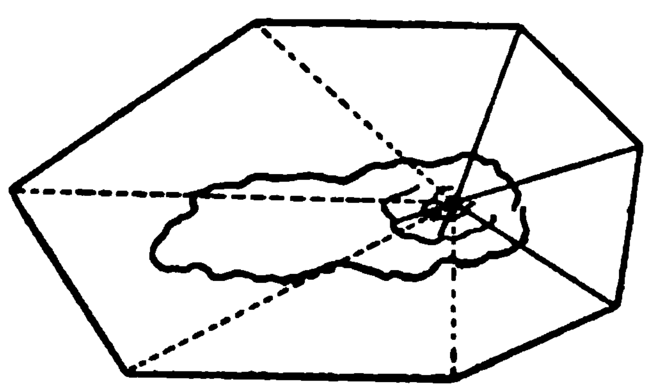


FIG. 224.

whether by M.H. angle; whether true bearings shall be taken from the shore or from the ship; if there is a natural object that will be visible from every 'fix' round the island; and whether it is advisable to erect such a mark. For an interesting and elaborate system of a 'running survey,' Wharton should be consulted.

The next example illustrates the method of 'shooting up' a number of objects on the shore from three positions, two being occupied by a ship, the other by a boat—in other words, triangulating from two plotting sides afloat—also, how a small island may be 'put in' by tangents; of what assistance sketches are; how to determine the height of a conspicuous hill; and how to find an 'absolute' position by astronomical observations by the 'St Hilaire' method, the method adopted and now practised in H.M. Navy since 1904.

Running Survey of the Outside Coast of an Island (Plate XI.).—The coast of the island runs in a north and south direction.

Example 1.—An islet surrounded by a reef lies off the west side of the island.

At position (1) the ship is at anchor. Observations place her in lat. 50° 20′ 20″ N., long. 159° 43′ 30″ E. Take the first position of the ship 4 inches from bottom and 4 inches from left of paper.

Scale, 1 inch = 1 mile = 6082 feet.

In coming to anchor, her head was north, and she gradually swung till she headed south, then swung again through west to north.

When at N. 20° E.,	the bearing of a	F.S. was	N. 43° E.
S. 70 E.	„	„	N. 46 E.
S. 20 W.	„	„	N. 42 E.
N. 70 W.	„	„	N. 39 E.

Accept the mean of these bearings as a correct magnetic bearing of F.S., and deduce the variation.

At about 7.45 P.M., when the sun was setting, the following observation was made:— $\underline{\odot}$ 97° 44′ F.S.

Declination, 20° 15′ N. S.D. 16′.

Angles taken from ship at position (1), 9 A.M., 9½ fathoms:—

Rock ϕ head of—

Green Bay	5° 30′	White Cliff ϕ F.S. on hill ϕ ⊢ White Cliff.	12° 30′	⊣ White Cliff.
⊣ Yellow Cliff	15 00	„ „	25 30	Stone at head of White Bay ϕ hill about 1 mile in.
Yellow Cliff	20 30	„ „	43 00	Log on Beach.
Canoe on Beach	25 20	„ „	53 00	South Point, White Bay.
Hut	30 45	„ „		
North Islet	35 00	„ „		

A steamboat is left here at anchor; the ship proceeds in a northerly direction to a position (2).

When the ship has anchored:

At steamboat, Ship 52° 30′ F.S.

Position (1) is then buoyed.

The steamboat then tows a patent log to ship's position (2), and also back to the buoy; the mean of the patent log readings is 2·98 miles.

Angles taken from ship at position (2), 11.30 A.M., 8 fathoms:—

F.S. 94° 00′ Steamboat (position (1)),

and

$\dashv$ Yellow Cliff	11°	40′	F.S.	19°	15′	$\vdash$ White Cliff.
Yellow Cliff	19	00	,,	24	30	Rock on Green Bay.
$\vdash$ Yellow Cliff	20	30	,,	37	30	White Cliff ϕ hill 1 mile inland. Elevation, 0° 47′ to shore horizon. Eye 30 feet. Time, noon.
Canoe	32	00	,,	45	30	$\dashv$ White Cliff.
Hut	42	00	,,	53	40	Stone.
North Islet	47	30	,,	72	00	South Point.
				66	30	Log.

The following sketch was made from ship's position (2) at noon:

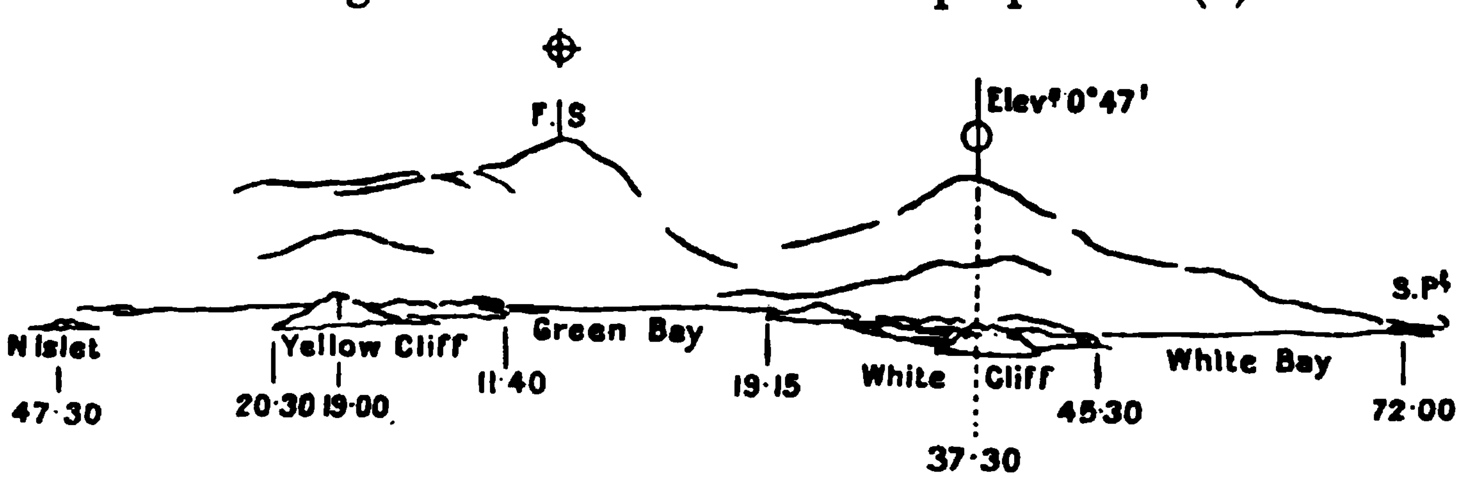


FIG. 225.

The steamboat then weighed the buoy, and anchored it to the northward of the ship.

At ship (position (2)), Steamboat 76° 00′ F.S.

Angles taken at steamboat (2), 1 P.M., 7½ fathoms:—

$$\left.\begin{array}{l}\text{F.S. } \phi \text{ North Islet} \\ \quad\ \ \phi \vdash \text{Yellow Cliff}\end{array}\right\} 69^\circ\ 30' \text{ Ship.}$$

and Canoe	10°	30′	F.S.	5°	45′	Yellow Cliff ϕ $\dashv$ Yellow Cliff.
Hut	12	00		13	00	head Green Bay
				23	00	$\vdash$ White Cliff.
				31	30	Rock.
				37	00	White Cliff.
				51	00	South Point.

From the buoy to ship (2), the mean of patent log readings, running both ways, was 2·6 miles.

The coast forms two bays, distinguished as Green Bay and White Bay.

The northern, Green Bay, terminates in a cliff, about $\frac{1}{2}$ mile in extent, at its north end, and, separating the two bays is a cliff about $1\frac{1}{2}$ miles long.

F.S. is on a summit which gradually rises from Yellow Cliff.

The small hill, 1 mile inland, is separated from F.S. hill by a valley.

North Islet joins the shore at L.W., and, between it and Yellow Cliff is a small sandy cove.

Enclose the whole in a frame as a Mercator chart.

For the off-lying islet to the westward.

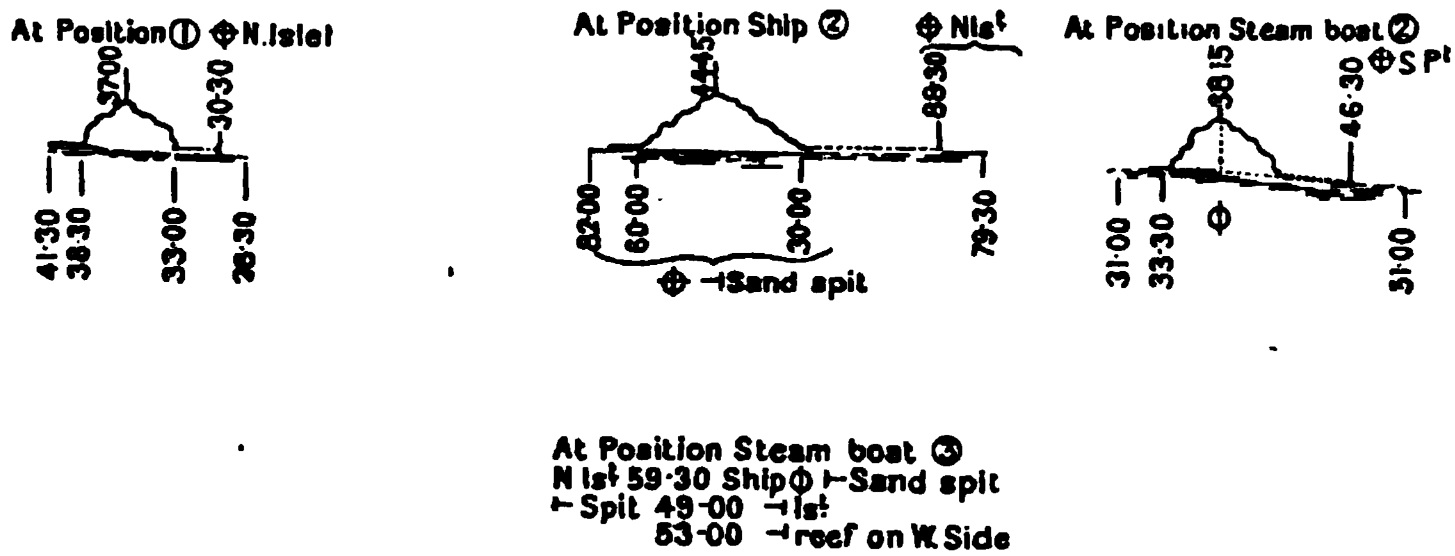


FIG. 226.

Insert soundings at each position.

H.W.F. and C., $7^h\ 4^m$.

Rise of springs, 20 feet.

Neaps rise, $14\frac{1}{2}$ feet.

Moon's meridional passage, $7^h\ 32^m$ (corrected).

Age of tide, $2\frac{1}{2}$ days.

Correction for age of tide and moon's meridional passage from table, Appendix, -62^m.

Hence lunitidal interval on that day

$$
\begin{array}{rrr}
 & = & 7^h\ 04^m \\
 & - & 1\ \ 02 \\
\hline
 & = & 6\ \ 02 \\
\text{adding moon's mer. pass.} & & 7\ \ 32 \\
\hline
\text{time of H.W.} & & 13\ \ 34 \\
\text{subtracting} & & 12\ \ 24 \\
\hline
\text{Then it is H.W. at} & & 1\ \ 10
\end{array}
$$

subtracting 12 24 for time from previous H.W.

Then it is H.W. at 1 10 P.M.

If the moon's meridional passage is $7^h\ 32^m$; then $7^h\ 32^m$ divided by 48^m gives roughly the age of the moon $= 9\frac{1}{2}$ days;

and the age of the tide is $2\frac{1}{2}$ „

Therefore the tide is 7 days from spring tides.

Spring tides rise 20 feet. Neaps, $14\frac{1}{2}$ feet.

Therefore the rise of the tide diminishes $5\frac{1}{2}$ feet from springs to neaps, that is, $7\frac{1}{2}$ days; or at the rate of $\frac{5\frac{1}{2}}{7\frac{1}{2}}$ per day.

In 7 days it will have diminished $7 \times \frac{5 \cdot 5}{7 \cdot 5} = 5\frac{1}{3}$ feet; and since springs rise 20 feet, then the rise of tide for the day will be $14\frac{2}{3}$ feet, and the range $9\frac{1}{3}$ feet $=$ 9 feet 4 inches.

If it is H.W. at 1.10 P.M.,

at I. 10 P.M.	reduction	$= 10^h + 4^h\ 8^m$	$= 14^h\ 8^m$	$= 2\frac{1}{2}$	fathoms.
at XI. 30 A.M.	„	$= 10 + 2\ \ 9$	$= 12\ \ 9$	$= 2$	„
at IX. 00 A.M.	„	$= 10 - 2\ \ 4$	$= 7\ \ 8$	$= 1\frac{1}{4}$	„

For elevation, $h =$ dist. tan elevn.

The distance from the ship to the foot of White Cliff is 2·4 inches of paper $=$ 2·4 miles.

The correction to elevations for a height of 30 feet and distance $2\frac{1}{2}$ miles is $-8'$ (see Table 14, *Inman's Tables*).

The corrected angle of elevation is $0°\ 47' - 8' = 0°\ 39'$.

The distance of the summit from the ship $=$ 4 miles.

$$\text{Then } h = 4 \times 6082 . \tan 0°\ 39'$$

$$\begin{aligned} \log 4 &= \ \cdot 602060 \\ \log 6082 &= 3 \cdot 784046 \\ \tan 0°\ 39' &= 8 \cdot 054809 \\ \hline \log 2 \cdot 440915 &= 276 \text{ feet.} \end{aligned}$$

Reduction to height, for tide:

From above, the rise of the tide at noon is 13 feet.

The observation for height was made at noon, when the tide was 13 feet above L.W.S.

Springs rise 20 feet; therefore the tide would rise another 7 feet to H.W.S. And since heights are shown as above H.W.S., then 7 feet must be subtracted from the height found; and $276 - 7$ $=$ height of the hill above H.W.S. $=$ 269 feet.

589. Example of 'Fixing' Astronomically, using the Sea Horizon.—Assume a latitude and longitude near about where the ship is supposed to be: in this case, let the lat. be 50° 18′ N., and long. 159° 40′ E.

On the 23rd July 1908, at about 7.10 A.M. mean time, the following sights were taken :—

watch time	0^h	00^m	28^s		obs. alt., 24° 21′ 10″; I.E., + 2′ 30″	
comparison fast	1	05	10		height of eye, 30 feet.	
chron. time	10	55	18			
error chron.	2	36	35	fast		
G.M.T.	8	18	43	(22nd)	dec. 20° 19′ 14″ N.	change 29·7
longitude	10	38	40	E.	− 4 26	8·3
M.T.	18	57	23	(22nd)	20 15 48	60 | 266·51
eq. time	0	6	13		90 0 0	= 4′ 26″
app. time	18	51	10	(22nd)	P.D. 69 45 12	
	24	0	0		eq. time 6^m 12^s·2	change ·1
					+ ·8	8·3
hour-angle	5	08	50		6 13·0	·83

With the above H.A. and the assumed lat. 50° 18′ N., calculate the sun's Z.D. and her T.B.

log hav. 5^h 08^m 50^s	9·590281
log cos 50° 18′ 0″	9·805343
log cos 20 14 48	9·972301
diff. 30 03 12	9·367925 hav. = 57° 45′ 55″

log vers. 57° 45′ 55″	0466611
log vers. 30 03 12	0134440
vers.	0601051 = 65° 29′ 16″ = Z.D.
	90 0 0
	24 30 44 = altitude (calculated).

To find the true bearing :

log sec lat. 50° 18′ 00″	10·194657
log sec alt. 24 30 24	10·028431
diff. 25 47 36	
P.D. 69 45 12	
log ½ hav. 95 32 48	4·869520
log ½ hav. 43 57 36	4·573200
log hav.	9·665808 = N. 85° 47′ E. = sun's T.B.

Then, at a given H.A. in lat. 50° 18′ N. and long. 159° 40′ E., the sun's Z.D. is 65° 29′ 16″, and her T.B. N. 85° 47′ E.

Let O° be the assumed position in latitude and longitude. If the sun bears N. 85° 47′ E., then, anywhere at right angles to this bearing, *i.e.* $\frac{N.}{S.}$ 4° 13′ $\frac{W.}{E.}$ line A B (fig. 227) for a limited distance, which varies as the altitude, or, say, between 12 and 20 miles, the sun will practically have the same bearing and the same altitude.

But, simultaneously with the 'time' of the above calculation, the sun's altitude was observed.

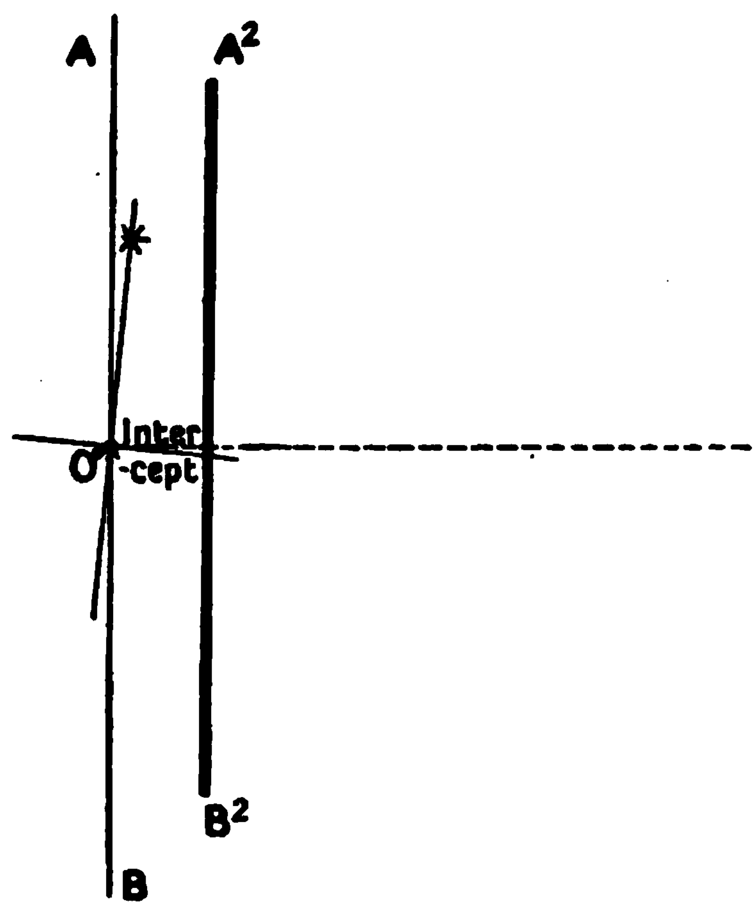


FIG. 227.

Now, if that altitude with the usual corrections applied, is found to be the same as that found by the calculations 'through time,' then the observer must be at O°; unless there has been an error made in either the latitude or the longitude, and the error of the one has compensated for the error in the other, which is very unlikely.

If, on the other hand, the altitude found by observation is not the same as that calculated, then the observer must be on a line parallel with A B, either nearer to the sun, or further, according as the altitude observed is greater or less than that calculated; and the distance, nearer or further, is denoted by the difference of the altitudes, and is called the 'intercept.'

The mean of the observed altitudes $\underline{\odot}$, when the watch showed, as above, $0^h\ 00^m\ 28^s$, was 24° 21′ 10″; I.E., + 2′ 30″; eye, 30 feet.

Obs. alt.	24°	21′	10″
I.E.		2	30
	24	23	40
dip		5	24
	24	18	16
ref. par.		2	00
	24	16	16
S.D.		15	46
true alt.	24	32	00
calc. alt.	24	30	44
diff.		1	16

At the instant of 'stop' by the watch, suppose the observer to make an error of ± 1′: it will probably be much more than that, but assuming it as 1, simplify the ratios which follow. When the hour-angle is small this error will be smaller.

diff. 1 16 = 1·27 miles ('intercept' *towards* the sun).

Since the true altitude is the greater, then the 'intercept' (the peculiar expression for this difference) is 1·27 miles nearer the sun, and the line A B is therefore moved in a direction N. 85° 47′ E. 1·27 miles—*i.e.* parallel to itself—to A^2B^2.

It is now known that either or both the assumed latitude and longitude are wrong; and the premise is that the observer is somewhere along the line A^2B^2.

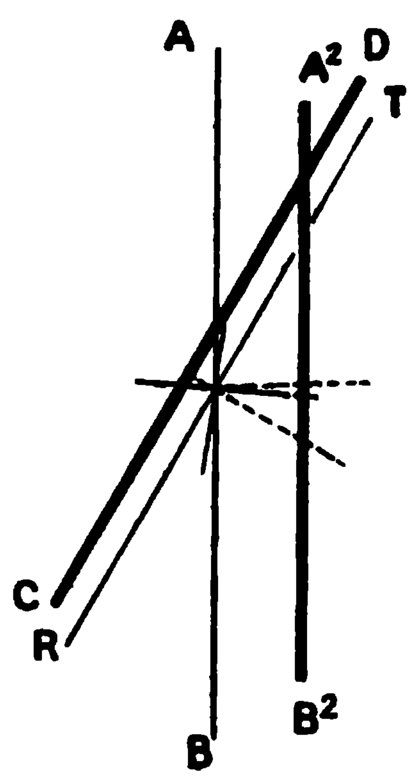


Fig. 228.

Owing to 'errors of observation,' it would probably be more accurate to draw this line about $\frac{1}{4}$ or $\frac{1}{2}$ inch—*i.e.* 1 mile—thick, depending on the scale used, probably never less than the former, unless one is 'fixing' on a chart of the world.

If now, by similar means, we can find another such line cutting the first at an angle of say not less than 30°, then, at the intersection of these broad bands will be a four-sided figure some miles in area within which the observer may consider himself to be. There will be a small difference in the size of this figure, and consequently in that of its maximum diagonal, according to the angle with which the bands cut each other; but again, if the angle of cut is greater than 60°, and both lines are on the same side of the meridian, the second line of position must be such that the sun will be fairly close to the meridian; in which case the width of the second band, representing, as it does, an 'error of observation,' might be very many more times the width of the first; because, with a small H.A., the corresponding error of observation to that

of the earlier altitude will be considerably larger; so that a problematic gain by a wider 'cut' will be more than compensated for by the very much enlarged width of the band.

If the lines of position are considered as lines such as are drawn with an ordinary sharp pencil, and with a ruler not boomerang shaped, and not with a carpenter's pencil used broadways on, having no definite edges, there is *practically* no difference obtained between two lines drawn cutting each other at 30° and at 60°, and certainly none between 60° and 90°, providing the ruler is straight and the point of the pencil sharp. This matter is therefore not taken into account. In order to illustrate the 'method' under discussion, another observation should be taken somewhere on a bearing nearly at right angles to the first; though in this case it will turn out to be on the same side of the meridian.

It should be clearly understood, that the two sights taken on the same side of the meridian do not eliminate any 'errors of observation,' at whatever angles the lines 'cut'; though, on the other hand, the nearer the meridian, the more accentuated do the errors become.

The following sight is introduced to form a means of comparison between the 30° and 90° 'cut.'

At about 9^h^ 30^m^ A.M. on the 23rd, when the sun had changed its bearing about 35 degrees, the following sight was taken:—

	h m s		° ′ ″		m s	
Watch time	2 30 50	Dec.	20 19 14 N.	Eq. time	6 12·2	·1
comp. fast	1 05 15		− 0 5 20		+ 1·1	10·8
	1 25 35		20 13 54		6 13·3	1·08
error chron. fast	2 36 35		90 00 00			
G.M.T.	10 49 00 (22nd)		69 46 06 P.D.			
long. in time	10 38 40 E.					
mean time	21 27 40 (22nd)					
eq. time	00 6 13·3					
hour-angle	2 38 33·3					

log hav. H.A.	$2^h\ 38^m\ 33^s{\cdot}3$	9·060546
log cos lat.	50° 18′ 00″	9·805343
log cos dec.	20 13 54	9·972344
diff.	30 04 06	8·838233 hav. = 30° 26′ 06″

log vers. 30° 26′ 06″	0·137796	
log vers. 30 04 06	0·134572	
log vers.	0·272368	43° 18′ 43″ = Z.D.
		90 00 00
calculated altitude		= 46 41 17

log sec lat. 50° 18′ 00″	10·194657
log sec alt. 46 41 17	10·163690
	3 36 43
P.D.	69 46 06

log ½ hav. 73 22 49	4·776325
log ½ hav. 66 09 23	4·737018

log hav. 9·871690 = 119° 14′ 07″ sun's T.B.

The obs. alt. was	46° 28′ 30″	
I.E. +	2 30	
	46 31 00	
dip −	5 24	
	46 25 36	
R.P. −	0 00 49	
	46 24 47	
S.D.	15 46	
true alt.	46 40 33	
calculated alt.	46 41 17	
'intercept'	00 44	= ·73 mile away from the sun.

At the moment of 'stop' by the watch, suppose the observer to make an error of ± 45″—its ratio to the first error being ·75. (See p. 372.)

The sun's T.B. is N. 119° 14′ 07″ E.

The line at right angles to this is $\frac{\text{N.}}{\text{S.}}$ 29° 14′ 07″ $\frac{\text{E.}}{\text{W.}}$

Draw this line through O, represented by R T in fig. 228; and, since the true altitude is less than the calculated, draw a line parallel to R T, ·73 mile further from the sun. Let C D represent this line.

Then the intersection of A^2B^2 and C D gives the position of the observer.

The result of these two sights probably gives a more correct latitude than longitude, and it would be better to take the next

sights at about the same hour-angle on the opposite side of the meridian, so as to obtain a more correct longitude; but, according to the system, the next sight will be taken near noon, when the sun's bearing will have changed about 60°.

At about noon:—

Watch time	5^h	05^m	10^s	
comp. fast	1	05	20	
chron. time	3	59	30	
error chron.	2	36	35	
G.M.T.	13	22	55	(22nd)
long.	10	38	40	
M.T.	24	01	35	(23rd)
eq. time –	0	6	13·5	
app. time	23	55	21·5	
hour-angle	0	04	38·5	

App. M.T.	0^h	00^m	00^s	(23rd)	
long.	10	38	00	E.	
app. G.M.T.	13	22	00	(22nd)	
Dec.	20^h	19^m	14^s	N	29·7
	00	6	37		13·4
	20	12	37		60|397·98
	90	00	00		6·37
P.D.	69	47	23		

log hav. H.A.	0°	04′	38″·5	6·010756
log cos lat.	50	18	00	9·805343
log cos dec.	20	13	37	9·972402
diff.	30	05	23	5·788501 hav. = 0° 53′ 09″

Eq. time	6^m	$12^s·2$	·1
	+	1·3	13·4
	6	13·5	1·34

log vers.	30°	05′	23″	0·134759
log vers.	0	53	09	0·000119

vers. 0·134878 = 30° 06′ 12″ = Z.D.
90 00 00

calculated altitude = 59 53 48

log sec lat.	50°	18′	00″	10·194657
log sec alt.	59	53	48	10·299676
	9	35	48	
P.D.	69	47	23	
log ½ hav.	79	23	11	4·805280
log ½ hav.	60	11	43	4·700265

hav. 9·999878   178° 05′ sun's T.B.

The mean of obs. alt.	59°	38′	56″
I.E.		2	30
	59	41	26
dip		5	24
	59	36	02
R.P.		00	30
	59	35	32
S.D.		15	46
true alt.	59	51	18
calculated alt.	59	53	48
'intercept'		2	30 = 2·5 miles away from the sun.

At the instant of 'stop' by the watch, suppose the observer to make an error of $\pm 10''$; its ratio to the other two errors being ·17. (See pp. 372, 374.)

As before, lay this off on the bearing of N. 178° 05′ E., through O, away from the sun, and draw a line at right angles to it N. 88° 05′ E. This, if the sights are right, should pass through the previous intersection; if it does not, there is a 'cocked hat.' Accept which you think are the most trustworthy lines. In the areas drawn in Plate XII., the 'cocked hat' is formed by joining the point of intersection of each diagonal: the centre of the inscribed circle is the accepted position. It would have been much better to have taken the third sight when the bearing was nearly opposite to the first; as has been stated, the first and second observation gave a better latitude than longitude, and the second, taken with a third at about the same hour-angle on the opposite side of the meridian, would have given a better longitude than latitude; and the results would have been far more satisfactory than those that were taken.

Plate XII. shows the plotting of all three position lines on 1 inch to a mile scale, and the errors are included in areas.

590. *Example* 2 (Plate XIII.).—The example that follows consists of, partly a regular triangulation of the shore, followed by a running survey of the coast. The regular part of the triangulation should require no explanation, beyond the fact that the measurement of the base took place the day after the triangulation was plotted, and while the anchorage was being sounded out.

591. **Plotting from an Arbitrary Length.**—It is intended as a demonstration of the fact, that it is of no consequence to the triangulation, what length in inches of paper is adopted for the plotting side.

Whatever length is laid down, every point of the triangulation

is relatively quite correct to that side; it is only a question of what the eventual scale turns out to be.

But since, as a rule, the survey should be plotted on about the scale desired, it will be necessary at the very beginning to obtain a rough distance between the points on which one intends to plot from.

It is of no consequence if the ultimate scale is a whole number or not; for, it must not be forgotten that, when published, it will probably be on a smaller scale than the original; and may be reduced to a 'whole number' scale: for instance, the work may be on a 4·23 inches to a mile, and possibly be reduced to a 3-inch scale.

To find the approximate distance between the ends of the plotting side. In this example, a steamboat towing a patent log ran between a point abreast of one end, to a second point nearly abreast of the other; this gave a rough measurement of the distance.

By adopting that distance as a commencement, the ultimate scale, derived from the measurement of a base, will be near what was originally intended.

For instance, it is stated that the distance between A and B is approximately 1·6 miles; and, since the work is intended to be on a scale of 4 inches, then the length of the plotting side is drawn 6·4 inches; and the whole work proceeds. At the close, a measurement is made elsewhere, and by calculation A B is found to be 9105 feet. On this basis, if the mile is as stated 6075 feet, A to B = 1·5 miles; and 6·4 inches of paper really equals 1·5 miles, not 1·6. The correct scale then will be $\frac{6 \cdot 4}{1 \cdot 5} = 4 \cdot 267$ inches and not 4 as originally intended.

In deducing this scale, the length of the piece representing the base measured, is not necessarily plotted; in fact, in the present instance it is not so; but the distance of the plotting side is separately calculated 'through' a number of triangles.

In plotting this example, considerable judgment must be exercised as to the 'point' and the line of reference from which the 'checking' angles should be projected; and it will be found that all the 'points' cannot be 'checked' from one place. And therefore a second place must be used from which the 'checking' may be effected.

592. All Angles Taken are not Plotted.—When angles are taken in a survey, every angle as it is observed is written down; but every angle is not necessarily plotted.

For example, take any triangle A B C. The three angles of the triangle have been observed, such as A 40° 30′, B 80° 30′, C 59° 00′; but if the triangle is constructed on the side A B, then, only angles A and B are projected: the third angle is not plotted, though it must be observed as a check to the correctness of the other two angles.

So then, looking down the angles observed at any place, some

will be for the purpose of completing a triangle, and the other will be projected; and each angle must be picked out for its particular requirement.

Take the following example. At F, the angles D 87° 30′ A, B 44° 30′ D, C 54° 40′ D, are not plotted anywhere.

Also, places from whence angles were taken are written in the order in which they were visited; but, in the plotting, the angles must be picked out as they are wanted, from each place; and this requires a constant reference from one to the other.

Since the plotting side is A to B, then the angles observed at these two places must be plotted first; and the intersection of the two lines projected will give positions C, D, E, and F.

Now for the direction of the 'check,' or third line. For C, it may be convenient to take the angle from D; and for E, it may be better to take the angle from C; and so on.

It is also intended to convey, that the whole of the triangulation was done in one day.

While, on the next day, one party was employed finishing the 'secondary' triangulation of the coast, as well as measuring a base, the ship steamed round the island, and 'shot up,' by a running survey, the north and the west sides.

The day following this, a boat, or boats, sounded out the anchorage.

Survey of an Isolated Island, partly by Regular and partly by Running Survey.—Total time occupied, three days.

Rough Sketch.

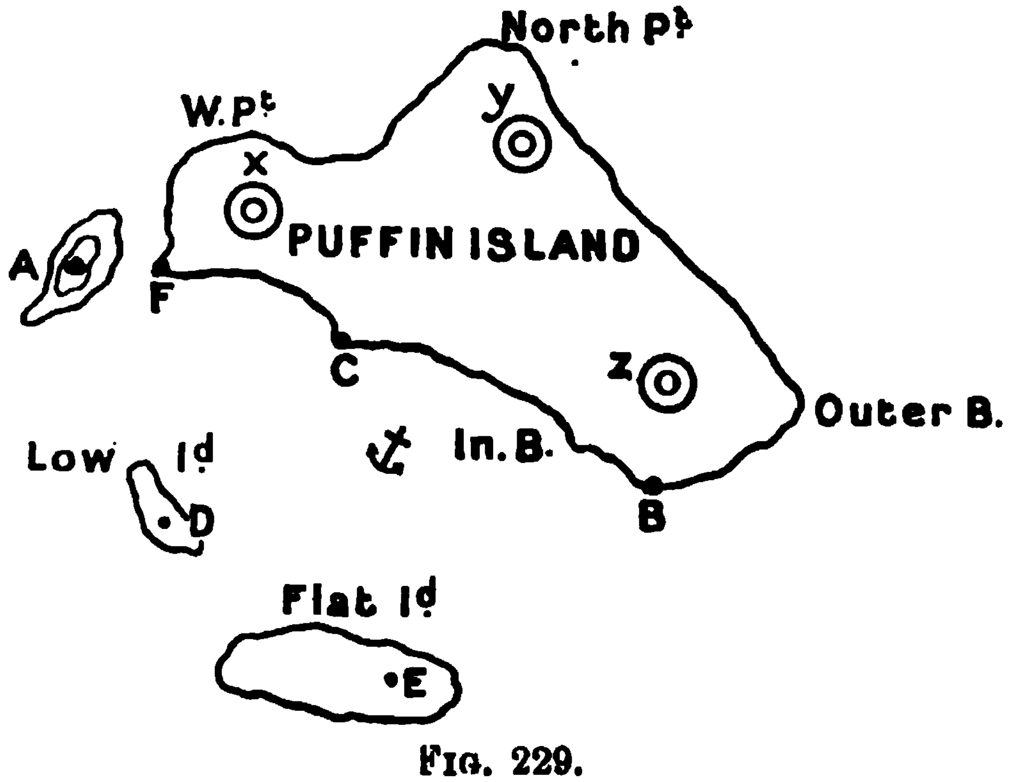


FIG. 229.

A, B, C, D, E, F are erected marks; x, y, z are three summits.

It is proposed to survey the anchorage by a regular triangulation, and the back of the island by running survey.

A to B is adopted as the plotting side: their distance apart is found to be roughly 1·6 miles. It is proposed to use a scale as near 4 inches as possible.

Take A 4 inches from the left, and 6 inches from the bottom of the paper.

Latitude is 54° 01′ N.; longitude, 56° 54′ W. 14th May 1907.

The mile = 6070 feet.

The T.B. is obtained at B, with a landing compass.

A.T. $6^h\ 49^m\ 20^s$ ○| ϕ A.
6 50 40 |○ ϕ A.

The magnetic bearing of A is N. 31° 40′ W.

Draw the meridians through A. Take the T.B. from *Burdwood's Tables.*

Observer at A.

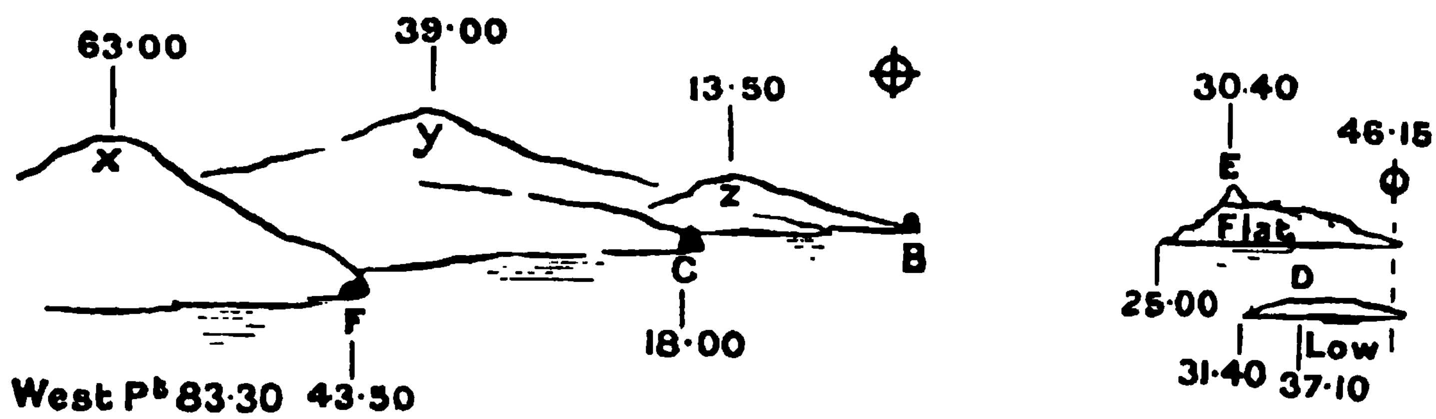


FIG. 230A.

Observer at D, depression to H.W.L. at B 0° 18′.

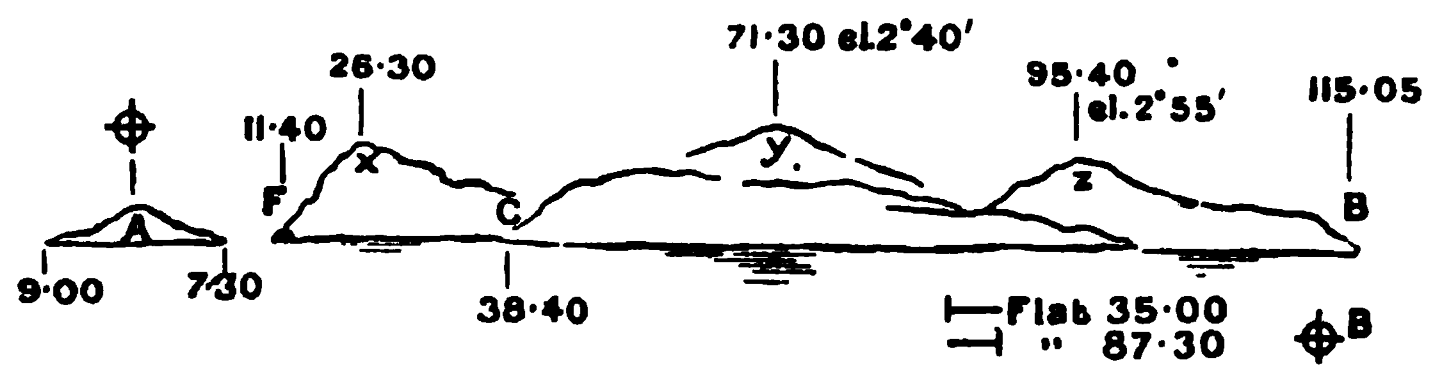


FIG. 230B.

Observer at E.

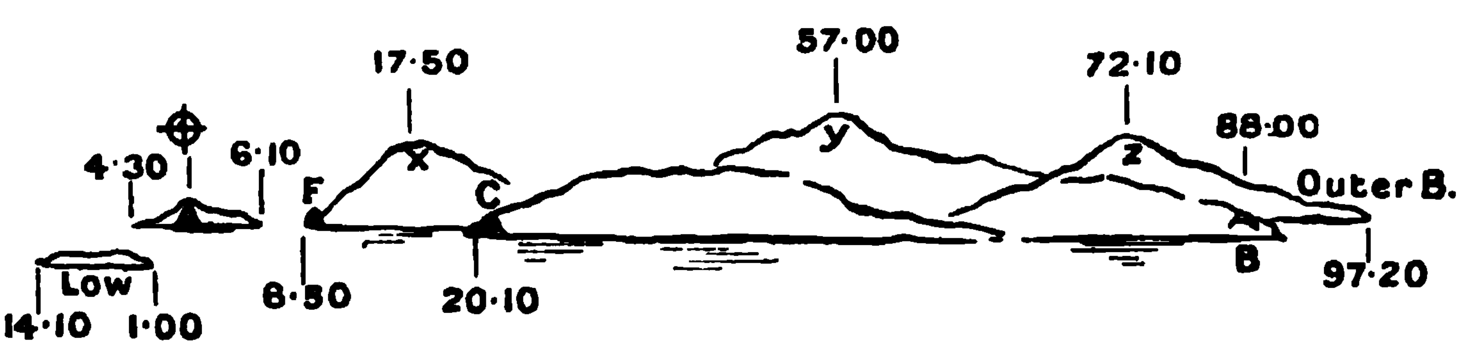


FIG. 230C.

Elevations (el.) are sextant angles taken to H.W. level. Height of eye, 5 feet 4 inches.

Angles taken at F.

		A	105° 30′	W. Pt. .
		⊢ A	80 00	⊣ A.
		D	87 30	A.
		D	68 40	⊢ A.
⊢ Low	6° 00′	D	12 00	⊣ Low Id.
⊢ Flat	19 30	D		
B	44 40	D		
C	54 40	D		
(Blob) *a*	85 30	D		

At Blob *a*. C 33° 30′ D 77° 10′ F.

Angles taken at C.

⊢ Low / ϕ ⊣ Flat }	7° 40′	D	18° 20′	⊣ Low.
E	25 08	D	76 10	⊢ A.
⊢ Flat	32 30	D	86 07	A.
B ϕ Inner B	67 35	D	98 20	F.
		D	105 30	⊣ A.
		F	66 30	Blob *a*.

At Blob *b*.	Inner B	80° 00′	D	61° 30′	C.
At Blob *c*.	Inner B	69 30	D	51 20	C.
At inner B.	E	44 00	D	50 00	C.
			D	67 20	Blob *b*.
			D	92 30	Blob *c*.

Blob *d*	28° 30′	B.
⊣ Low	32 00	C.
⊢ Low	53 30	C.
⊣ Flat	70 30	C.
⊢ Flat	105 00	C.

Inner B is a secondary station (⊙).

Blobs *a*, *b*, *c*, *d* are whitewashes on the high-water line of the coast, for sounding marks.

NOTE.—*b* and *c* should be fixed by calculated angles.

Angles taken at B.

E	61° 20′	A	4° 30′	F.
⊢ Flat	74 00	A	7 30	⊣ A.
⊣ Flat	45 00	A	8 20	C ϕ Inner B.
⊢ Low	30 30	A	16 30	*x*.
D	27 45	A	38 30	Blob *d*.
⊣ Low	14 20	A		
⊢ A	4 10	A		

While the above triangulation was being completed, the ship weighed, and 'shot up' the north-west and north-east sides of the island, having 'taken off' the 'main points' on a plotting sheet of paper.

A boat was left at the anchorage, with a lead line over the side; and registered the following soundings:—

	fms.	ft.
IX. 00 A.M.	2	0
X. 00 „	1	4
XI. 00 „	1	0
Noon	0	4 L.W.

The estimated rise of springs was 20 feet.

IX. 00 A.M.

Fix.		Fms.	Reduction.
(1)	Pat. log 0 B 109° 30′ D 70° 30′ A .	6	$1\frac{3}{4}$
	6 × $5\frac{1}{2}$		
(2)	Pat. log ·4 E 51° 30′ 33 00 . .	6	
	$6\frac{1}{2}$ 8		
(3)	Pat. log ·7 D 79° 00′ z ϕ B . .	10	$1\frac{1}{2}$
	9 8 7 8 7		
(4)	Pat. log 1·3 z 28° 30′ y		
	E 74 20 . .	7	
	Outer B ϕ z		

X. 00 A.M.

Fix.		Fms.	Reduction.
	B 60° 30′ White		
	67 00 10-ft. Rock		$1\frac{1}{4}$
	63 00 Blue		
	$6\frac{1}{2}$ × 8		
(5)	Pat. log 1·7 z 43° 30′ y		
	E 64 10 y . .	9	
	z ϕ White . .	$9\frac{1}{4}$	
	Outer B 92° 30′ y		
	78 00 10-ft. Rock		
	96 30 Blue		
	99 00 Green		
	103 30 N. Pt.		

Find the length in inches of paper of the patent log mile, and apply this measure to the readings which follow.

Reset patent log—reading 0.
Shaped course N. 11° E. mag.
8 × × $8\frac{1}{2}$ ×

(6) Pat. log ·6 . . . . 9 1
Green 17° 00′ N. Pt. $9\frac{1}{4}$
y ϕ Blue 45 40 N. Pt.
10-ft. Rock 93 10 N. Pt.
White 99 40 N. Pt.
Outer B 110 50 N. Pt.

Altered course to N. $21\frac{1}{2}$ W. mag.
10 14 16

XI. 00 A.M.
(7) Pat. log 1·1 . . . . 10 $\frac{3}{4}$
$10\frac{1}{2}$
Green 39° 30′ N. Pt. 8° 00 Red
y 62 30 ϕ
Blue 69 40 Yellow
10-ft. Rock 80 50

Altered course to N. $49\frac{1}{2}$ W. mag.
11 12 17

(8) Pat. log 1·6 . . . . 16 $\frac{1}{2}$
y 73° 10′ W. Pt. ϕ A $16\frac{3}{4}$
Yellow 22° 00′ A
Cove 41 30 A
Red 62 20 A
N. Pt. 83 00 A
16 15 13 15 14

(9) Pat. log 2·25 . . . 12 $\frac{1}{2}$
$12\frac{3}{4}$
y 28° 00′ *x* 57° 00′. A—Yellow ϕ Cove. ϕ West Pt. 101° 00′ A.

(10) A 53° 00′ D 21° 40′ E.
$\vdash$ A 55 00 D 14 00 $\vdash$ E.
$\vdash$ D 17 30 D 35 00 $\dashv$ E.

Noon.

NOTE.—Red, white, blue, green, and yellow, are distinguishing marks along the coast.

Cove is at the head of a small sandy cove. The remainder of the coast is cliff.

Inside the harbour, from C to B is sandy beach, the remainder is a hard rocky coast.

Ship returned to anchorage.

Lead line measured	3 ft.	too long at	5 fms.,	*i.e.*	1	foot	per hour.
	4	„ „	7 „		$1\frac{1}{3}$	„	„
	6	„ „	10 „		2	feet	„
	9	„ „	15 „		3	„	„

The lower of the two soundings given at each fix, is corrected for error of line.

A base was measured on Flat Island:

E	was the east end of the base	
W	„ west „ „	
At W	A ϕ D	
	A 129° 30′ E.	
At E	W 44 00 A.	

The measured distance from E to W was 1137·2 feet.

This base forms no part of the triangulation, and W is not plotted: it is only a means of obtaining the true length of A B.

Calculation for scale:

In Δ A W E (fig. 231)—

W	129°	30′
E	44	00
A	6	30 (see sketch A, angle between E and D)
	180	00

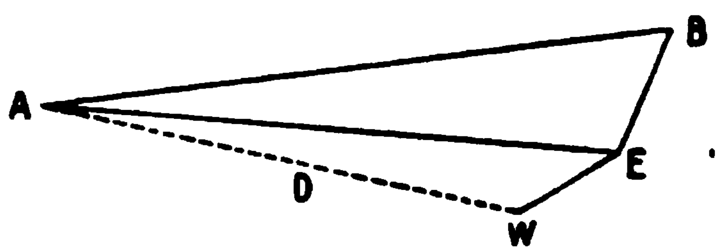


Fig. 231.

and W E = 1137·2.

A E : E W :: sin W : sin A

$\therefore$ A E = E W sin W cosec A.

log E W = 3·055837
log sin W = 9·887406
log cosec A = 10·946141

log 3·889384

A E = 7751·5 feet.

A B : A E :: sin E : sin B

In Δ A B E—

A	30°	40′
B	61	20
E	88	00
	180	00

$\therefore$ A B = A E, sin E, cosec B
log A E = 3·889384
log sin E = 9·999735
log cosec B = 10·056790

log 3·945909 = 8829 feet = A B.

If 8829 feet = 6·4 inches, how many inches = 1 mile = 6070 feet?

$$8829 : 6070 :: 6{\cdot}4 : x. \qquad x = \frac{6070 \times 6{\cdot}4}{8829}$$

$$\begin{aligned} \log 6070 &= 3{\cdot}783189 \\ \log 6{\cdot}4 &= \underline{\ \ {\cdot}806180} \\ &\ \ \ 4{\cdot}589369 \\ \log 8829 &= \underline{3{\cdot}945909} \\ &\ \ \ {\cdot}643460 = 4{\cdot}4 \text{ inches} = \text{correct scale.} \end{aligned}$$

To complete the work, the following soundings were taken the next day:—

VIII. 00 A.M.

50 yards off F ϕ C 5
14
(1) X 47° 05′ F 56° 50′ 5
(2) X 22 40 F 87 05 D. . . . . 5_h
towards F
12
50 yards off F 5
(3) 50 yards off F ϕ X 5
10
(4) X 49° 50′ C . . . . . . $5\frac{1}{4}$
111 40 D
$3\frac{1}{4}$ 3 $2\frac{1}{2}$
(5) X 37° 02′ C . . . . . . $3\frac{1}{2}$
105 54 D
4 $5\frac{1}{2}$ 9

IX. 00 A.M.

(6) X 56° 20′ C . . . . . . 10_h
124 30 D
$5\frac{1}{4}$ $3\frac{1}{2}$
(7) C 9° 00′ Inn. B . . . . . $1\frac{1}{4}$
C 89 35
(8) C 103 40 A 5° 00′ F . . . . $\frac{1}{4}r$
D 78 20
E 107 10
$\frac{3}{4}$ 1 3 $4\frac{1}{4}$ 5
(9) C 34° 35′ Inn. B 9
X 83 15
12 11 9
(10) C 75° 35′ D O E 10
(11) C 84 40 D O E . . . . . . 13_h
14 5 $4\frac{1}{4}$
(12) F 84° 30′ C 46° 30′ Inn. B. . . . 3
3 $3\frac{1}{2}$ $2\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$
(13) A 64° 30′ X . . . . . . $\frac{1}{4}$
6 20 F

X. 00 A.M.

(14) 50 yards off X, toward D . . . . $2\frac{1}{4}$

$3 \quad 3\frac{3}{4} \quad 4\frac{1}{2} \quad 2\frac{3}{4} \quad 1\frac{1}{2}$

(15) F 64° 00′ C 62° 00′ Inn. B . . . . $\frac{1}{4}_r$

$\frac{1}{4} \quad 1 \quad 3 \quad 4\frac{1}{4} \quad 5$

(16) C 94° 30′ D O E . . . . . . 14_h

$5 \quad 4$

(17) A 84° 20′ C 52° 30′ Inn. B . . . . $3\frac{1}{2}$

$2 \quad 1 \quad 2\frac{1}{2}$

(18) F 108° 05′ C 59° 00 Inn. B . . . $3\frac{1}{2}$

$3\frac{1}{2} \quad 2\frac{1}{2}$

(19) E 31° 10′ D 80° 20′ A 1

$2\frac{1}{2} \quad 3\frac{1}{2} \quad 5$

(20) E 31° 00′ D 105° 00′ A 5

$5 \quad 3 \quad \frac{1}{2}$

(21) A 67° 05′ C 82° 20′ B. . . . . $\frac{1}{4}$

XI. 00 A.M.

(22) A 56° 20′ C 89° 30′ . . . . . $\frac{1}{4}_r$

$1 \quad 3 \quad 5$

(23) A 53° 40′ C 112° 00′ B . . . . 6_h

$6\frac{1}{2} \quad 5\frac{1}{2} \quad 3\frac{1}{2} \quad 2\frac{3}{4} \quad 1\frac{1}{2}$

(24) D 78° 10′ C O F 1

(25) D 70 20 C O F . . . . . . $\frac{1}{2}$

$1\frac{1}{2} \quad 2\frac{1}{2} \quad 3\frac{3}{4} \quad 5\frac{1}{2} \quad 7$

(26) A 39° 50′ C 125° 20′ B . . . . $6\frac{1}{2}_h$

$5 \quad 3 \quad 1$

(27) A 44° 30′ C 93° 20′ B. . . . . $\frac{1}{4}$

(28) A 38 00 C 89 20 B. . . . . $\frac{1}{4}_r$

$1\frac{1}{2} \quad 2\frac{1}{2} \quad 3\frac{3}{4} \quad 5 \quad 7$

(29) A 32° 00′ C 119° 40′ Inn. B . . . $\frac{1}{4}_r$

Noon.

(30) E 36° 200′ D 54° 20 A . . . . $\frac{1}{4}$

And so on. Lead line O.K.

The remainder of the sounding was finished in the afternoon, and the ship ran the three lines shown in dots. (See Plate XIII.)

The position of E was fixed astronomically with the artificial horizon, using ex-meridians of sun and of Pole Star for latitude, while the longitude was obtained by equal altitudes.

593. Example of Equal Altitudes taken in Different Days.—The first set of observations was taken on 19th September at about II. 30 P.M., and the corresponding altitudes were taken on 21st September at about IX. 30 A.M.

The latitude assumed was 54° 00′ N., and longitude 56° 50′ W.

Comparison of the watch P.M. 19th was $1^h\ 59^m\ 41^s{\cdot}9$ slow on chronometer.

Comparison of the watch A.M. 21st was 1^h 59^m $32^s{\cdot}8$ slow on chronometer.

Error of the chronometer on G.M.T., 8^h 06^m $45^s{\cdot}9$ fast.

Sept. 21st, 9^h 30^m A.M. is
Sept. 20th, 21 30 is
Sept. 19th, 45 30 And, since the first sights were taken on
Sept. 19th, 2 30

43 hours is, roughly, the time that has elapsed.

The following were the observations taken :—

	Time by Watch, A.M.	Time by Watch, P.M.	Sum of Secs.
	7^h 20^m $28^s{\cdot}5$	0^h 22^m $07^s{\cdot}2$	35·7
	0 21 16·5	0 21 23·5	40
	0 22 02	0 20 38·0	40
	0 22 45·8	0 19 53·8	39·6
	0 23 32·5	0 19 07	39·5
	0 24 15·5	0 18 21·2	36·7
	0 25 00·5	0 17 36	36·5
	0 18 11·3	0 69 06·7	
mean	7 22 45·9	0 19 52·4	
A.M. comp. / P.M.	1 59 32·8	1 59 43·3	
chron. time	9 22 18·7	2 19 35·7	

	h	m	s
19th Sept., chron. time, P.M.	2^h	19^m	$35^s{\cdot}7$
19th Sept., chron. time, A.M.	45	22	18·7
sum	47	11	53·4
diff. (el. time)	43	02	42·0
½ sum (mid. time chron.)	23	50	56·7
½ diff. (½ el. time)	21	31	21

App. noon 0° 00′ 20th Sept. dec. A.N. 1° 99′ 07″ N. ch. dec. 58·3
long. W. 3 47 − 0 3 41 3·8

App. G.A.T. 3 47 1 05 26 60|221·54
P.D. 88 54 34 3·41

eq. of time − 6 31·17 ch. ·88 increasing. 58·31
+ 3·25 3·7 21·52 (½ el. time).

6 34·4 3·256 15|1249·8312
83·322

$$\text{'Eqn. of equal alt.'} = d \cdot \frac{p}{2} \cdot (\tan 1 \cdot \overset{-}{\operatorname{cosec} h} - \overset{+}{\cot p} \cdot \overset{-}{\cot h})$$

The proper signs are affixed to p and h as above (see Appendix III.).

log tan lat.	10·139005	log tan dec.	8 280020
log cosec $21^h\,31^m\,21^s$ (sec 3 31 21)	10·218907	log cot $21^h\,31^m\,21^s$ (tan 3 31 21)	10·120325
log 83·322	1·920749	log 83·322	1·920749
log	2·278661	log	·221094
	190·00		1·66

- 190·00
- 1·66

60 | 188·34

− 3 08·3

Mid. T. by chron.	$23^h\,50^m\,56^s{\cdot}7$	app. noon	$24^h\,00^m\,00^s$
eq. of equal alt. −	3 08·3	eq. time −	6 34·4
chron. T.A.N.	23 47 48·4	M.T.A.N.	23 53 25·6 (19th)
		G.M.T.A.N.	3 41 02·5 (20th)
error chron. fast	8 06 45·9	long.	3 47 36·9 W. = 56° 54′ 13″.
G.M.T.A.N.	15 41 02·5		
	3 41 02·5 (20th)		

Suppose that during the long interval between P.M. and A.M. sight the chronometer has changed its rate ± 2 seconds, then the middle time by chronometer is ± 1 second in error; and the longitude is therefore ± 15″ in error.

To Find the Latitude.

594. Ex-meridian by Sun Observations.—By ex-meridian of the sun taken with the artificial horizon the following observations were taken $\underline{\underline{\odot}}$. Lat. 54° 00′ N.; long. 56° 54′ W.

watch time	alt.	
$9^h\,43^m\,42^s$	73° 43′ 10″	I.E. − 2′ 35″
46 50	43 30	
48 10	44 00	
50 33	44 30	
52 18	43 50	
53 41	43 40	
	mean 73 43 47	

Comparison of watch, $1^h\,59^m\,39^s{\cdot}5$ slow on chronometer.
Error of chronometer, $8^h\,06^m\,45^s{\cdot}9$ fast on Greenwich.

	h	m	s		
app. noon	0	00	00·0	A.N.	0^h 00^m
eq. time –		6	34·4	long.	3 47
M.T.A.N.	11	53	25·6	G.T.	3 47 apparent.
long.	3	47	37·0 W.		
G.M.T.A.N.	15	41	02·6		
error chron. fast	8	06	45·9		
chron. T.A.N.	23	47	48·5		
comparison of W.	1	59	39·5		
watch T.A.N.	21	48	08		

dec. G.A.N. 1° 09′ 13″	ch. 58·3	eq. time 6^m 31^s·07	·88
–0 3 41	3·8	–0 3·34	3·8
1 05 32	60)221·54	6 34·4	3·344
	3·41		

Watch Times.	Watch T.A.N.	H.A.	Vers. H.A. / Sin 1″	(See *Inman's Tables.*)
9^h 43^m 42^s	9^h 48^m 08^s	4^m 26^s	38	
46 50		1 18	3	
48 10		0 02	0	
50 33		1 25	3	
52 18		4 10	34	
53 41		5 33	60	
			138	
		mean	23	

Delambre's formula for the reduction to the meridian is:

$$\text{Reduction (in seconds of arc)} = \frac{\cos \text{lat.} \cdot \cos \text{dec.}}{\sin \text{Z.D.}} \cdot \frac{\text{vers. H.A.}}{\sin 1''}$$

The mean of the obs. alt.	73°	43′	47″ ⊙
I.E. –		2	35
	73	41	12
	36	50	56
refr. par.		1	10
	36	49	46
S.D.		15	57
true alt.	37	05	43
	90	0	0
Z.D.	52	54	17

log cos lat.	9·769045
log cos dec.	9·999921
log cosec Z.D.	10·098200
log 23	1·361728 $\left(\text{the mean value of } \dfrac{\text{vers. H.A.}}{\sin 1''}\right)$
log	1·228894 = 16·94 (17 nearly).

The true Z.D.	52°	54′	17″	
reduction to meridian –		0	17	or + to altitude.
mer. Z.D.	52	54	00 N.	
declination	1	05	32 N.	
latitude	53	59	32 N.	

Probable error ± 1′.

595. Ex-meridian of Polaris.—Also the following ex-meridian of Polaris was observed in the artificial horizon, 19th September 1908, long. 56° 54′ W.

Time by watch, $11^h\ 20^m\ 10^s$. Alt., 110° 31′ 00″ I.E., – 2′ 35″

M.T. mer. passage = R.A. (Ursa Minoris) – R.A.M.S. (sid. time)
= $1^h\ 26^m - 11^h\ 52^m$ (see *Nautical Almanac*) = $13^h\ 34^m$
long. 3 47 W.
G.M.T. 17 21

Time by watch	11^h	20^m	10^s
comparison	1	59	39
chron. time	13	19	49
error chron.	8	06	46
G.M.T.	5	13	03
	12	00	00
19th Sept.	17	13	03
long.	3	47	37 W.
mean time	13	25	26

Sid. time G.M.N.	11^h	51^m	57^s
corr. for G.M.T.	0	2	49
mean time	13	25	26
	1	20	26 (sid. time of obs.).

Observed alt.	110°	31′	00″
I.E. –		2	35
2 |	110	28	25
	55	14	12
refr. –		00	41
	55	13	31
subtract		01	00
reduced alt.	55	12	31

1st correction	− 1	10	58	(see *Nautical Almanac*, Table 1, p. 54).
	54	01	33	
2nd „	+	00	1	
	54	01	34	
3rd „	+	00	57	
	54	02	31	latitude. Probable error, ± 50″.

The lat. by sun ex-mer. was 53° 59′ 54″
„ „ Polaris „ 54 02 31

accepted mean 54 01 12 N.; or giving 6 values to Polaris and 5 to sun, according to the probable error in results, gives a latitude = 54° 01′ 19″.

Wharton does not recommend meaning sun with star observations, on account of the difference in instrumental error that will be due to temperature, and also on account of the difference in refraction.

These are, however, given here as examples of the methods of finding a latitude, and a probable value has been assigned to each.

Survey by a Number of Torpedo Boats of a River Entrance.—A torpedo boat flotilla is required to sound out the Ray Sand and Whitaker Channels, and the Swin Spitway, in the Thames Estuary (fig. 231*a*).

The work is to be on a scale of 1·35 inches = 1 mile.

Take Tillingham 4 inches from the bottom of the paper, and 2 inches from the left edge.

Draw the true meridian through 1° of longitude E.

On 18th March 1908.

The boats are distinguished in their first positions as A, B, C, D, and E.

A is sent to Holliwell Point; erects a mark named Hole, on the sea-wall, using three long ship's boat-hooks, surmounted at the top, as high up as possible, with a whitewashed deck-cloth—the mark being well stayed.

B moors close to the West Buxey Buoy.

C moors close to the Buxey Beacon, lashes a tide-pole to the beacon, and sends in a boat to erect a mark at the old buildings of Tillingham Coast Guard, and whitewashes a large patch on the building.

D took up a position to the northward of the Ray Sand Channel,

where B and C are in transit, and as near as possible to the 3-fathom line off Bachelor Spit.

E moored close to South Buxey Buoy.

When all the boats were in position they hoisted the letter T flag.

The following is the approximate position of each mark, and of each boat:—

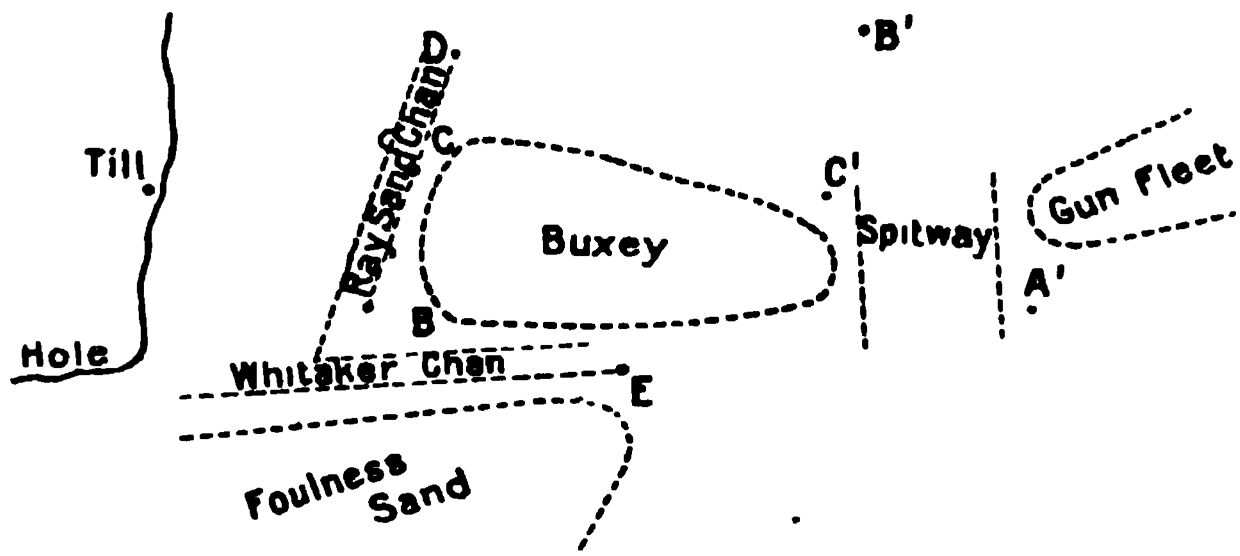


FIG. 231A.

The dotted lines show these lines of soundings given in this example. In actual practice a greater number would have been run.

First, it is required to plot the various marks and positions.

Torpedo boat F ran a patent log backwards and forwards from B to D, repeated twice, and found the distance = 3·4 miles.

Till. to E is the side adopted to plot from.

The following angles were taken:—

At Holl.

B 9° 55′ E
C 24 40
D 30 50
Till. 66 20

At B (West Buxey Buoy). R.W.H.S.

Till. 96° 10′ C ϕ D 57° 30′ E
Holl. 44 50 Till.

At D.

Holl. 16° 40′ Till.
C ϕ B 35 30
E 97 20
N. Buxey Buoy 36 00 E

At E (South Buxey Buoy). R.

Holl. 8° 45′ B
19 50 Till.
41 20 C
69 00 D
D 119 40 Whitaker Spit Buoy.

At Till.

D 15° 20′ C
34 00 E
49 10 B
127 20 Holl.

Lat. 51° 39′ 57″ N., long. 0° 56′ E. Dec. 0° 55′ S.

App. T. VII. 00 A.M. $\underline{\odot}$ 75° 24′ Holl. S.D. 16′ I.E. 0°.

From the above, pick out the angles of the triangles

T E H, T E D, H E D, T B D, B D E;

and adjust the angles of each triangle so that the sum of the angles shall = 180°.

Then plot the positions from the side T E.

Every angle must be laid off by chords, and the corrected triangles must be shown in the work sent in.

A and B were then supplied with a tracing of each position, and undertook the sounding. Soundings are in feet.

By A. VII. 00 A.M.

(1) Holl. ϕ Till. 51° 10′ B 38

38 ● 33 29 31 32 30

(2) Till. 69° 40′ B 18° 00′ E 33

40 37 31 35 34 33 34

(3) Till. 74° 40′ B 36° 10′ E 34

29 25 22 22 23 25 27

(4) Till. 60° 30′ B 66° 50′ E 29

29 ● ● ● ● 31 32

(5) Till. ϕ B 72° 20′ D

33 ● 34 35 36

(6) Holl. 17° 40′ B 80° 30′ D 37

D 79 40 E

This fix was at Ridge Buoy. R.W. Cheq.

The boat continued on this line until she arrived at E, then turned to port, and ran another line parallel with the first.

VII. 30 A.M.

(7) Alongside South Buxey Buoy, 1 cable N. of E. 30

34 36 38 40 41 42 *

(8) Holl. 10° 35′ B 71° 40′ D 43

Continuing along this line until she arrived at—

(9) Till. 78° 10′ B ϕ D 23

Turned to Starboard.

(10) Till. 90° 10′ B 42° 20′ E 25

(11) Abreast of B ϕ W. Buxey on starboard beam 25

(B now got under weigh.)

VIII. 00 A.M.

24 25 ● 26 22 ● ●

(12) Till. 119° 10′ D 71° 20′ E 22

22 ● ● ● ● 23 25

(13) Abreast of C ϕ E, dist. 1 cable . . . 24

And so on to the end of the line, to abreast of D, and back on another line parallel with the first and 2 cables from it.

At the West Buxey Buoy, turned to port, and ran a line to E, and went on to the Whitaker Spit Buoy (*see* further on) at X. 00 A.M.

Do not overcrowd the soundings on the paper; put down only such as will be distinctly legible.

Lead line in error by a negligible quantity.

VIII. 30 A.M.

B sounded a line to D, then proceeded to a position B′ in the Runch Shoal, where she moored in 25 feet, distant about 5 miles in a N.E. direction from D.

C left a dinghy at Buxey Beacon to watch the tide-pole, then she weighed.

IX. 00 A.M.

C at North Buxey Buoy (this must be fixed by calculated angles). Sounding, 48 feet.

E 94° 40′ Till. 40° 20′ D

IX. 30 A.M.

Then C took up a position C′, about 3½ miles in an E.N.E. direction from North Buxey, and moored there in 36 feet.

X. 00 A.M.

A proceeded to the Whitaker Spit Buoy. Sounding at buoy, 31 feet. B.W.H.S.

C′ 10° 50′ B′
E 107 10

(This buoy must be fixed by calculated angles.)

XI. 00 A.M.

A afterwards took up a position A′ at the Swin Spitway Buoy, about 5 miles in a N.E. direction of E. Sounding at buoy, 37 feet.

A′, B′, C′ *is an extended triangulation from* D *and* E, *for the purpose of sounding the Spitway. Every angle must be plotted by chords.*

At D.
B′ 21° 20′ C′
25 00 A′
74 15 E

At E.
C′ 21° 20′ A
B′ 28 50
D 100 45

At A′.
D 7° 00′ C′
58 00 B′
E 29 50 D

At B′.
E 34° 20′ D
46 50 C′
97 40 A′

At C′.
A′ 122° 40′ E 47° 20′ D
D 112 20 B′
B′ 77 40 A′

And, having fixed positions A′, B′, C′, the Spitway Shoal was

sounded out. The necessary triangles required to plot A′, B′, C′ must be adjusted, and the adjustment shown (see p. 397).

At position D a true bearing is obtained of St Osyth Church, N. 18° 40′ E.

At position B′ a true bearing is obtained of St Osyth Church, N. 47° 30′ W.

St Osyth Church—Lat. 51° 47′ 52″ N. } from published chart.
Long. 1 04 45 E. }

These bearings connected the positions B′ and D with a determined object on the shore, were a check to them, and the amendments could be inserted on the published chart.

Tide-pole readings:—

		ft.	ins.		ft.	ins.	
H.W.	VII. 00	15	4	X. 30	7	10	
	VII. 30	15	2	XI. 00	6	4	
	VIII. 00	14	6	XI. 30	5	1	
	VIII. 30	13	7	Noon	4	2	
	IX. 00	12	4	XII. 30	3	6	
	IX. 30	10	10	I. 10	3	3	L.W.
	X. 00	9	4				

Rise of springs, 14 feet.

Before plotting, the following calculations are necessary:—

1. *To correct the various triangles required.*

Angles Observed.	Corrections.	Corrected Triangles.	Angles Observed.	Corrections.	Corrected Triangles.
Δ TED			Δ DTH		
T 34° 00′	− 6′	33° 54′	D 16° 40′	+ 5′	16° 45′
E 49 10	− 9	49 01	T 127 20	+15	127 35
D 97 20	−15	97 05	H 35 30	+10	35 40
180 30		180 00	179 30		180 00
Δ TEH			Δ DTB		
T 93° 20′	+15	93 35	D 35° 30′	−10	35 20
E 19 50	+ 3	19 53	T 49 10	−15	48 55
H 66 20	+12	66 32	B 96 10	−25	95 45
179 30		180 00	180 50		180 00
Δ TEB			Δ BDH		
T 15° 10′	+ 1	15 11	B 141° 00′	−22	140 38
E 11 05	+ 1	11 06	D 18 50	−11	18 39
B 153 40	+ 3	153 43	H 20 55	−12	20 43
179 55		180 00	180 45		180 00

2. To find the T.B. of long side.

log hav. H.A.	5^h	0^m	0^s	9·568894
log cos lat.	51	39	57	9·792557
log cos dec.	0	55	00	9·999944
	52	34	57	9·361395 = θ.

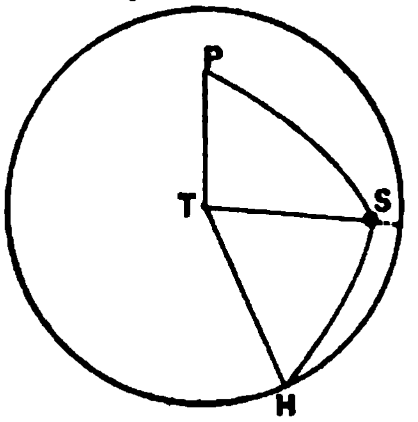


FIG. 281B.

nat. hav.	52° 34′ 55″	196195
„ „	θ	229819

log nat. hav. 426014 = 81° 29′ 24″ = T.S. = sun's Z.D.
90 00 00

8° 30′ 36″ = sun's altitude.

log sec. lat.	51°	39′	57″	10·207430
log sec. alt.	8	30	36	10·004804
	43	09	21	
P.D.	90	55	00	
log ½ hav.	134	04	21	4·964140
log ½ hav.	47	45	39	4·607280

log hav. 9·783658 = 102° 26′
Sun's T.B. = N. 102° 26′ E.

In Δ TSH, TS = 81° 29′ 24″.
TH = 90
HS = 75 24 (observed angle) + 16′ = 75° 40′.

Required the angle STH, the horizontal angle between the sun and the object.

$$\cos . STH = \frac{\cos HS}{\sin TS}$$

log cos 75° 40′ = 9·393685
log sin 81° 29′ 24″ = 9·995190

log cos 9·398495 = 75° 30′.

T.B. ☾,	N. 102° 26′ E.		
Horizontal angle to H =	75 30	to the right.	
T.B. of H =	N. 177 56 E.		
Angle H T E =	93 35	E being to the left of H.	
T.B. of E,	N. 84 21 E.		

3. *For the length of long side.*

The measured length of D B = 3·4 miles.

The scale being 1·35 inches = 1 mile, length of D B in inches of paper = 3·4 × 1·35.

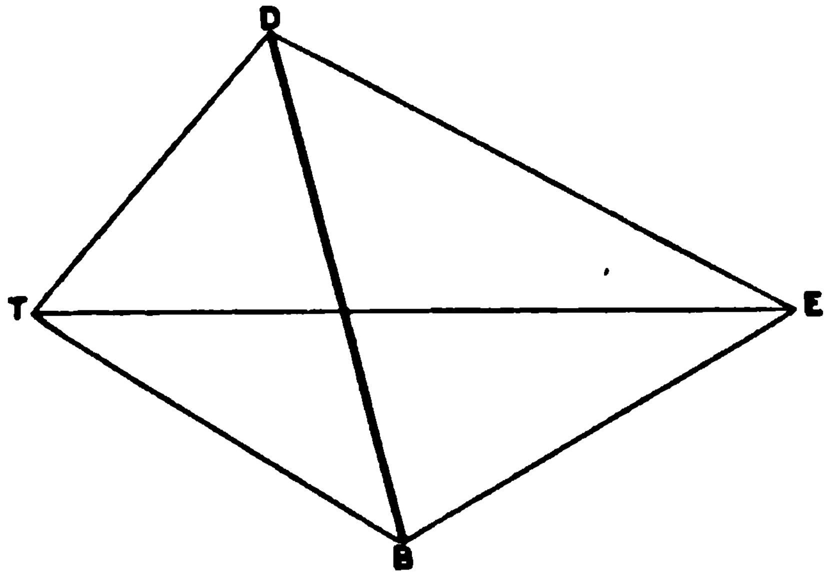


Fig. 231c.

Calculate the length of T E :—(*a*) through triangle D B T ; and (*b*) through Δ D E B.

(*a*)

E T = T D . sin 97·45 . cosec 49·01.

T D = B D . sin 95·45 . cosec 48·55.

∴ E T = 3·4 × 1·35 . sin 97·45 . sin 95·45 . cosec 49·01 . cosec 48·55.

log 3·4 =	·531479
log 1·35 =	·130334
log sin 97·45 =	9·996673
log sin 95·45 =	9·997809
log cosec 49·01 =	10·122110
log cosec 48·55 =	10·122770
	log ·901175 = 7·965 inches.

(*b*)

Angles Observed.			Corrections.	Corrected Triangle.	
Δ DEB					
D	61°	50′	+9	61°	59′
E	60	15	+9	60	24
B	57	30	+7	57	37
	179	35			

$$TE = DE \,.\, \sin 97{\cdot}05 \,.\, \operatorname{cosec} 33{\cdot}54.$$
$$DE = BD \,.\, \sin 57{\cdot}37 \,.\, \operatorname{cosec} 60{\cdot}24.$$
$$\therefore\ TE = 3{\cdot}4 + 1{\cdot}35 \,.\, \sin 97{\cdot}05 \,.\, \sin 57{\cdot}37 \,.\, \operatorname{cosec} 33{\cdot}54 \,.\, \operatorname{cosec} 60{\cdot}24.$$

$$\begin{aligned}
\log\ 3{\cdot}4 &= \ {\cdot}531479 \\
\log\ 1{\cdot}35 &= \ {\cdot}130334 \\
\log \sin\ 97{\cdot}05 &= 9{\cdot}996673 \\
\log \sin\ 57{\cdot}37 &= 9{\cdot}926591 \\
\log \operatorname{cosec}\ 33{\cdot}54 &= 10{\cdot}060733 \\
\log \operatorname{cosec}\ 60{\cdot}24 &= 10{\cdot}253564 \\
\hline
\log {\cdot}899374 &= 7{\cdot}932 \text{ inches.}
\end{aligned}$$

by (*a*) $TE = 7{\cdot}965$ inches.
„ (*b*) $TE = 7{\cdot}932$ „

mean 7·948 inches.

4. *For the triangulation extended to A′, D′, C′.*

Angles Observed.			Corrections.	Corrected Triangles.		Angles Observed.			Corrections.	Corrected Tringles.	
Δ DEB′						Δ DA′B′					
D	74°	15′	−12	74°	03′	D	25°	00′	− 6	24°	54′
E	71	55	−12	71	43	A′	58	00	−14	57	46
B′	34	20	− 6	34	14	B′	97	40	−20	97	20
	180	30		180	00		180	40		180	00
Δ DEA′						Δ EA′B′					
D	49°	15′	+ 3	49	18	E	28°	50′			
E	100	45	+ 5	100	50	A′	87	50			
A′	29	50	+ 2	29	52	B′	63	20			
	179	50		180	00		180	00			
Δ DEC′						Δ EB′C′					
D	52°	55′	+ 6	53	01	E	7°	30′	+ 4	7	34
E	99	25	+ 9	79	34	B′	12	30	+ 6	12	36
C′	47	20	+ 5	47	25	C′	159	40	+10	159	50
	179	40		180	00		179	40		180	00

The position of St Osyth must be plotted from Tillingham in its relative astronomical position.

lat. T	51° 39′ 57″	long. T	0° 56′ 00″ E.
lat. O	51 47 52	long O	1 04 45 E.
diff. lat.	7 55	diff. long	8 45 = 8′·75.
	= 7′·917		dep. = 8′·75 × 1·35 × cos 51° 44′

scale, 1·35 inches log 8·75 = ·942008

log 1·35 = ·130334

diff. lat. in inches, 10·6879 cos 51° 44′ = 9·791917

log ·864259

= 7·316 inches (dep.)

$$\frac{\text{dep.}}{\text{diff. lat.}} = \text{tan bearing.}$$

log dep. 7·316 = ·864274

log diff. lat. 10·688 = 1·028897

log tan 9·835377

= 34° 23′ (T.B. of O from T, N. 34° 23′ E.)

dist. = diff. lat. sec. *bearing*.

log. diff. lat. = 1·028897

log sec bearing = 10·083443

1·112340 = 12·952 inches.

From this position of St Osyth, the true bearings taken at D and B′ are projected: the lines so projected should go through D and B′.

If there is an appreciable error, reverse the process; that is, lay off the true bearings from D and B′: at the intersection of these lines will be the position of St Osyth by triangulation.

Compared with the astronomical position, the bearing from T will be so much in error in bearing and so much in distance; and these errors must be adjusted by a constant error in the bearings of each position from T, and a proportional error in distance. (See Appendix V.)

596. Sextant Angle Survey with Notes.—1. It is intended to spend a couple of days in surveying an anchorage by sextant angles (Plate XIV.).

Steaming round the harbour, from a boat, a rough sketch is made—

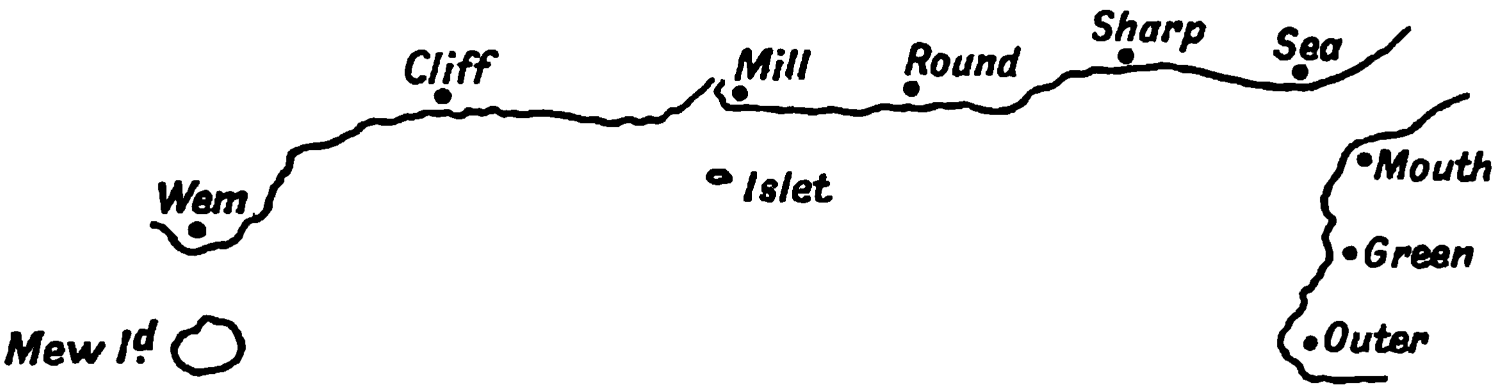


FIG. 232.

2. Whitewashes, or other marks, are put up in the positions indicated in the sketch; all being placed so as to be visible from Mew and Green, and, if possible, also from Mill.

3. It is decided to use Mew to Green as the plotting side; because from these two all the other marks are visible, and also because at these marks the 'receiving' angles are 'well conditioned.'

4. Observer 'A' lands at Mew with a landing compass, and makes the following sketch:—

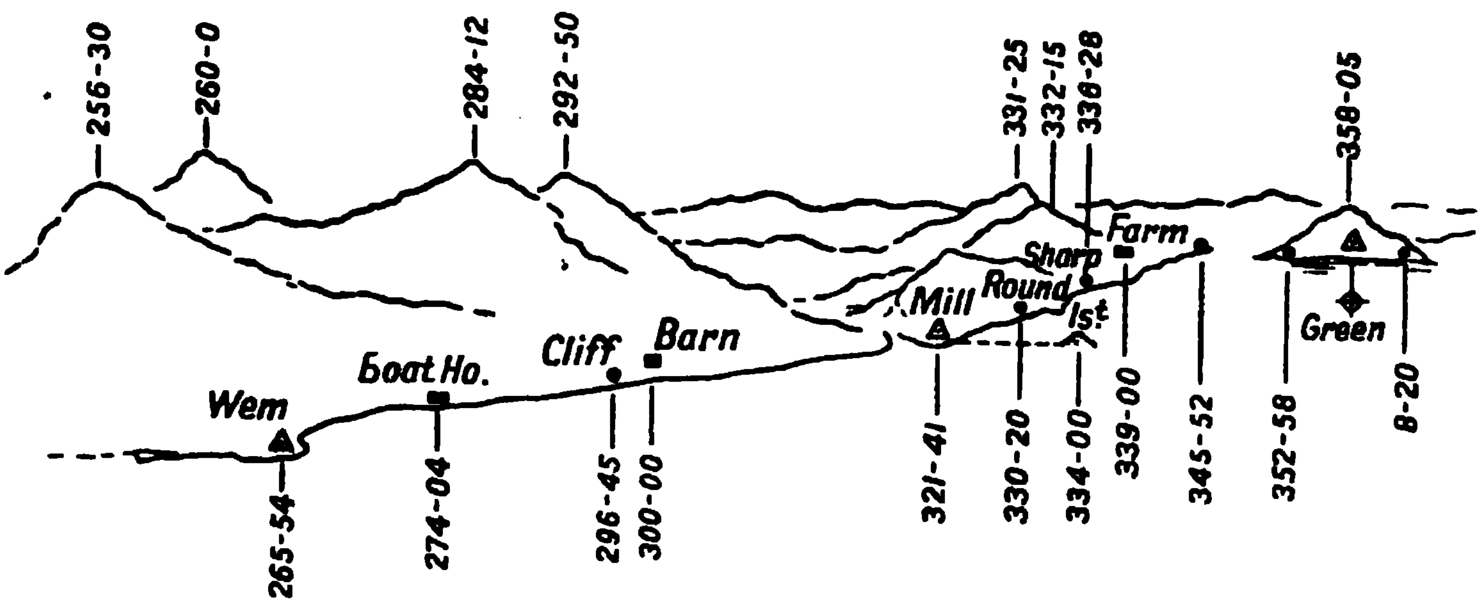


FIG. 233.

5. For true bearing; at Mew.

Time by watch, $9^h\ 46^m\ 10^s$; watch slow on chronometer, $2^h\ 55^m\ 16^s$.

Error of chronometer, $9^h\ 13^m\ 38^s$ slow on G.M.T.

Latitude, 50° 20′ N.; longitude, 4° 00′ W.; declination, 1° 30′ S.; Eq. T., $8^m\ 56^s$ + to M.T.

$\oplus$ Green 360° 0′ $\odot|$ 48° 57′ . . S.D. 16′

from which deduce the T.B. of Green.

Also the following bearings were taken with the compass.

|O	S. 23° 10′.
O|	S. 22 40 E.
Green	S. 71 10 E.
Mill	N. 70 40 E.
Wem	N. 15 00 E.

From these observations find the variation.

6. 'A' then proceeded to Green △ by steamboat, towing a patent log; and found the distance between Mew and Green to be about 1 mile.

7. The scale adopted is intended to be 5 inches = 1 mile = 6083 feet.

8. 'A' then made the following sketch from Green :—

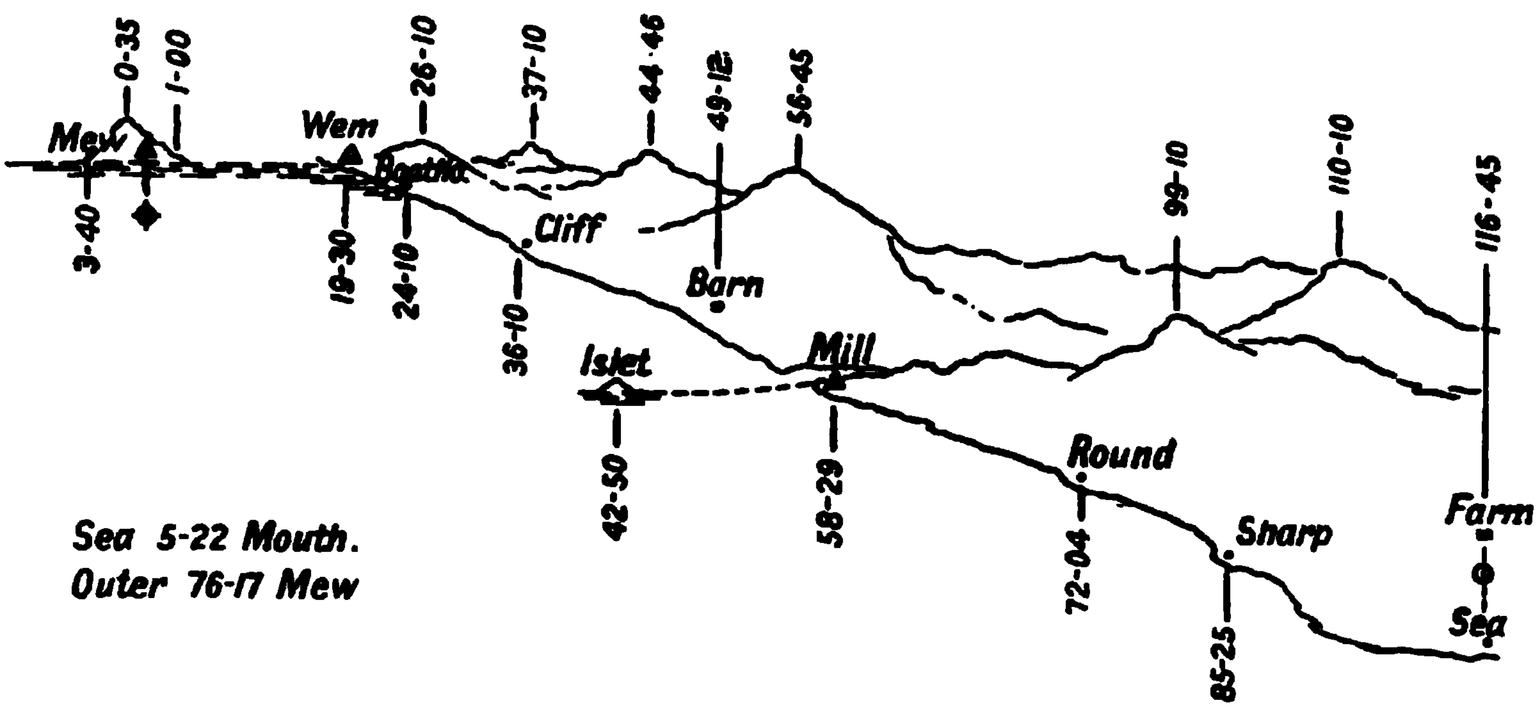


FIG. 234.

9. Observer 'B,' after putting up the marks on the coast, took angles at Mill △ :—

Islet	46°	54′	Mew	24°	58′	Wem.
Outer	75	00	Mew	38	10	Cliff.
Green	83	12	Mew	70	20	Barn.
Summit Green	94	00				
Mouth	98	10				
Sea	117	35	Mew	2	30	Mew Summit.
⊢ Mew	1	30	Mew	4	30	⊣ Mew.
Sharp	38	28	Green.			
Round	42	48				

10. There is a reef, dry at L.W., connecting Islet with the coast off Mill △.

'B' measured by chain the distance from Mill △ to Islet and found it 1224 feet.

At Islet, Mill △ 128° 3′ Green.
Sea 40 15 Green.

11. After the points have been plotted on the assumed scale, from the above measurement deduce the true scale of the plan.

12. 'B' erected a tide-pole at Islet—

(*a*) and it was found that the estimated H.W. spring mark on the Islet was level with the 16 feet 6 inches mark on the pole;

(*b*) the reading at H.W. on the pole was 13 feet 2 inches,
" " L.W. " " 4 " 4 "

from which deduce a datum mark on the pole corresponding with L.W.S.

A datum mark $\overline{\wedge}$ is cut on the rocks 4 feet 10 inches above H.W. as a future reference, but representing 6 inches only above H.W.S., and this was the eventual height of springs decided on.

13. 'C' did the sounding:—

	VIII. 30 A.M.							
Fix (1)	Off Mew 100 yards						. . . . .	$2\frac{3}{4}$
	$1\frac{1}{4}$	$2\frac{1}{4}$	×					
(2)	Barn	46°	20′	Islet	24°	00′	Green	2
	$1\frac{1}{4}$	$1\frac{3}{4}$	1					
(3)		57	00		16	20	. . .	1_r
(4)		68	50		14	40	. . .	1_r
	$1\frac{3}{4}$	$3\frac{1}{4}$	$4\frac{1}{4}$					
(5)	Barn	58	00	Islet	26	50	Green	$4\frac{1}{4}$
	$4\frac{3}{4}$	4	3					
	IX. 00 A.M.							
(6)	Off Mew 100 yards						. . . . .	$2\frac{3}{4}$
	$3\frac{1}{2}$	$5\frac{3}{4}$	$6\frac{1}{2}$					
(7)	Barn	60	10	Islet	39	30	Green	9
	$5\frac{3}{4}$	5	$3\frac{1}{4}$					
(8)		97	50		21	00	Outer	1_r
(9)	Mill	50	30		14	20	. . .	1_r
	IX. 30 A.M.							
	$4\frac{1}{2}$	$5\frac{3}{4}$	6					
(10)	Barn	52	10		59	40	Green	$6\frac{1}{4}$
	7	$8\frac{1}{4}$	$7\frac{3}{4}$					
(11)		41	30		46	30	. . .	$6\frac{1}{4}$
	$9\frac{1}{4}$	$8\frac{1}{4}$	$7\frac{1}{4}$					
(12)		23	00		98	30	. . .	$8\frac{1}{4}$
	$6\frac{1}{4}$	$4\frac{1}{4}$	5					
(13)	Off Islet 100 yards						. . . . .	$3\frac{1}{4}$
	X. 00 A.M.							
	3	$3\frac{3}{4}$						
(14)	Green	85	50	Islet	69	30	Barn	$2\frac{1}{2}$
	$5\frac{1}{2}$	$6\frac{1}{4}$	5					
(15)	Mew	59	30	Islet ϕ Barn			. . .	$5\frac{1}{4}$
	$4\frac{1}{4}$							

(16) Off Green 100 yards . . . . . . $1\frac{1}{2}$

$1\frac{1}{2}$

(17) Off Mouth 100 yards . . . . . . $1\frac{1}{2}$

$1\frac{1}{2}$ $4\frac{1}{4}$

(18) Mew 20 20 Islet 102 40 Farm . $5\frac{3}{4}$

$5\frac{3}{4}$ 5 4

(19) Green 73 40 8 50 Wem . 3

$2\frac{1}{2}$

(20) Off Sea Point 100 yards 2

$3\frac{1}{4}$ $4\frac{1}{4}$

(21) Off Mouth 100 yards. 2

Noon.—Returned on board.

Measured lead line, O.K.

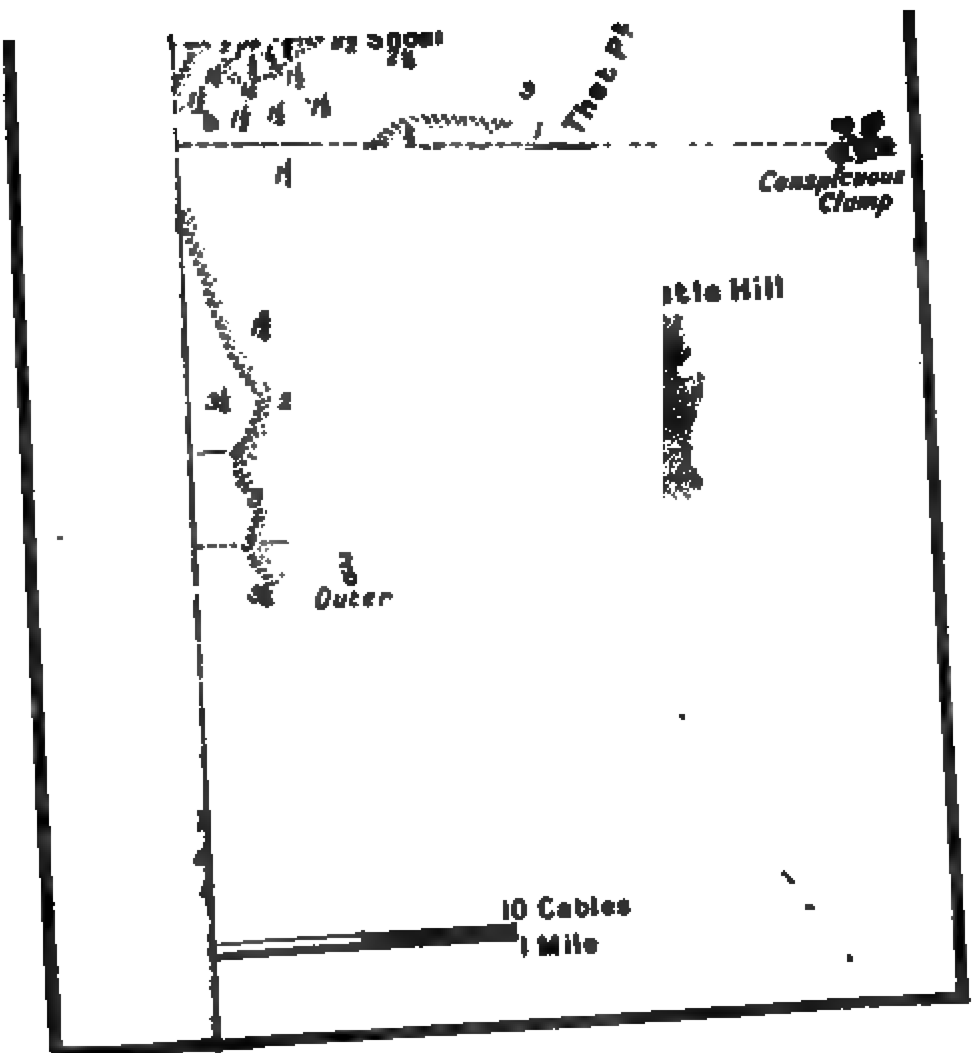
The soundings were resumed on the next occasion of visiting the anchorage.

14. Tide-pole register :—

	ft.	in.
VIII. 00	4	11
IX. 00	6	7
X. 00	8	10
XI. 00	11	11
Noon	12	9

15. 'B' put in the coast-line—

At (*a*)	Sea 7° 00′ Islet 81° 10′ Mew.
	Mew ϕ $\vdash$ Mew 24° 00′ Boat-house.
	7° 45′ St Mew.
	11° 30′ $\dashv$ Mew.
	Sandy beach to Wem. Cliff to Cliff $\odot$.
At (*b*)	Sea 27° 10′ Islet 90° 10′ Mew.
	Coast back to Cliff, cliffy—about 50 feet high.
	Coast on 67° 00′ Islet.
	Hard rock for 200 yards, then shingle.
At Mill	Mew 75° 30′. Coast back, shingle.
At (*c*)	Sea 90° 10′ Islet 29° 20′ Mew.
	Coast back to Mill, rocky.
At Round $\odot$	Coast back to (*c*), rocky.
	A stone 17° 45′ Sharp.
	blackwood 28° 10′ Sharp.
At Sharp $\odot$	A great many off-lying rocks off the coast, covering at H.W.
	Round 63° 30′ a stone.
	38 30 blackwood.
	Kelp 56° 00′ Sea.



Shoal
Conspicuous
Clump
Outer
10 Cables
1 Mile

At (*d*)	Green 70° 10′ Islet 8° 30′ Wem.
	Islet 99° 00′ Kelp.
	Coast back, rocky.
	Stick 49° 00′ Sea.
	Coast on, 200 yards shingle.
At Sea	Sharp 38° 30′ Stick.
	Coast back, sand, then gravel.

Sounded back to ship.

16. At ship, make a sketch, for topography, at 10 A.M.

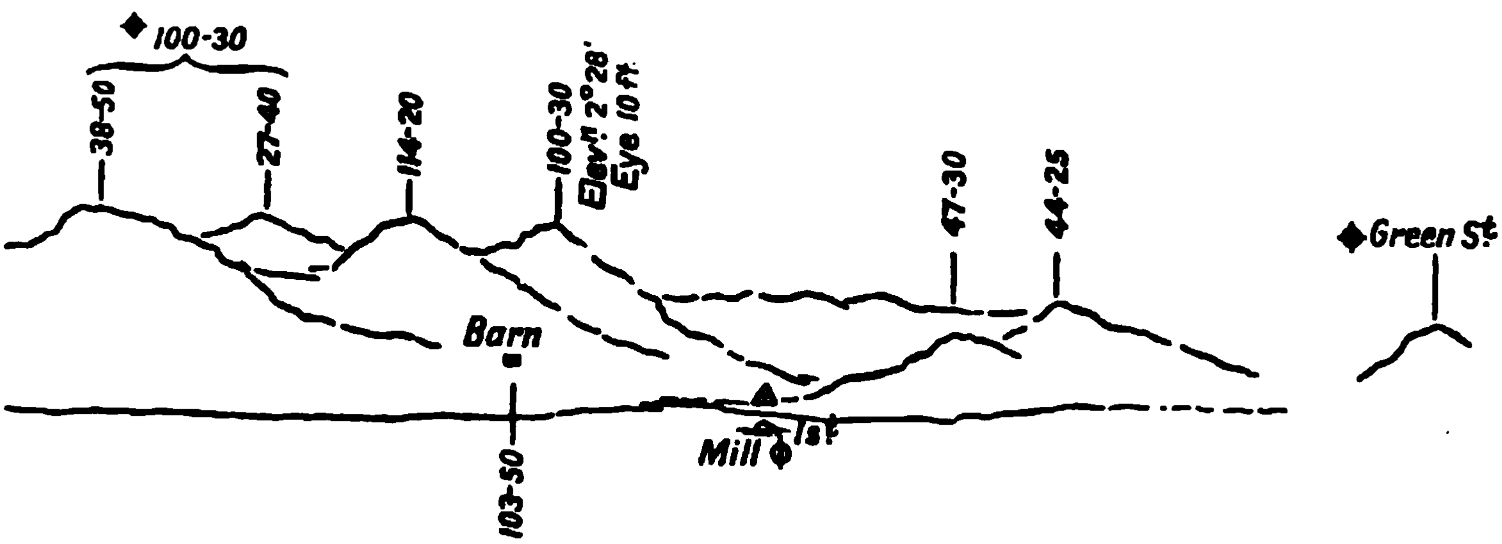


FIG. 235.

Fix at Ship, Mew 96° 00′ Mill ⌽ Islet 84° 00′ Green.

17. Sea, Point, and Mouth are at the entrance to a creek, shallow water stretching across.

Mouth round to Green and thence to Outer is a steep bank fronted by numerous ledges of rock, drying at L.W.

They extend ½ cable off the coast.

18. The low land to the west of Mill is cultivated.

The hills are wooded, and about 200 feet high, their height being found by elevations taken at the foot of the ship's gangway.

19. Draw the true and magnetic meridians through Mew, whence they were obtained.

20. State, in title, name of port; who did the survey; latitude and longitude of observation spot (50° 19′ 25″ N., 4° 06′ 20″ W.); the assumed rise of springs; the position of the datum marks and what it represents; state soundings in feet or fathoms, and the date; and scale.

Show by a pencil line where you think additional lines of soundings should be run.

597. Plan of a Harbour by Theodolite Angles.—It is intended to make a plan of a harbour in which the ship would be detained for a few days (Plate XV.).

A general sketch is made from a steamboat.

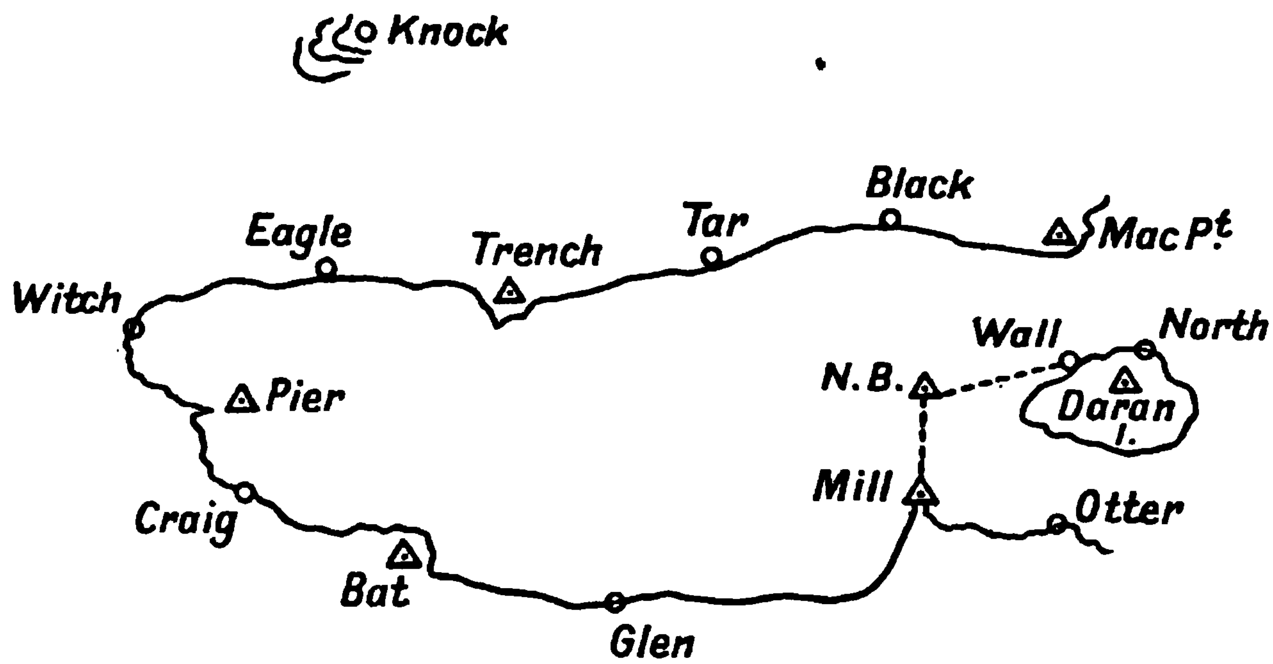


FIG. 236.

By patent log, towed by the steamboat, from abreast of Bat △ to abreast of Daran △ the distance found is roughly 1·6 miles.

Principal stations (△) are erected as shown in sketch; and later, an assistant went round the coast, and put up subsidiary marks (⊙) as shown.

It is proposed to use Bat to Daran as the plotting side, from which the other main stations are plotted in the following order:—

(1) Trench, (2) Mac, (3) Mill, (4) Pier, (5) Knock.

The scale adopted is to be about 4·5 inches = 1 mile = 6070 feet.

From Bat △, Daran △ bears east by compass—the approximate variation is 21° W.

At Bat △ the following sketch was made, and angles taken:—

⊕ Daran	360°	00′	φ N. Base.
Mill △	12	50	
Glen ⊙	43	10	
Craig ⊙	236	40	
Pier △	255	38	
φ Witch ⊙			
Eagle ⊙	284	06	
Knock Summit . .	289	17	
Trench △	322	26	
Tar ⊙	336	30	
Mac Point △ . .	340	46	
Wall ⊙	358	52	
→ Daran φ North ⊙ .	357	30	
Highest Peak Daran .	8	14	Elevation 1° 43′
→ Daran	11	40	

Z.O.K.

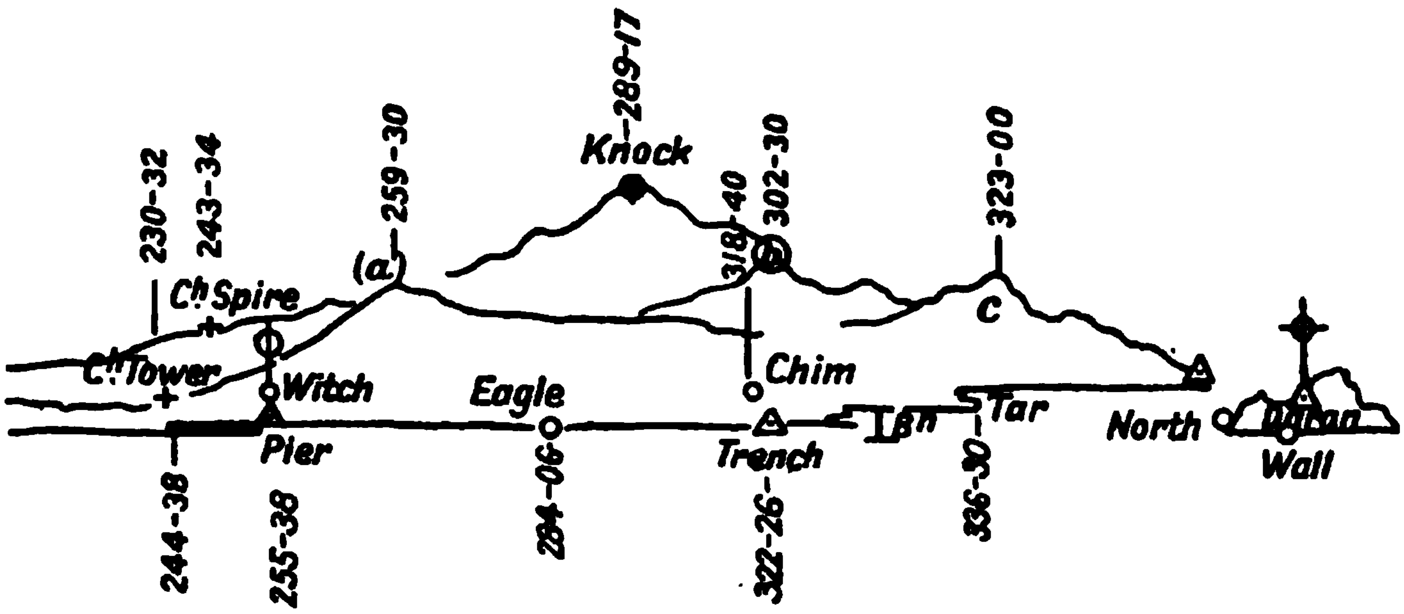


FIG. 237.

Taken with landing compass :—

Knock △ . .	N. 23° 40′ W.
Mac △ . . .	N. 28 10 E.
Glen ⊙ . . .	N. 71 05 E.
Craig ⊙ . . .	N. 76 05 W.

For true bearing ⊕ Knock, latitude 55° 20′ 10″ N., longitude 7° 10′ 12″ W. :—

	95° 37′	
	96 09	Dec. 18° N.
	95 40	Appt. T.P. VII. 12 A.M.
	96 06	

Draw the true and magnetic meridian through Bat △.

At Daran △ :—

⊕ Bat. ϕ N. Base dep. to W.L. (2° 44′). Error − 3′

Trench △ . . .	15° 00′
Pier △	17 02
Knock Summit . .	50 22
Mac Point △ . .	79 24
Mill △ . . .	347 04
Black ⊙ . . .	48 20

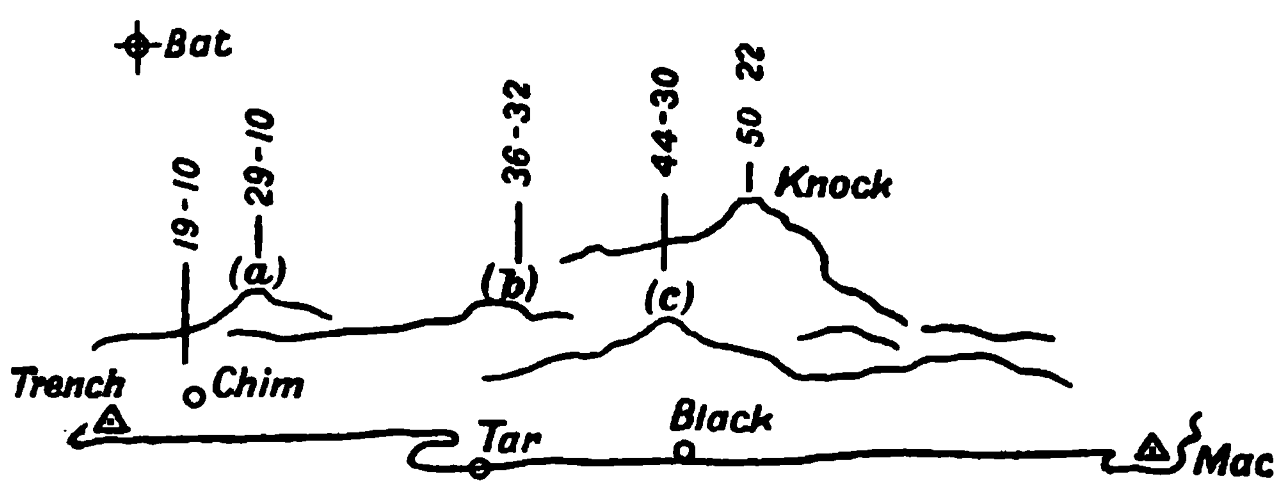


FIG. 238.

At Mac Point △ ⊕ Bat △ :—

Trench △ . . .	8° 36′		
Mill △ . . .	333 30		
N. Base . . .	339 43		
Summit (*d*) . . .	321 40	Elevation 3° 04′	Error + 2′
„ (*e*) . . .	354 18	„ 3 47	„ + 2
Highest Summit Daran	293 30	„ 3 27	„ + 1
North ⊙ . . .	272 00		
Wall ⊙ φ . . .	315 42		
Otter ⊙			
φ → Daran Island			
← Daran Island .	264 50		
Tar ⊙ . . .	6 09		
Black ⊙ . . .	20 20		
Daran △ . . .	278 38		

Z.O K.

At Trench △ ⊕ Daran △ :—

N. Base . . .	20° 30′	
Mill △ . . .	46 06	→ Daran 16° 00′ ← „ 355 30
Bar △ . . .	127 27	φ
Pier △ . . .	195 33	North ⊙
(*d*) Summit . .	69 00	
(*e*) „ . . .	127 30	

Z.O.K.

At Mill △ ⊕ Bat :—

Pier . . .	24° 02′
N. Base . . .	95 05
Chim	44 45
Knock Summit . .	66 04
← Daran . . .	142 30
φ Wall ⊙	
→ Daran . . .	178 00
Otter ⊙ . . .	196 35

Z.O.K.

A base is measured at L.W. on the dry shingle beach between N.B. and Mill △ = 1223 feet :—

In Δ B N M—

B = 12° 50′
N = 72 05
M = 95 05 } Find B N.

In Δ D N M—

D = 12° 56′
N = 107 55
M = 59 09 } Find N D.

B N + N D = length in feet of plotting side, from which deduce the true scale of plan.

At Pier △ ⊕ Daran △ :—

(d) Summit	48°	50′
(e) "	98	30
Base of Pier.		
ϕ Church (Tower)	140	00
Church (Spire)	221	30
(a) Summit	247	30
Knock Summit	290	24
(b) Summit	319	30
(c) "	334	20

Pier is 660 feet long, 60 feet wide.

Z.O.K.

Put up tide-pole, on north-west side of Pier end; at L.W.—reading 2 feet.

H.W. spring mark was, from local information, said to reach within 3 feet 6 inches of top of Pier, and this mark was level with the 11 feet 4 inches mark of the pole.

H.W. reading, 9 feet 10 inches.

29th.	ft.	in.	30th.	ft.	in.
10.0 A.M. L.W.	2	0	8.0 A.M.	6	2
11 0	3	6	9.0	4	11
Noon	5	2	10.0	3	2
1.0 P.M.	6	8	11.0	1	11
2.0	8	1	Noon	3	5
3.0	9	0	1.0 P.M.	5	1
4.0	9	9	2.0	6	7
5.0	9	6	3.0	8	0
6.0	8	4	4.0	8	11
			5.0	9	8

Deduce the datum reading, and hence the reductions for soundings.

Bench mark was cut on a building 5 feet above the level of the Pier.

To fix the position of secondary stations ⊙.

After they are erected, sextant angles are taken at them.

At Otter ⊙ :—

Black ⊙	23°	06′	Mac △ ϕ Wall ⊙ ϕ ← Daran I.
Tar ⊙	42	44	" " "
Trench △	72	36	" " "
Mac △ ϕ Wall	60	30	→ Daran (Hutch. Point).

At Glen ⊙ :—

Trench △	45°	18′	Mac △
Bat △	103	15	
Trend on 300 yds.	128	00	
Mac △	50	00	Trend back 400 yards.

At Craig ⊙ :—			
Trench △ ϕ Mac △	66°	10′	Bat △
Pier △ . .	77	35	Trench △
Glen ⊙ . .	6	58	Bat △
Base of Pier . .	18	25	Pier △
At Witch ⊙ Pier △ ϕ Bat △ :—			
Trench △ . .	39°	53′	Bat △
Bat △ . . .	9	13	Craig △
At Eagle ⊙ :—			
Trench △ . .	53°	52′	Bat △
Bat △ . . .	22	00	Craig ⊙
	58	02	Pier △
	96	35	Witch ⊙
	97	30	Spire △
At Tar ⊙ :—			
Mill △ . .	60°	30′	Bat △
Mac △ . .	109	02	Mill △
Wall ⊙ . .	53	30	,,
North ⊙ . .	75	50	,,
← Daran . .	76	00	
→ ,, . .	43	44	
Coast on to→Blk. Pt.	16	45	Mac △
Trench △ ϕ . .			Coast back 200 yards.
At Black ⊙ :—			
Mac △ . .	113°	06′	Mill △
Wall ⊙ . .	37	42	,,
North ⊙ . .	72	04	,,
→ Daran ϕ Wall ⊙			
← ,, . .	75	35	
Coast on 400 yds. .	30	00	Mac △
Mill △ . .	34	20	Tar ⊙

Coast-line.

The north coast of Daran Island is rocky, sand at L.W. drying out 50 yards.

The N.E. Point is rocky, with a rocky ledge drying at L.W. extending 100 yards north-east of it.

The west coast is rocky.

The highest point of the island is on the south side, and forms part of the ridge on which Daran △ is erected.

Between Wall ⊙ on Daran Island and N. Base there is a narrow wall of shingle and stones, awash at H.W.; this same wall continues from N. Base to Mill △.

At boat . . . N.B. ϕ Wall ⊙ 37° 30′ Mac △.
Hutch. Point 30 20 N.B. ϕ Wall.

Mill △ is at the extremity of a sharp point between it and Otter ⊙; the coast is rocky, fronted by stones drying at L.W.

The whole space enclosed by—

The W. side of Daran Island—the shingle wall joining Wall ⊙ and N. Base, the continued wall between N. Base and Mill △, and the coast between Mill △ and Otter ⊙—is a sand-flat dry at L.W. from 1 foot along the eastern edge to 3 feet in centre and 7 feet along the shingle wall.

This sand-flat connects Daran Island to the main at L.W.

From Mill △ to Glen ⊙ the coast is rocky, forming an indentation about 400 yards deep from the line joining the two marks; half-way from Mill △ the coast is fronted by sand and stones drying at 100 yards, thence to Glen ⊙ a dangerous reef extends out 150 yards.

From Glen ⊙ to Bat △ the coast is rocky, forming a continuation of the curve of the coast joining Mill and Glen.

There is a F.S. just at the back of Bat △.

Bat to Craig, and thence on to the base of Pier △, is a straight, rocky coast, fronted by sand and stones dry at L.W. 100 yards, the dry portion gradually extending to within 20 yards of the end of the Pier.

Between the Pier and Witch ⊙ there is a township, and thence on to Eagle ⊙ is a rocky coast, forming a small bight.

Trench △ is on the extremity of a narrow spit and joining Eagle ⊙ by a low rocky coast; here to the back there is cultivated land. The conspicuous chimney is on the east end of a large building.

To the east of Trench Point there is a sand-flat drying 1 foot—with a red beacon on east extremity; its position is

Wall 41° 45′ Mill 39° 38′ Bat △.

From Trench Point to the eastward, through Tar ⊙ and Black ⊙, the coast is low and rocky and fronted by dry sand and stones, and rocks in places.

From Black ⊙ to Mac △ there is a small bight, all fronted by reef.

Mac △ is on the south extremity of a narrow spit, and is fronted by three small rocks sonth-east of the point, the outermost being 80 yards out and 10 feet high.

Topography.

(*a*) Summit, (*b*) Summit, and (*c*) Summit are part of the same range, all covered with trees, and sloping to the coast.

Between (*b*) and (*c*) Summits there is a deep valley, from which the head of a stream runs to the sea at a point midway between Trench △ and Tar ⊙; it is called Barask Burn.

Knock △ is on the summit of a high peak, 704 feet, gradually sloping to the south-east and south-west, (*a*), (*b*), and (*c*) being connected to it.

(*d*) is a peak on the south shore, bare of trees, and with a very gradual slope towards Otter ⊙.

(*e*) is the highest peak visible, and is steep on the south side—leaving the land between it and Bat △ and Craig ⊙, a marsh with numerous streams running into the sea.

SOUNDINGS (IN FATHOMS).

I. 00 P.M. 29TH MARCH.

(1) North 97° 10′ Tar ϕ Trench 4
14 18 21
(2) E. ϕ N.B. 83° 15′ Mac 21
18 16 5½
(3) North ϕ Wall 77° 20′ Mac 1
1 s 1 s
(4) Wall ϕ E 98° 34′ 3
13 17 20
(5) N.B. ϕ Glen 118° 20′ 23
21 18 5
(6) Wall 69° 45′ Black . . . . . . ¾
(*d*) 61 03 Black
Is this a good fix?
1¼
(7) North 95° 50′ Bat ϕ Tar . . . . . 1 s
2½ 4¼ 13 18 21
(8) North 111° 30′ N.B. ϕ Glen 22
18 14 6¼
(9) Wall 107° 00′ Mac 78° 52′ North . . . ¾ s
Is this a good fix?
(10) Wall ϕ Mill 87° 35′ Black 2
11 18

II. 00 P.M. 29TH MARCH.

(11) Glen ϕ N.B. 82° 30′ 21
21 18
(12) Wall 76° 15′ Trench ϕ Tar 11
4½ 2½
(13) Wall 60° 00′ (*e*) ϕ Tar . . . . . ½ s st
(14) North 77° 35′ N.B. 35° 00′ Bat . . . 1¾
2¾ 4¼ 9¼
(15) North 56° 50′ Wall 55° 50′ N.B. . . . 18
20 17 12
(16) Otter ϕ Wall 55° 40′ N.B. 47° 10′ Tar . . 1¾ s
(17) Otter 61° 50′ N.B. 32° 40′ Trench . . 1 s st
12 13
(18) Otter 40° 20′ N.B. 48° 20′ Trench . . 10¼
5½ 3½ 2¾
(19) Wall 86° 10′ Bat ϕ Tar 1½

III. 00 P.M. 29TH MARCH.

(20) Wall 97° 20′ Bat ϕ Tar . . . . . $\frac{1}{2}$ s st
2½ 3¼ 4¼
(21) Otter 41° 00′ N.B. 58° 20′ Trench . . 10
12 12
(22) Otter 68° 10′ N.B. 41° 00′ Trench . . 2½
(23) Otter 72 30 N.B. 53 20 Trench . . ½
9½
(24) Otter 41° 20′ N.B. 73° 40′ Trench . . 12
3¾ 3 2¼
(25) N.B. 72° 00′ Trench ϕ Tar . . . . . ¾ s
(26) Otter 16° 10′ N.B. 88° 50′ Trench . . ¾ s st
Is this the best fix?
2¾ 2½ 4¼
(27) Otter 23° 10′ N.B. 102° 40′ Trench . . 8½
10 10
(28) Wall 105° 50′ Mill 96° 30′ 1½

IV. 00 P.M. 29TH MARCH.

(29) Wall 94° 10′ Mill 111° 20′ 4½
(30) Otter 48° 40′ Mill 85° 10′ Bat . . . 4½
7½
(31) Pier ϕ Trench 92° 0′ Tar 12
4½ 2¼
(32) North 57° 10′ N.B. 83° 00′ Bat . . . 1¼ s
(33) North 44 30 N.B. 92 00 Bat . . . 1¾
2¼ 4½ 11½
(34) North 68° 40′ Mill 101° 10′ 11½
10½ 10½ 9½
(35) Trench 99° 10′ Wall ϕ Mill 1¼
(36) Bat 102° 00′ N.B. 13° 30′ Mac . . . 1¾
8½ 8½ 10½
(37) Tar 64° 30′ N.B. 28° 10′ Otter . . . 10½
Is this fix good?
11½ 11½
(38) Mac 59° 50′ N.B. 72° 10′ Glen . . . 1

V. 00 P.M. 29TH MARCH.

(39) Mac 51° 20′ N.B. 69° 30′ Glen . . . 1
10½ 11½
(40) Mac 38° 50′ N.B. 107° 50′ 7¼
8½ 9½ 8½ 9½ 4¾
(41) Bat 101° 40′ N.B. 8° 30′ Mac . . . 1¾
(42) Pier 96° 30′ N.B. 19° 20′ Daran . . . 1¾
Is this fix good?
3 7½ 9½ 9½ 9½

(43) Mac 32° 00′ N.B. 102° 40′ Glen . . . $9\frac{1}{2}$
$10\frac{1}{2}$ $11\frac{1}{2}$ $10\frac{1}{2}$
(44) North 23° 40′ N.B. 112° 00′ Bat . . . $1\frac{1}{4}$
$10\frac{1}{2}$ $10\frac{1}{2}$
(45) Glen 72° 10′ Bat 52° 10′ Pier . . . $9\frac{1}{2}$
$9\frac{1}{2}$ $9\frac{1}{2}$
(46) Pier 47° 10′ Trench 51° 20′ Mac . . . $7\frac{1}{4}$
$8\frac{1}{2}$
(47) Pier 40° 00′ Trench 46° 30′ Mac . . . $1\frac{1}{4}$
2 2

VI. 00 P.M. 29TH MARCH.

(48) Pier 49° 30′ Trench 32° 40′ Mac . . . 2
7 $8\frac{1}{2}$ $9\frac{1}{2}$
(49) Pier 79° 20′ Trench 33° 40′ Mac . . . $9\frac{1}{2}$
9 10 9
(50) (*d*) 52° 00′ Bat 61° 40′ Pier . . . . $1\frac{1}{2}$
(51) (*d*) 44 50 Bat 66 00 Pier . . . . 1 s st
(52) (*d*) 41 10 Bat 63 30 Pier . . . . 1 s st
$4\frac{3}{4}$ 9 10
(53) (*d*) 58° 10′ Bat 92° 20′ Pier 9
9 9 $4\frac{1}{4}$
(54) Knock 23° 20′ Trench 56° 30′ Mill . . 1 s
(55) Knock 30 30 Trench 54 20 Mill . . 1 s
(56) Knock 34 40 Trench 50 20 Mill . . 1 s st
$6\frac{1}{4}$ 9 10
(57) Mill 73° 20′ Bat 106° 40′ Pier . . . 10
12 $6\frac{1}{2}$
(58) Trench ϕ Mill 57° 40′ Bat . . . . $1\frac{1}{2}$

Line measured and found correct.

VIII. 00 A.M. 30TH MARCH.

(59) Bat 62° 00′ Pier 38° 30′ Witch . . . 1
9 10
(60) Glen 69° 20′ Craig 75° 30′ Pier . . . 10
10 8 $5\frac{1}{2}$
(61) Pier 80° 20′ Trench 7° 20′ Mac . . . 2 *
1
(62) Pier 88° 30′ Trench 16° 10′ Wall . . . 1 s st
$1\frac{3}{4}$ 6
(63) Witch 81° 10′ Eagle 59° 00′ Trench . . 9
10 8
(64) Bat 49° 10′ Pier 59° 00′ Witch . . $\frac{1}{2}$ s
$1\frac{1}{4}$

IX. 00 A.M. 30TH MARCH.

(65) Trench 70° 40′ Pier 76° 50′ 1

6½ 2¾

(66) Witch 95° 40′ Eagle 37° 20′ Trench . . 4

(67) Witch 80 00 Eagle 30 50 Trench . . ½ s

Is this good?

1¾

(68) Eagle 70° 00′ Pier 43° 00′ Church Tower . ½

(69) Eagle 53 40 Pier 59 30 Church Tower . ½

2½ 7¾

(70) Bat 68° 30′ Church Tower ϕ Pier . . 9

6¾ 1½

(71) Church Tower 77° 00′ Pier 43° 30′ Eagle . ½ s

Where required, state whether fix is good or bad, and suggest, if necessary, any better objects.

Draw the 1, 3, 5, and 10 fathoms lines.

State where, in your opinion, any additional lines of soundings are required; show the directions these lines should take.

Give a clearing line for a ship drawing 22 feet, to avoid the shoal water off Tar ⊙; and state a danger angle for the same shoal water.

Position of ships at anchor:—

(*d*) 52° 00′ (*e*)
Knock 79° 20′ Chim 54° 20′ (*d*)
Is this fix good?

In a title, state:—

1. Approximately the rise of springs.
2. What is the datum for the heights shown.
3. The variation.
4. Latitude and longitude of observation spot.
5. Whether soundings in feet or fathoms: and state what steps you would take to leave a permanent record of the datum to which your soundings have been reduced.
6. Draw a scale for the measurement of—

   (*a*) latitude and distance;
   (*b*) longitude;

and state the natural scale of the chart.

N.B.—Attention is called to the angle of slope of the sea floor off Tar Point.

The soundings of 2¼ fathoms and 10 fathoms are ½ cable apart, *i.e.* 100 yards. The bottom then changes its depth 60 feet − 13 feet = 47 feet in a distance of 300 feet, and the angle of slope is about 9°. If the lines of soundings are 1 cable (200 yards) apart, a detached shoal would have to be of that length so that

any indications of it would be found; and it would suggest that lines 200 yards apart do not allow sufficient margin, and that they should not have been more than 100 yards, as is shown by the above.

MAIN TRIANGLES (CORRECTED).

(1) △ DBT		(2) △ DBM		(3) △ DBL (Mill).	
D	15° 00′	D	79° 24′	D	12° 56′
B	37 34	B	19 14	B	12 50
T	127 26	M	81 22	L	154 14

(4) △ BTM		(5) △ BLM		(6) △ BDK	
B	18° 20′	B	32° 04′	B	70° 23′
T	153 04	L	121 44	D	50 28
M	8 36	M	26 12	K	59 09

(7) △ BKL		(8) △ BKP		(9) △ KLP	
B	82° 56′	B	34° 02′	K	48° 16′
K	31 46	K	17 08	L	41 26
L	65 18	P	128 50	P	90 18

(10) △ BTL		(11) △ DPB	
B	50° 24′	D	17° 02′
T	91 21	P	58 36
L	38 15	B	104 22

and other necessary triangles.

CHAPTER XII.

MERIDIAN DISTANCES.

597*a*. Definition.—Meridian distance is the distance between two meridians measured on the equator, and is therefore difference of longitude; it is always given in hours and minutes: for instance, the difference of longitude being 30°, the meridian distance is exactly 2 hours; and this again means, that the time elapsed between the passage of the sun over two meridians being 2 hours, their difference of longitude is 30°.

598. Purpose of Meridian Distance.—The purpose for which difference of longitude is required has been explained in various places in this work: it is, combined with the difference of latitude, to eventually find the distance between two places by astronomical observations; for, given the difference of latitude, and meridian distance between any two points, the bearing and distance from each other can be calculated (see Appendix V., p. 453).

The purpose for which this bearing and distance is required is best explained in *River Survey*, p. 254; and for a full, comprehensive application of it, see Appendix V. Suffice it here to say that it is for the purpose of checking the bearing and distance which has been obtained by triangulation, between the same two points.

A meridian distance as a *difference* of longitude may be applied to one of the longitudes, and the other deduced.

First, the longitude of some principal observatory will be deduced from that of Greenwich; then other meridians be deduced from the principal one; and if such one is sufficiently well fixed, and is raised to the dignity of a secondary meridian, then a meridian distance can be obtained from it by observations.

599. In fig. 239, let X, Y, Z, represent the meridians of three places in their respective positions, as here shown, east and west of each other. Now suppose the sun to be on the meridian at X, then it is noon (apparent noon) there. Before reaching the meridian of X it has passed over that of Z, and therefore when it is noon at X, it is some time after noon at Z: suppose it is

$12^h\ 30^m$; and similarly, not having yet reached Y, it is not yet noon there—let it be $11^h\ 15^m$.

Following the same figure, since the meridian distance or difference of longitude is the time that has elapsed between the passage of the sun over two meridians, then the meridian distance between X and Z must be ½ hour, and between X and Y ¾ hour, and between Z and Y it is 1¼ hours.

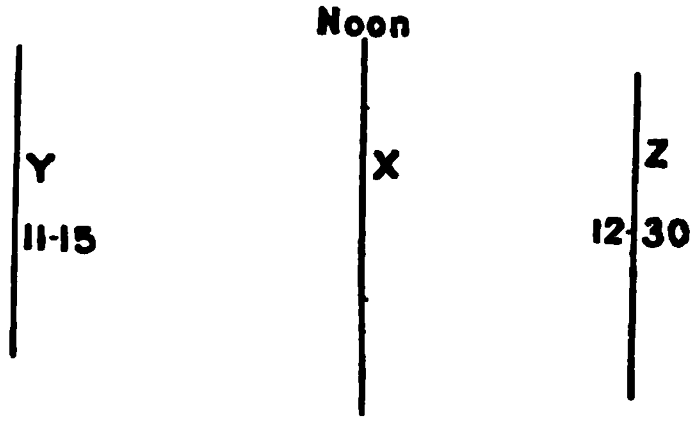


FIG. 239.

But to obtain this difference, we must know what the 'apparent' time is at Z when it is noon at X; or at Y when it is either noon at X, or $12^h\ 30^m$ at Z; or, generally, we must know at any particular instant what the apparent time is at X and Y, or Y and Z; or, which is the same thing, the mean time at any instant at any two places; and the difference between these mean times of place (M.T.P.) at the two places is the meridian distance, or difference of longitude.

So that a meridian distance is a difference of longitude. As a meridian distance, it is the difference in 'time' at any instant between the M.T.P. at two places (see fig. 239 above); it is $11^h\ 15^m$ at Y, and $12^h\ 30^m$ at Z; and the meridian distance between Y and Z is $1^h\ 15^m$; or, what comes to the same thing, it is the difference between the error of the same chronometer at two places at a given instant.

600. Position of Places relative to the Chronometer Meridian.—For, suppose a chronometer keeps exact local 'apparent' time—in other words, has a meridian of its own—its error is $0^h\ 0^m\ 0^s$; then there must be some meridian to which the chronometer belongs, some spot where the chronometer could be placed, and at noon there show $0^h\ 0^m\ 0^s$.

In fig. 240, let the wavy line represent the position of such a meridian. If the chronometer is suddenly moved to another place, and it is slow on M.T.A.N. there, the meridian of that place must be east of the chronometer meridian, because, as is shown in fig. 239, the time is later the further east the place is. Let X represent such a place; if, again, the chronometer is shifted to another place, and is fast on M.T.A.N. there, then

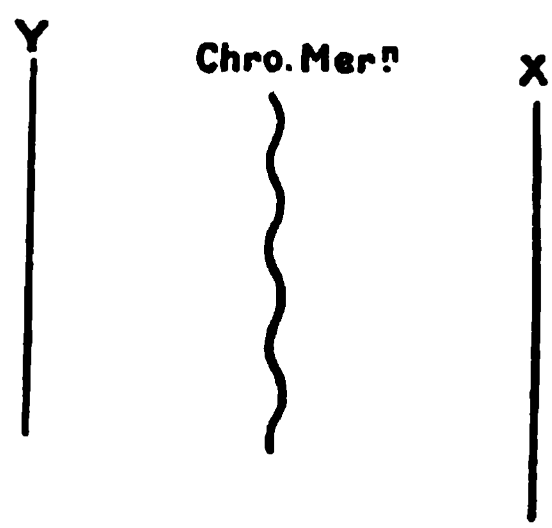


FIG. 240.

the meridian of that place is to the west of the chronometer meridian, as at Y; and the meridian distance between the two places is the difference between the error of the same chronometer on M.T.A.N. at the two places, where 'apparent noon' is the zero.

If it is slow on both places, the meridian distance is the arithmetical difference between the errors; if it is slow at one place and fast at another, as shown in fig. 240, it is the arithmetical sum of the errors, or algebraic difference; but the errors of chronometer are usually stated as always slow, or always fast, so that the meridian distance is the arithmetical difference between the errors of the same chronometer on 'apparent noon' or 'apparent midnight' at two places.

For time obtained by transit instruments, the surveyor had better refer to works specially and wholly devoted to the subject.

Now observations of equal altitudes taken with a sextant, give the error of the chronometer on M.T. apparent noon, or M.T. apparent midnight (see examples, pp. 441, 459), depending upon whether the sights are taken on each side of the 'superior' meridian (noon), or the 'inferior' meridian (midnight). One set of observations is by 'superiors,' the other by 'inferiors.'

Therefore either is the adopted instant or zero; and if the error of the chronometer is found at a place X, for instance, in the above figure, and if it were possible to have the same chronometer at the same moment at Y, then the difference between the errors of the same chronometer at the instant of noon at both places, if the chronometer had no rate, would give the meridian distance between the places: for suppose the chronometer at noon at X to show $0^h\ 0^m$; if it was instantaneously placed at Y, the sun would be another $\frac{3}{4}$ hour before it reaches noon there, and by that time the chronometer will appear to be $0^h\ 45^m$ slow (supposing it to have no rate); then the difference, viz. 45^m, is the meridian distance between X and Y.

In one case the error of the chronometer at noon was 0^h, in the other it was $0^h\ 45^m$ fast; hence the meridian distance would be the difference between the error of the one chronometer at two places, the zero for the error being M.T.A.N. (mean time apparent noon).

We cannot, however, have the chronometer at two places at the same time, unless we could transport it by telegraph.

601. But instead of transporting the chronometer by telegraph, why not transfer *its* time, by wire or wireless, to another chronometer; therefore, if there is another chronometer at Y, for instance, calling the one at X, A, and the one at Y, B, then A chronometer can be compared with B chronometer, and the comparison between them applied to B's time; that will practically make the observations taken with the two chronometers the same as if A had been

used simultaneously at both places; thus, if the comparison between the chronometers is:

by A,	11^h	52^m	$10^s{\cdot}2$
by B,	10	46	05·8
	1	06	04·4

then if $1^h\ 06^m\ 04^s{\cdot}4$ is added to B's observed time it will be the same as if A was used at X and at Y.

In transferring A's time we need an instantaneous means of communication between X and Y.

The following simple expedient was used for many years (see fig. 241). Let A chronometer be at the meridian of X, and B at the meridian of Y. The means here of communicating between them was through the medium of rockets fired from the ship

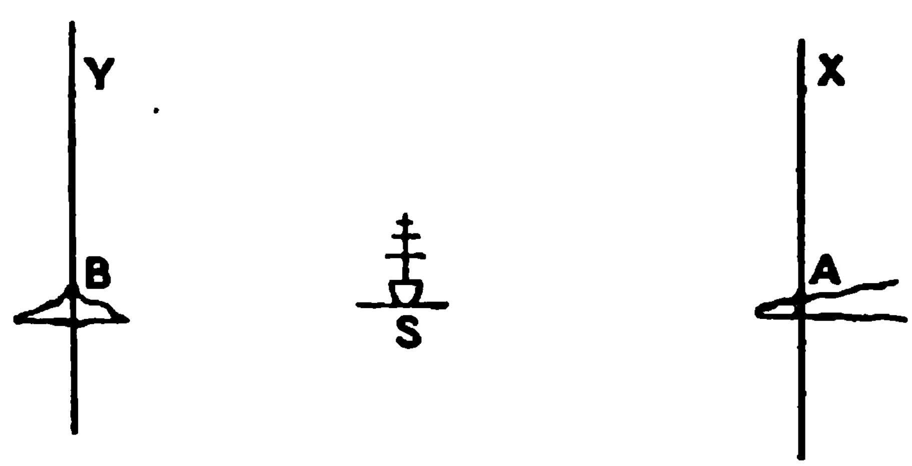


FIG. 241.

(daylight at noon rather reduces the efficacy of this), X and Y being out of sight of each other.

At the burst of the rockets, both observers at X and Y took time by their chronometers; this evidently gave a 'stop' for both chronometers and was a means of 'comparing' the error of B on A.

Now both X and Y observe 'equal altitudes'; resulting in the error of A chronometer on M.T.A.N. or Mid. at X, and B's error on M.T.A.N. or Mid. at Y.

If now the comparison between A and B chronometers be applied to B's error of chronometer, the result will be the error of A chronometer on both X and Y at M.T.A.N. or midnight; and the difference between these errors is the meridian distance between X and Y.

For example, suppose the error on M.T.A N. at X by A chronometer is $0^h\ 26^m\ 10^s{\cdot}7$ slow, and suppose that by B chronometer the error on M.T.A.N at Y is $1^h\ 15^m\ 47^s{\cdot}3$ slow; by applying the same comparison as above, B's time is $1^h\ 06^m\ 04^s{\cdot}4$ slow on A chronometer; therefore the error of A on Y meridian would be $1^h\ 15^m\ 47^s{\cdot}3$ (B's error) $-$ $1^h\ 06^m\ 04^s{\cdot}4$ (comparison) $= 0^h\ 09^m\ 42^s{\cdot}9$; and the meridian distance between X and Y would be the differ-

ence between A's error at X meridian ($0^h\ 25^m\ 10^s{\cdot}7$) and its error on Y meridian ($0^h\ 09^m\ 42^s{\cdot}9$) = $16^m\ 17^s{\cdot}8$.

602. Comparisons made at Time of Sights.—Now it will be noticed that since the sun did not arrive at Y till 16 minutes after X, it is possible that the observer at Y did not begin his observations till 16 minutes after A started his.

The comparison between the chronometers only holds good for the moment the comparisons were made; for suppose the rate of A chronometer to be gaining 12 seconds a day, and B to be losing 12 seconds a day, both extreme cases; the times by these chronometers are separating at the rate of 1 second an hour, and in 16 minutes they will have separated roughly ·3 of a second.

Then the comparison that will be required is at the time B was observing; and it should be found both immediately before his first sight was taken, and immediately after his last.

In the afternoon sights, the results of the difference of rates is still more marked; for the comparisons then will have changed 6 seconds from what they were in the forenoon; therefore, again, the comparisons must be made in the afternoon as well.

The comparison that may be made at noon is merely a check.

This is the case when A and B are two chronometers; but supposing them to be deck watches which have to be compared with a chronometer on board, the calculations become more complicated in order to bring the errors of the watches to that of the one chronometer.

In the case of a ship firing rockets, the distance between X and Y is limited to the distance the burst of the rocket is visible, and therefore the signal is made from a spot somewhere midway between them, to give effect to the greatest distance.

603. *Example.*—Two places X and Y are on the same parallel of latitude, 60° 30′ N. 1 mile = 6092·84 feet.

At X the error of watch A on M.T.P. is $5^h\ 02^m\ 55^s$ slow.

At Y on same date the error of watch B on M.T.P. is $7^h\ 02^m\ 51^s{\cdot}2$ slow.

By simultaneous signal from the ship at about noon M.T.:[1]—

Time by B, $5^h\ 02^m\ 25^s$.
" A, $7^h\ 03^m\ 23^s{\cdot}2$.

Allow personal error of receiving the signal in each case to be ·5 second. What is the distance in feet between X and Y, and what is their relative position?

Time by A, $7^h\ 03^m\ 23^s{\cdot}2$.
" B, 5 02 25

B 2 00 58·2 slow on A.

[1] It is only suggested here for the sake of the example that the comparison at noon is the same as the comparisons at A.M. and P.M. sights; for more detailed comparison see example, p. 460.

At Y, error of B on M.T.P. is $7^h\ 02^m\ 57^s{\cdot}2$ slow
error of B slow on A is 2 00 58·2

$\therefore$ error of A on M.T.P. at Y = 5 00 53·0
and observed error of A on M.T.P. at X = 5 02 55·0;

therefore meridian distance = 2 min. 02 sec.
To convert this into arc multiply by 15

diff. long. between X and Y = 30′ 30″ = 30·5 miles.

Dep. (or distance in this case, because X and Y are on the same parallel) = d. long. . cos latitude

departure or distance in miles = 30·5 . cos 60° 30′
distance in feet = 30·5 . cos 60 30 . 6092·84

log 30·5 = 1·482874
log cos 60° 30′ = 9·692339
log 6092·84 = 3·784760
57
3

4·960033 = 91,208 feet, probable error ± 200 feet.[1]
959995

38

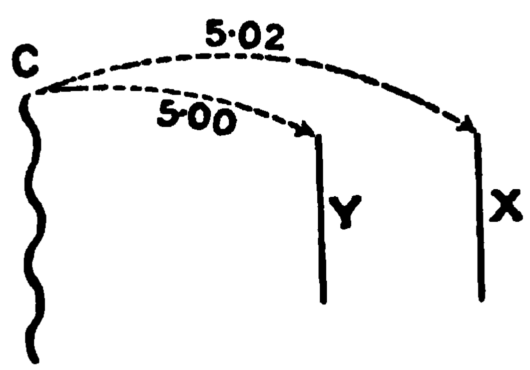


Fig. 242.

As regards their relative positions, C is the meridian of the watch A (in this case the standard); since it was slow on M.T.P. when at X, X must be $5^h\ 02^m$ to the east of it. At Y it was $5^h\ 00^m$ slow: therefore Y is west of X.

604. Comparing by Telegraph.—Eliminate the ship and rockets, and substitute a telegraph wire, or the wireless, connecting A direct with B; then by comparing one chronometer with the other, by a series of 'ticks' the same end is obtained as before, and with probably better results; although even by telegraph there is still the same human fallibility, both in pressing the button of the transmitter, and in receiving a 'stop,' as well as the time in travelling over the wire.

Yet it is not so large an error as that caused by the waiting and watching and straining of the eye while a watch is held to the ear, or the ear to a clock. There are devices by which the transmitter of a 'stop' actually registers his signal on a piece of paper, on which is also registered the beats of the chronometer compared; the whole thing is automatic, simplified, and more correct.

[1] This example is only given as an illustration, not for any value it may have as a means of finding a correct distance.

On p. 459 there is a complete example of a telegraphic meridian distance (Appendix VI.):—

Possibilities are further enhanced with wireless telegraphy or telephony, with the aid of which meridian distances between the two ends of a survey will easily be carried out; and for that purpose will supersede the following methods, where chronometers have to be carried from place to place.

605. Carrying Chronometers from Place to Place for Meridian Distances.—The next point considered is the case where there is neither wire nor wireless telegraphy.

The objective is the same; viz., to find the error of the same chronometer on M.T.A.N. or midnight on two places; but there being no connection between the places to enable comparisons to be made, the chronometer has to be carried from one place to the other; and since this will be a matter of days or of at least one day between the observations, it will be necessary to know the rate of the chronometer.

For example, taking the error of chronometer on M.T.A.N. at X as $0^h\ 26^m\ 10^s{\cdot}7$ slow on 1st September; and supposing it to arrive at Y two days afterwards, viz. 3rd September; or not used there till then; and that its error on M.T.A.N. at Y is $0^h\ 09^m\ 50^s$.

Now this so-called A chronometer at Y is practically not the same A as was used at X, because in the meantime it has changed its error; it may be called A′ chronometer, and the same calculation recurs as before, to bring these two chronometers down to the one, and that one is usually the last one used, or A′.

The only way we can compare A with A′ is by applying the rate of A to A′; and if the accumulated rate for two days is applied to the errors of A at X meridian, then we shall have the error of A′ on X meridian on the 1st September, and the error of A′ on Y meridian on the 3rd September. But we have the same consideration as before, that the sights taken by A′ will be taken not exactly two days after those taken by A, but two days ± diff. longitude in time between the places. If travelling east it will be −; and if west +; and supposing the diff. longitude to be 1 hour to the west, then the rate of A multiplied by 2 days 1 hour, will have to be applied to A's error, to bring it up to A′ chronometer.

606. Rates of Chronometers.—The next point will then be, What is the rate of A chronometer?

There are several separate works written on chronometers, but suffice it here to say, that chronometers resemble human beings in some respects: their works are affected by change of air or temperature; it unsteadies them to be violently shaken; they will not assimilate dirt or dust, and must be attended to regularly; and they may be erratic in spite of all care.

Putting aside the last failing, and supposing all possible care is taken of a chronometer, there still remains change of temperature, which will affect the rate; and if when carried about from place to place it receives a sudden blow or is violently shaken, both the error and rate will be affected.

Therefore, as regards these chronometers, and especially such as are carried on board a ship, it is known that they may have a more or less steady rate in harbour, only affected apparently by temperature; and immediately they are moved to another place, in the process of moving they develop another rate, independent of the thermometer: when brought to rest again, they may or may not continue the previous rate, though, if rested long enough, they will probably do so.

When a chronometer is moved from place to place, the rate we want is that during this period, not what it had in a port, or while at rest; and the problem is to find this rate as accurately as circumstances permit.

Case 1. (See fig. 243.)

Let A be a known meridian, *i.e.* the longitude of which is known, and let X also be another known meridian.

Suppose the error of the chronometer at A on the 1st September is found to be $2^h\ 10^m\ 2^s$ slow.

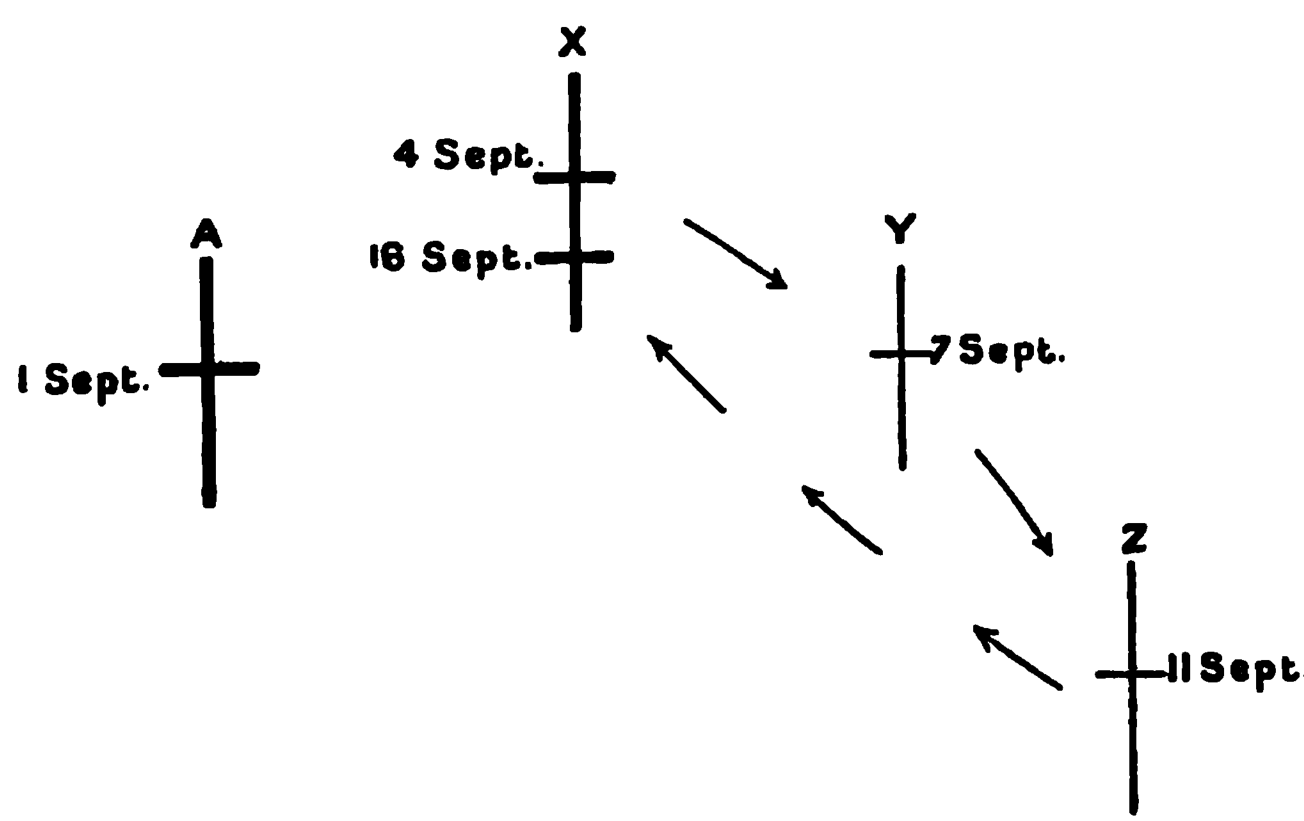


FIG. 243.

The chronometer is now carried to X, and on arriving there on the 4th, the error of the chronometer is found to be $3^h\ 25^m\ 10^s$ slow. Let the difference of longitude between A and X be $1^h\ 15^m$.

Since the error of the chronometer on A is $2^h\ 10^m\ 0^s$ slow, and

the difference of longitude between A and X is $1^h\ 15^m$, then the error of the chronometer on X is $3^h\ 25^m\ 0^s$ slow. On arrival at X the error of the chronometer on X is found to be $3^h\ 25^m\ 10^s$ slow.

Therefore the chronometer must have changed its error 10 secs. during the period at sea; and since the ship was 2 days going, and there was an interval of $1^d\ 22^h\ 45^m$ between the observations, then 10 divided by $1^d\ 22^h\ 45^m$ (1·95 days) = the rate of the chronometer while at sea.

It is proposed to continue the journey with the intention of running a meridian distance on to Y; then $\frac{10}{1 \cdot 95}$ will be the daily sea rate of the chronometer accepted.

Suppose the ship arrives at Y on the 7th September, and finds the error on M.T.A.N. $4^h\ 10^m\ 13^s{\cdot}2$.

Then since the error on the 4th September at X is $3^h\ 25^m\ 10^s$, and the losing rate of the chronometer was $\frac{10}{1 \cdot 95}$ between the 4th and 7th, then on the 7th, $3^d\ 0^h\ 44^m$, it will have lost $\frac{2 \cdot 97 \times 10}{1 \cdot 95}$ seconds = 10·62 seconds; and on the 7th its error will be $3^h\ 25^m\ 10^s + 10^s{\cdot}62$ on M.T.P. at X = $3^h\ 25^m\ 20^s{\cdot}62$; and since on the 7th at Y it was $4^h\ 10^m\ 13^s{\cdot}2$, therefore the meridian distance between X and Y =

$$\begin{array}{rrr} 4^h & 10^m & 13^s{\cdot}2 \\ 3 & 25 & 20{\cdot}62 \\ \hline 0^h & 44^m & 52^s{\cdot}58 \end{array}$$

and since the chronometer was slower at Y than at X (see fig. 243), then Y must be east of X.

Of course, there will be a better result if the ship or traveller runs back to X again, or if he has to go on to Z, etc., with a continuance of the same rate, but eventually does return to X, still travelling with the same assumed rate.

At X, the error of the chronometer is again found, and applying it to the previous error obtained there, the rate is known for the whole period; if this rate differs from the one used for the meridian distance to Y, Z, etc. (which it is sure to do), the whole thing will have to be re-adjusted; how, depends upon one's own judgment.

It is not possible to state here definitely how the adjustments will be made, though consideration will have to be given to the state of the weather, the change of temperature, and the performance of each individual chronometer. The fact is, the distances X to Y, and Y to Z, are incorrect, and the error is inextricably mixed up in the chronometer and the observer.

But these matters are mainly connected with a fully equipped surveying vessel; they are only mentioned here because the

subject of this form of meridian distance would be incomplete without such an explanation.

Again, if the ship is detained for a day or two at any one place the whole subject becomes still more complicated; because in the total error, a harbour error is mixed up with a sea error. (See *Case* 3, p. 427.)

607. The following simple example, if followed out, will explain the subject:—

On the 10th January.
At A, long. 1° 5′ 54″ W. Chron. $2^h\ 03^m\ 45^s{\cdot}6$ slow on M.T.A.N.

On the 14th January.
At X, long. 16° 53′ 54″ W. Chron. $1^h\ 00^m\ 54^s{\cdot}4$ slow on M.T.A.N.

1. Required the sea rate of the chronometer.

Long. A	1°	05′	54″	W.
„ X	16	53	54	W.
diff. long.	15	48	00	$= 1^h\ 03^m\ 12^s$.

Error chron. at A	2^h	03^m	$45^s{\cdot}6$	slow on 10th January.
diff. long.	1	03	12 W.	
error chron. on X	1	00	33·6	slow on 10th January
and error on X is	1	00	54·4	slow on 14th January
change of error	0	00	20·8	

The time elapsed from A to X is 4 days 1 hour = 4·04 days; and since the error changed 20·8 seconds, therefore

$$\text{rate of chron. at sea} = \frac{20{\cdot}8}{4{\cdot}04} = 5{\cdot}015 \text{ losing.}$$

The ship then proceeded first to Y, next to Z, and then on to J.

At Y, 18th January, error found	11^h	35^m	$32^s{\cdot}8$	slow.
„ Z, 20th „ „ „	11	14	56·2	
„ J, 23rd „ „ „	11	45	15·7	

Finally returning to X on 2nd February.

At X, 2nd February, error found, $1^h\ 01^m\ 23^s{\cdot}6$ slow.

2. Required the meridian distance between each place.

Error, 14th January, at X,	1^h	00^m	$54^s{\cdot}4$
„ 2nd February, at X,	1	01	29·6
change of error for whole period	0	1	35·2 in 19 days.

Therefore rate while travelling $= \frac{95{\cdot}2}{19} = 5{\cdot}01$ losing, and this agrees with the previous rate found in another way.

Error, 14th January, at X,	1^h	00^m	$54^s{\cdot}4$	slow.
Interval of time from 14th to 18th January travelling westward $= 4^d\ 1^h\ 25^m$; rate per day, 5·01 losing.				
∴ 4·05 days × 5·01 = amount lost =	0	00	20·3	
error on 18th at X	1	01	14·7	slow
„ „ Y	11	35	32·8	slow
meridian distance, X to Y	1	25	41·9	

Error on 18th at Y	11^h	35^m	$32^s{\cdot}8$	slow.
Interval of time from 18th to 20th going westward $= 2^d\ 0^h\ 20^m = 2{\cdot}001$.				
∴ 2·001 × 5·01 = amount lost =	0	00	10·2	
error 20th at Y	11	35	42·8	
„ „ Z	11	14	56·2	
meridian distance Y to Z	0	20	46·6	

Error 20th at Z	11^h	14^m	$56^s{\cdot}2$	slow.
Interval of time from 20th to 23rd going eastward $= 3^d - 0^h\ 30^m = 2^d\ 23^h\ 30^m = 2{\cdot}99$ days × 5·01 =	0	0	14·98	
error 23rd at Z	11	15	11·18	slow
„ „ J	11	45	15·7	„
meridian distance between Z and J	0	30	04·5	

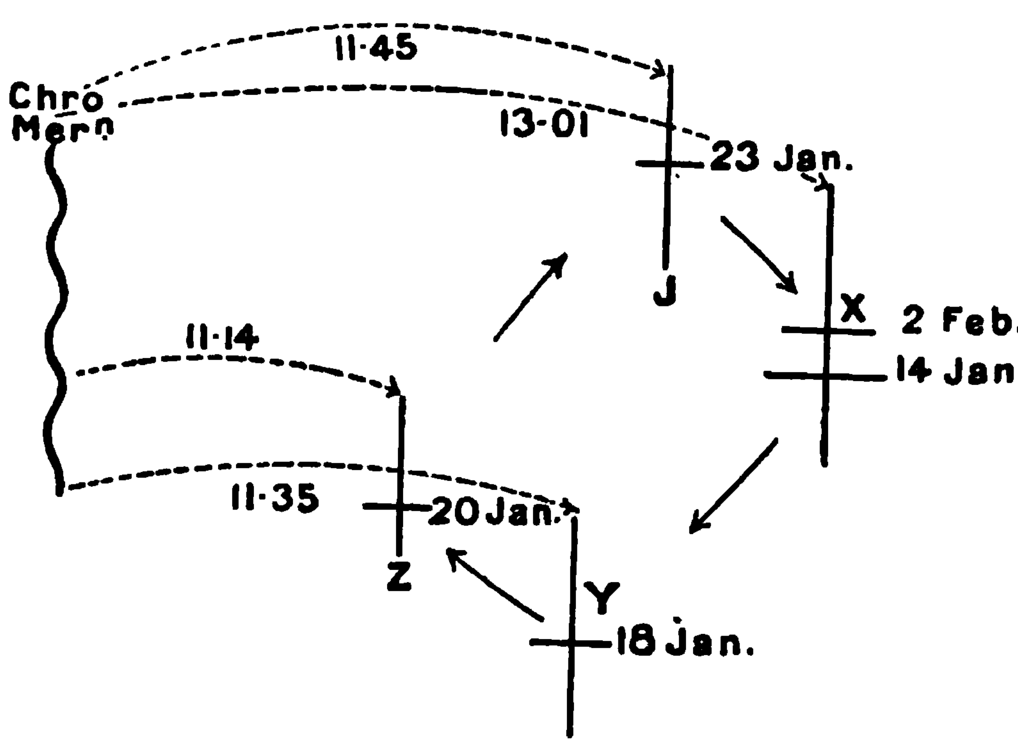


FIG. 244.

Since the chronometer was $13^h\ 01^m$ slow on X, and $11^h\ 35^m$ slow on Y, therefore Y is west of X.

Similarly, Z is west of X, and so is J.

By a similar analysis, Z is west of Y, but J is east of Z and of Y.

Case 2.

Suppose now we have not a handy 'back meridian,' such as A, wherewith to find a sea or moving rate.

The simplest thing that suggests itself is that the chronometer should be taken out for a 'trot'; and, leaving X, the ship goes to sea for a few days and returns again. The difference between the errors obtained just before leaving, and immediately on return, divided by the number of days away, would give the sea rate.

But why not apply this to some useful end? Therefore, on this trial trip, why not go as far as Y and call there, staying just sufficiently long to take observations for error? (see fig. 245).

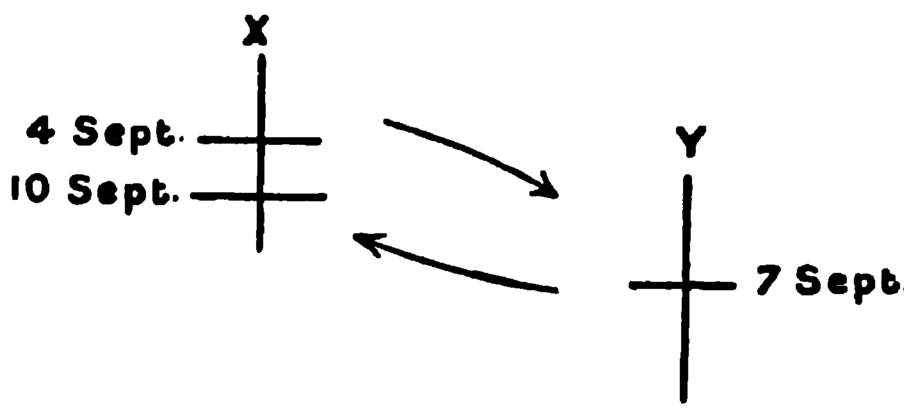


FIG. 245.

The chronometer itself will not be landed, but will not have time to recover from its journey.

So observations are taken at Y on the 7th, and the ship returns immediately to X, and again finds the error at X. As before, the difference of the errors at X divided by 6 days (3 days going and 3 days returning) = rate of chronometer while at sea. Then repeat the calculations as before given.

If 3 days of this rate is applied to the error at X on the 4th, it will give what the error would have been there on the 7th; and since the error was found at Y on the 7th, the difference between these two errors on the same date at the two places gives the meridian distance from X to Y.

Failing the telegraph meridian distance, a number of results obtained by this means is the most accurate method, where chronometers are carried from place to place.

Example of Case 2.—This is a very simple case. Three chronometers will be employed: at X, chronometers all fast.

	A.	B.	C.
1 May	$5^h\ 53^m\ 42^s{\cdot}1$	$11^h\ 12^m\ 23^s{\cdot}4$	$1^h\ 00^m\ 00^s{\cdot}2$
[1] $6\frac{1}{2}$ May	5 54 12·7	11 12 23·4	0 59 19·5

At Y, all chronometers fast.

4 May	$3^h\ 54^m\ 01^s{\cdot}1$	$9^h\ 12^m\ 25^s{\cdot}9$	$10^h\ 59^m\ 56^s{\cdot}7$

[1] This means that the error was obtained by 'inferiors,' *i.e.* at midnight on the 6th.

Required the meridian distance between X and Y.

	A.	B.	C.
1 May	$5^h\ 53^m\ 42^s{\cdot}1$	$11^h\ 12^m\ 23^s{\cdot}4$	$1^h\ 00^m\ 00^s{\cdot}2$
6½ May	5 54 12·7	11 12 23·4	0 59 19·5
change in 5½ days	0 0 30·6	0 0 0·0	0 0 40·7
rate per day	$5^s{\cdot}564$ gaining	0	$7^s{\cdot}4$ losing.

Interval of time from departure from X, 1 May to 4 May, date of arrival at Y, 3 days − 2 hours = 2 days 22 hours = 2·917 days (travelling eastward from X to Y).

	A.	B.	C.
days	2·917	2·917	2·917
rate	$5^s{\cdot}564$	0	$7^s{\cdot}4$
total change	$16^s{\cdot}22$ gained	0	$21^s{\cdot}58$ lost.

	A. h m s		B. h m s	C. h m s
Error 1 May	5 53 42·1	fast	11 12 23·4	1 00 00·2
change in 2·917 days	0 00 16·22	gained	no change	0 00 21·6 lost.
at X, error 4 May	5 53 58·3		11 12 23·4	0 59 38·6
at Y, „ „	3 54 01·1		9 12 25·9	10 59 56·7
meridian distance	1 59 57·2		1 59 57·5	1 59 41·9

M.D. by A $1^h\ 59^m\ 57^s{\cdot}2$
„ B 1 59 57·5
„ C 1 59 41·9 (discarded, something gone wrong with C chronometer).

mean of A and B 1 59 57·3

Therefore Y is east of X (fig. 246).

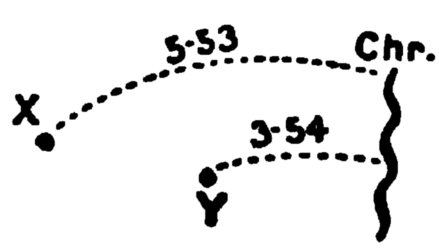


FIG. 246.

Case 3.

It may occur, and in a number of cases does arise, that on arrival at Y, there are impediments in the way of taking observations for equal altitudes. Either there is no sun, or it fails in the second half of the sights, and the ship may be detained at Y for

a few days (see example of equal altitudes taken part one day and part on another day, pp. 385, 441). If this occurs, the continuity of the sea rate is interfered with by a compulsory detention in the harbour, and the introduction of a harbour rate. Fig. 247 graphically shows the course of events. An error was taken at X on the 4th September; the ship proceeded to Y, arriving there on the 7th; was detained there till the 11th, obtained an error on that day, and returned to X in time for observations on the 16th.

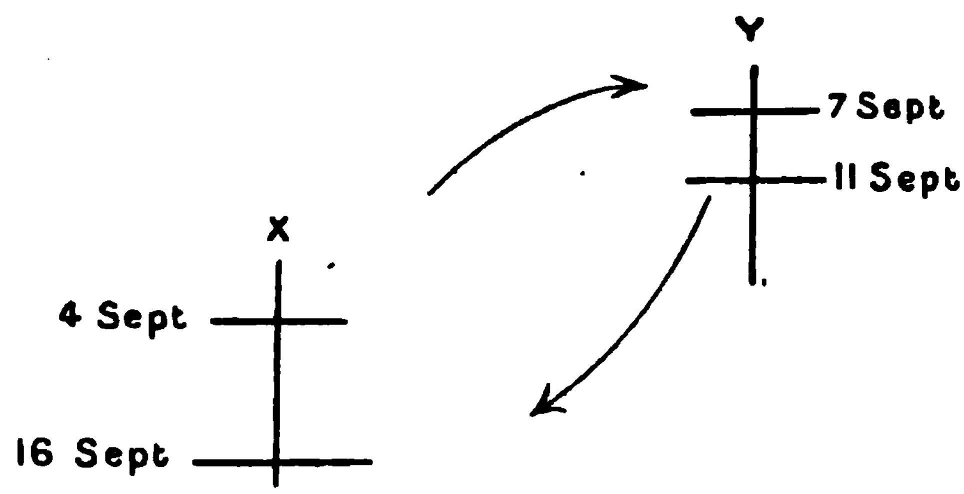


FIG. 247.

Now the difference between the error at X on the 4th, and the error at X on the 16th, is the total change of the error of the chronometer between these dates.

During part of the time from the 4th to the 7th, and from the 11th to the 16th, the vessel was at sea; while for the period between the 7th and 11th she was in harbour.

If, from the total error, we exclude the amount of this harbour error, what remains can be said to belong to the sea error.

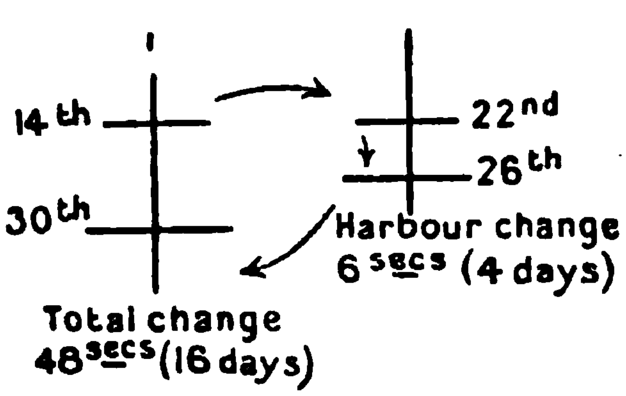


FIG. 248.

There is absolutely nothing to guide us, as to what proportion of this error belongs to the 3 days going, and what to the 5 days returning, except it be in the ratio of the number of days.

One thing may suggest if anything has gone very wrong, and that is the difference between the rate for the total period, as compared with the rate at Y; but that is all it can suggest.

Then this remainder of error while at sea is divided by 8, and the sea rate assumed to be the same both going and coming: this, in contradistinction to Cases 1 and 2, is called a travelling rate.

The result has a value as compared with Case 1, and also as compared with Case 4 which follows.

Example of Case 3.—At X (fig. 248) the following chronometers were slow on M.T.A.N. :—

	A.	B.
	h m s	h m s
14th March	8 13 9	10 21 53
Arrived at Y on 22nd March . . .	8 48 47	10 57 15
Remained here till 26th March . .	8 48 53	10 57 17
Then returned to X 30th March . .	8 14 07	10 22 25

	h m s	h m s
Error at X, 14th	8 13 19	10 21 53
„ X, 30th	8 14 07	10 22 25
Total change (16 days) . . .	lost 48	lost 32
Error at Y, 22nd	8 48 47	10 57 15
„ Y, 26th	8 48 53	10 57 17
Change in harbour (4 days) . . .	lost 6	lost 2
∴ change at sea = total − harbour =	lost 42	lost 30

Time at sea, going, is	7·91 days.
„ „ returning, is . . .	4·09 „
Total time at sea	12·00 days

∴ rate at sea A chronometer is assumed to be $\frac{42}{12} = 3^s{\cdot}5$ losing.

„ „ B „ „ „ $\frac{30}{12} = 2^s{\cdot}5$ losing.

A.				B.			
Error, 14th	8^h	13^m	19^s		10^h	21^m	53^s
7·91 days' rate	0	00	27·7 lost	7·91 days'rate	00	00	19·8 lost
error 22nd at X	8	13	46·7	22nd at X	10	22	13·8
„ „ Y	8	48	47	„ Y	10	57	15
meridian dist.	0	35	00·3	M.D.	0	35	02·2

Mean result, meridian distance $35^m\ 01^s{\cdot}25$. Y is east of X.

An analysis of the performance of the chronometers shows that the rate of A for the whole time was $3^s{\cdot}5$ losing, while its rate in harbour at Y was $1^s{\cdot}5$. For B, the total rate was $2^s{\cdot}5$ losing, while the rate in harbour Y was $\cdot 5^s$.

The chronometer was 8 days on its journey from X to Y and only 4 days returning; the cause of the longer journey probably being bad weather, which affected both chronometers equally.

There are so many possibilities, that no attempt will be made to disentangle them. The meridian distance is not a good one—that is the conclusion.

Case 4.

The last case is a still further modification of Cases 2 and 3.

The following diagrams (figs. 249–251) illustrate all three; the numbers attached show the *order of taking* the errors.

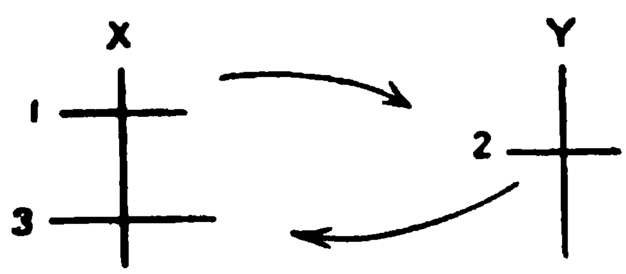


FIG. 249.

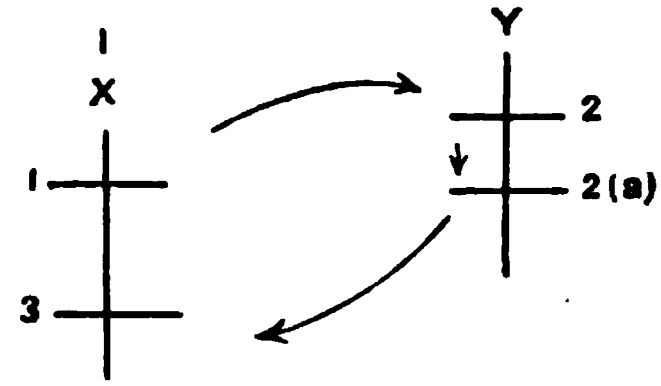


FIG. 250.

In Case 4, a *harbour* rate is obtained from sights 1 (a) and 1 (b) at X (fig. 251): the chronometers are then transported to Y, and sights 2 and 2 (a) are obtained.

On returning again to X, the errors 3 (a) and 3 (b) are found.

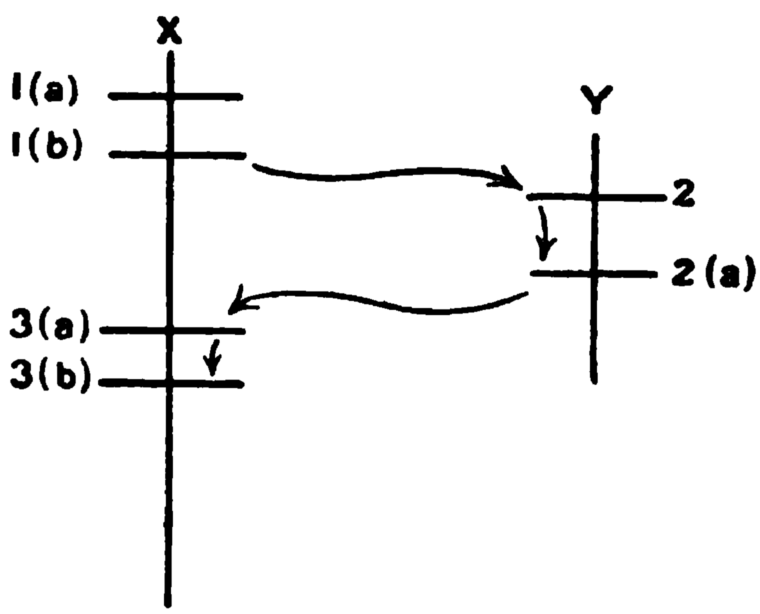


FIG. 251.

The harbour rates then, between each set of sights, are obtained at both places X and Y.

Shadwell, *On Chronometers*, states that, "observing for harbour rates, the time elapsed between the observations for errors should be between 4 and 7 days"; and there is a greater element of accuracy, if the time elapsed between these observations is the same at both places.

What we have to rely on is a harbour rate at X to start with; the rate between observations 1 (a) and 1 (b); and another harbour rate on arrival at Y between 2 and 2 (a).

From these foundations, the only workable assumption is, that the chronometer gradually changed its rate from the harbour rate at X to the harbour rate at Y; and therefore the mean of those two rates is adopted as the sea rate.

There will be two results obtained by this means, the one going (1 to 2), and the other returning (2 to 3); and each may have a better value than the other, depending upon the conditions of travel; for there might have been a fair wind and smooth sea, and an equable temperature in the one case, and the contrary in the other.

It should be noticed that, besides these two meridian distances

calculated by the mean of harbour rates, there is sandwiched in between them an independent meridian distance, which, commencing at 1 (*b*), goes to 2, then to 2 (*a*), and returns to 3 (*a*). It is, in fact, Case 3. So that we have three meridian distances, and each of the three will have a separate value (see following *Example*).

Example of Case 4.—At a place X, the chronometer was slow on G.M.T. by time-ball:

$$\begin{array}{llll} \text{15th June,} & 0^h\ 47^m\ 45^s{\cdot}8 & (1\,(a)) \\ \text{20th}\ \ \text{,,} & 0\ \ 47\ \ 44{\cdot}6 & (1\,(b)) \end{array}$$

At Y, by equal altitudes, the error of the chronometer was slow on M.T.P.:

$$\begin{array}{llll} \text{22rd June,} & 0^h\ 08^m\ 27^s{\cdot}2 & (2) \\ \text{28th}\ \ \text{,,} & 0\ \ 03\ \ 26{\cdot}0 & (2\,(a)) \end{array}$$

On returning to X, the error of the chronometer on G.M.T. by time-ball was:

$$\begin{array}{llll} \text{3rd July,} & 0^h\ 47^m\ 24^s{\cdot}6 & (3\,(a)) \\ \text{8th}\ \ \text{,,} & 0\ \ 47\ \ 25{\cdot}8 & (3\,(b)) \end{array}$$

Deduce the longitude of Y:

(*a*) by mean rates;
(*b*) by travelling rates.

Discuss which of the two you consider of greater value, and in finding the longitude of Y, give a value for the results obtained by each method.

First, the relative positions or the meridians had better be explained. The wavy line is the meridian of the chronometer; and since it is slow on G.M.T. the meridian of Greenwich is $0^h\ 47^m$ to the east of it.

The chronometer is also slow on Y, $0^h\ 08^m$. Therefore the meridian of Y is to the east of the chronometer meridian, and to the west of Greenwich; and the longitude of Y is the meridian distance G.Y.

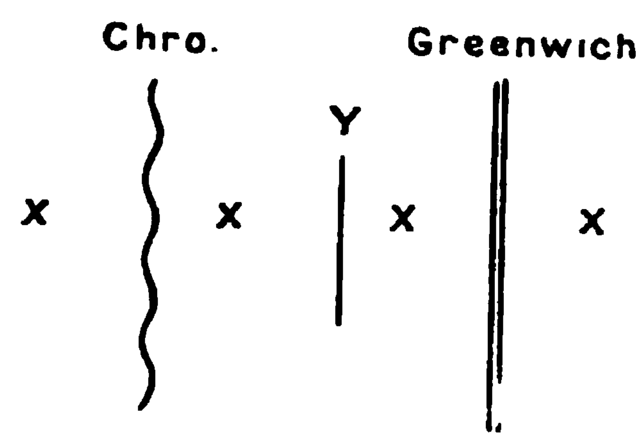


FIG. 252.

Where X is does not matter; it may be at any of the positions shown in the diagram: but since we require it, to know whether the chronometer in going to Y is travelling east or west, assume it to be somewhere between C and Y.

$$\begin{array}{llll} \text{15th June,} & 0^h\ 47^m\ 45^s{\cdot}8 & (1\,(a)) \\ \text{20th}\ \ \text{,,} & 0\ \ 47\ \ 44{\cdot}6 & (1\,(b)) \\ \hline & \qquad 5|1{\cdot}2 & \\ \end{array}$$

harbour rate = $\overline{{\cdot}24}$ gaining.

Now this harbour rate does not belong to the 15th nor to the 20th, but to the middle date, *i.e.* 17½ June; and the error on the 17½ June is the mean between that on the 15th and 20th = $0^h\ 47^m\ 45^s{\cdot}2$. Therefore at X, error 17½ June is $0^h\ 47^m\ 45^s{\cdot}2$; rate = ·24 gaining.

At Y, 23rd June, $0^h\ 08^m\ 27^s{\cdot}2$ slow (2)
28th „ 0 08 26·8 slow (2 (*a*))

5|1·2
harbour rate = ·24 gaining.

For the same reason as above, therefore, error on 25½ June $= 0^h\ 08^m\ 26^s{\cdot}6$, and the rate is $\cdot 24^s$ gaining.

It has to be assumed that the rate of the chronometer gradually changed from its rate at X to its rate at Y. Since in this case they are the same, then the sea rate adopted must be $\cdot 24^s$ gaining, *i.e.* the mean of the two.

Then error at X 17½ June $0^h\ 47^m\ 45^s{\cdot}2$ slow
8 days' rate, neglecting the odd minutes 0 00 19·2 gaining

error at X 25½ June = 0 47 26·0 (C to G)
„ „ Y 25½ „ = 0 08 26·6 (C to Y)

(1) . . . . . mer. dist. = 0 39 59·4 (G to Y)

3rd July, at X = $0^h\ 47^m\ 24^s{\cdot}6$ slow (3 (*a*))
8th „ „ = 0 47 25·8 slow (3 (*b*))

5|1·2 losing
harbour rate = ·24 losing.

Error on 5½ July at X, $0^h\ 47^m\ 25^s{\cdot}2$; rate, $\cdot 24^s$ *losing*.

At Y, the harbour rate on 25½ June was $\cdot 24^s$ *gaining*. Therefore the *mean* rate between these dates is 0.

The error on 25½ June at Y is $0^h\ 08^m\ 26^s{\cdot}6$
change to 5th July 0 00 00

∴ error on 5½ July at Y = 0 08 26·6 (C to Y)
„ 5½ „ X = 0 47 25·2 (C to G)

(2) . . . . . M.D. = 0 39 58·6

The third result obtained is by travelling rates (see Case 3).

X.	Y.
20th June $0^h\ 47^m\ 44^s{\cdot}6$ (1 (*b*))	23rd June $0^h\ 08^m\ 27^s{\cdot}2$ slow (2)
3rd July 0 47 24·6 (3 (*a*))	28th June 0 08 26·0 slow (2 (*a*))
total error = 20 *gained.*	harbour error = 1·2 *gained.*

Therefore the chronometer must have gained $18^s\cdot8$ while at sea from the 20th to 23rd June + from 28th June to 3rd July = 8 days; and the sea rate or travelling rate will be $\frac{18\cdot8}{8} = 2^s\cdot35$ *gaining.*

Error on 20th June =	0^h	47^m	$24^s\cdot6$	
3 days' rate =	0	00	7·05 gained	
error 23rd at X =	0	47	16·55	(C to G)
" " Y −	0	08	27·2	(C to Y)
(3) . . . M.D.	0	39	49·35	

Result by (1) . .	39^m	$59^s\cdot4$
" (2) . .	39	58·6
" (3) . .	39	49·35

Referring to fig. 251 (Case 4):

From 1 (*a*) to 1 (*b*) the rate of the chronometer was ·24 gaining.
" 2 " 2 (*a*) " " " " ·24 gaining.
" 3 (*a*) " 3 (*b*) " " " " ·24 losing.

Also from 1 (*b*) to 2 occupied 3 days, while from 2 (*a*) to 3 (*a*) occupied 5 days.

It is probable then that bad weather influenced the rate from 2 (*a*) to 3 (*a*), and therefore that the rate used between these two is not so reliable as that from 1 (*b*) to 2; and only an erratic rate from 3 (*a*) to 3 (*b*) gave the meridian distance near to *result* (1).

From 1 (*b*) to 3 (*a*) the chronometer showed a change of 20^s in 14 days = $1^s\cdot33$ gaining, whereas its previous rate from 1 (*a*) to 1 (*b*) was only $\cdot24^s$; showing again that it was from 2(*a*) to 3 (*a*) that the mischief occurred.

It is probable, then, that *result* (1) has a far greater value than either of the others: *result* (2) might be expunged, and *result* (3) might have $\frac{1}{3}$ of the value of (1).

Hence, giving *result* (1) three values = 3 × 39^m $59^s\cdot4$ =	119^m	$59^s\cdot2$
" *result* (3) one value = 39 49·35 =	39	49·35
	4|159	48·55
accepted result =	39	57·14

N.B.—Result (3) is retained, because there is a remote possibility that some of the error might have crept in while at Y.

As a meridian distance, it is unsatisfactory, and would be listed with those perhaps derived at some future period, and then again given a value: and, moreover, errors obtained in the one case

by time-ball, and in the other by observations, will, on account of the different form of error in each, considerably reduce the value as a meridian distance.

The mer. dist. between Greenwich and Y being $0^h\ 39^m\ 57^s{\cdot}14$, the longitude of Y is $0^h\ 39^m\ 57^s{\cdot}14 = 9°\ 59'\ 17''{\cdot}2$ W.

Where a number of chronometers are used, they will not necessarily be all affected in the same manner, and the performance of each will be analysed in the same way as above, and they in turn will be given a value in the general mean.

In all the examples given, it must be understood that the errors are found by 'equal altitudes,' superiors or inferiors; and therefore in each case the zero is M.T.A.N or midnight precisely. The error of chronometer may be found by the 'mean of absolutes' of very nearly equal altitudes, though calculated in a different manner: (see *Example*, p. 443). This would suffice, providing the altitude at the second half of the observations is taken within a quarter of an hour of the time that the equal altitude has passed, and also that the rate of the chronometer is known exactly, for the purpose of bringing up the error to that it will have at apparent noon at both places. This involves a further calculation (see p. 39, par. 115): it is not recommended.

It should not be forgotten that the meridian distance required is not for finding an absolute longitude, but for a distance in so many feet; and that, for instance, ·1 sec. in error in lat. 30° = 200 feet.

608. Comparing Chronometers.—Most chronometers beat ½ seconds; but some of the earlier type still in use beat 5 times in 2 seconds: starting from an even second the fractions are ·4, ·8, 1·2, 1·6, 0. When comparing these two types of chronometers with each other, there will be found, during any two seconds, a beat which is simultaneous on both. The synchronisation is easily distinguished, and the reading at the instant is ascertained in the following manner. Repeating 4, 8, 2, 6, 0, the audible ticks of the one, the eye watches the beats on the other, while the ear detects the tick on which both synchronise.

APPENDICES.

APPENDIX I.

'Parallax' in a Sextant, and Position of the Receiving Angle with Consequent Error.

Let H be the horizon glass, and I the index glass; E is the position of the eye at the end of the *long* telescope.

R is the right object, whose reflection enters the glass at I, is reflected to H, and thence to the eye at E. Join R I, and produce it to meet H E at O.

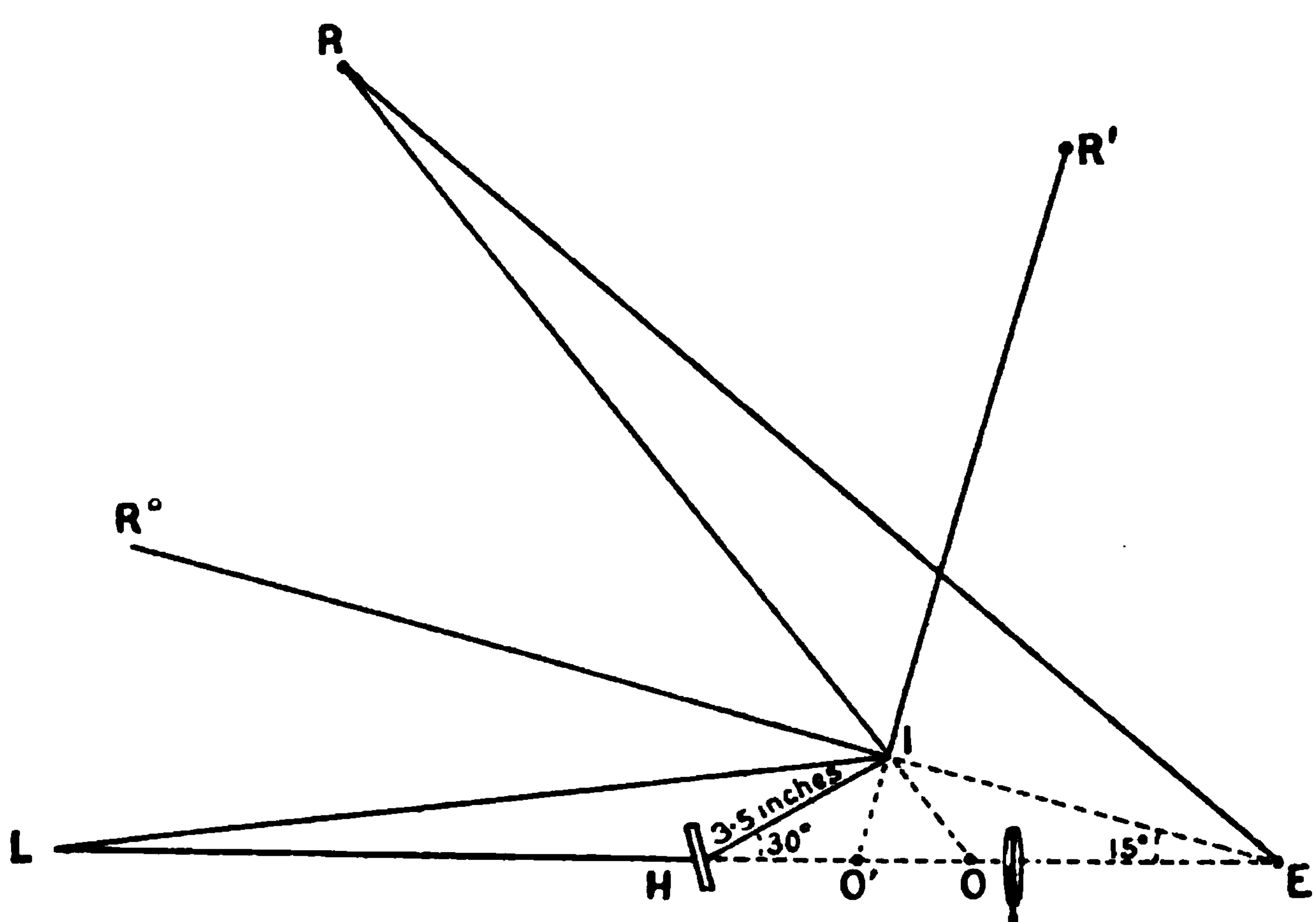


Fig. 253.

Then R E L is the observed angle; and R O L = true angle. R O L = R E L + O R E. ∴ O R E = the error in the angle R E L, at a distance R E.

If R is at R°, R°I E = 180°, *i.e.* R°, I, E are in one line, and the

error vanishes; and if R is at R′, where R′I E = 90°, the error is a maximum.

In most sextants H I = 3·5 inches; I H E = 30°, and I E H = 15°, when the long telescope is used.

If R′I E = 90°, E I O′ = 90°, and I O′E = 90° − 15° = 75°.

Therefore I O′L = 180° − 75° = 105°.

Hence the error is a minimum when R E L = 15°,
and a maximum when R O′L = 105°.

The amount of the error will vary as the distance, and for any angle $\frac{EI}{RI}=\frac{\sin ERO}{\sin REI}=\frac{\sin ERO}{\sin (REL-15^\circ)}$.

$$(1) \quad . \quad \therefore \quad \sin ERO=\frac{EI}{RI}=\sin (REL-15^\circ),$$

$$\frac{EI}{HI}=\frac{\sin EHI}{\sin IEH}=\frac{\sin 30^\circ}{\sin 15^\circ}.$$

$$\therefore \quad EI=HI\frac{\sin 30^\circ}{\sin 15^\circ}=\frac{3{\cdot}5 \sin 30^\circ}{\sin 15^\circ};$$

and substituting the value of E I in equation (1),

then $$\sin ERO=\frac{3{\cdot}5 \sin 30^\circ}{RI \sin 15^\circ}\sin (REL-15^\circ).$$

In the case of the maximum angle 105°, for a distance of 100 yards, R I = 100 yards = 3600 inches;

$$\sin ERO=\frac{3{\cdot}5 \sin 30^\circ}{3600 \sin 15^\circ}\sin (105^\circ-15^\circ)$$

log 3°·5 =	·544068	log 3600 =	3·556302
log sin 30° =	9·698970	sin 15° =	9·412996
	·243038		2·969298
log sin =	2·969298		
	7·273740		
	= 6′ 27″.		

Hence for a distance of 100 yards, when the observed angle is 105°, the error in the angle is 6′ 27″.

The error will vary inversely as the distance, and at 3800 yards would $=\frac{6' 27''}{38}=10''$ nearly.

Therefore, at an angle of 105°, the object R should not be less than nearly 2 miles.

For any angle at 100 yards, the error will be 6′ 27″ × sin (angle − 15°): if, as in taking index errors, the angle is 0°, then error at 100 yards = 6′ 27″ sin 15°.

(Note the − sign disappears, because the position of the receiving angle is then at L along the line E H produced; hence

the error of parallax, *i.e.* the distance between the glasses, is evidently the source of the error.)

$$= 6' \, 27'' \, . \cdot 26$$
$$= 6{\cdot}45' \, . \cdot 26 = 1'{\cdot}68 = 1' \, 40''.$$

So that this error shall $= 10''$, the distance must be 1000 yards.

Then at 1000 yards, error $= \dfrac{1' \, 40''}{10} = 10''$.

Therefore, to find index error, the object should never be less than 1000 yards or ½ mile.

The further fact is deduced, that a sextant cannot be used to determine accurately the three angles of a triangle so that their sum shall equal 180°, unless the point O is placed exactly at each point of the triangle; and this is impracticable; also that the distance of the objects should not be within the distance of the maximum error shown, *i.e.* 3800 yards, or roughly 2 miles.

Corrections for Parallax in Angles measured with Sextant when using the Long Telescope.

Applicable where the measurements of distances are concerned.

Angle observed.	Distance in yards.											
	100	200	300	400	500	600	700	800	900	1000	1100	1200
'Off' the arc 5°	2' 12"	1' 06"	0' 44"	0' 33"	0' 26"	0' 22"	0' 19"	0' 17"	0' 15"	0' 13"	0' 12"	0' 11"
0	1 40	0 50	0 33	0 25	0 20	0 17	0 14	0 12	0 11	0 10		
'On' arc 5°	1 07	0 33	0 22	0 17	0 13	0 11	0 09					
10	0 34	0 17	0 11									
					+ to a	ngles	above	this	line			
15	0 00											
					− ,,	,,	below	,,	,,			
20	0 34	0 17	0 11									
30	1 40	0 50	0 33	0 25	0 20	0 16	0 14	0 12	0 11	0 10		
40	2 44	1 22	0 55	0 41	0 33	0 27	0 23	0 20	0 18	0 16	0 15	0 14
50	3 42	1 51	1 14	0 55	0 44	0 37	0 31	0 28	0 25	0 22	0 20	0 18
60	4 34	2 17	1 31	1 08	0 55	0 45	0 39	0 34	0 30	0 27	0 25	0 23
70	5 17	2 38	1 46	1 19	1 03	0 53	0 45	0 39	0 35	0 32	0 29	0 26
80	5 51	2 55	1 57	1 27	1 10	0 58	0 50	0 44	0 39	0 35	0 32	0 29
90	6 14	3 07	2 05	1 33	1 15	1 02	0 53	0 47	0 42	0 37	0 34	0 31
100	6 26	3 13	2 09	1 36	1 17	1 04	0 55	0 48	0 43	0 39	0 35	0 32
105	6 27	Ma	ximu	m								
110	6 26	3 13	2 09	1 36	1 17	1 04	0 55	0 48	0 43	0 39	0 35	0 32
120	6 14	3 07	2 05	1 33	1 15	1 02	0 53	0 47	0 42	0 37	0 34	0 31
130	5 51	2 55	1 57	1 27	1 10	0 58	0 50	0 44	0 39	0 35	0 32	0 29
140	5 17	2 38	1 46	1 19	1 03	0 53	0 45	0 39	0 35	0 32	0 29	0 26

Correction to reading is + to angles less than 15°.
,, ,, − ,, above 15°.

APPENDIX II.

Supposing an error of position, it is required to project a line of reference to an object. Which is the best zero to select?

Let O be the true position of an observer. Let the shaded circle represent the area of erroneous positions anywhere on the circumference of which the observer supposes himself to be. Due to an erroneous 'fix,' or to drift between the time of 'fixing' and 'shooting up' an object, he supposes himself at O′, whereas he is at O.

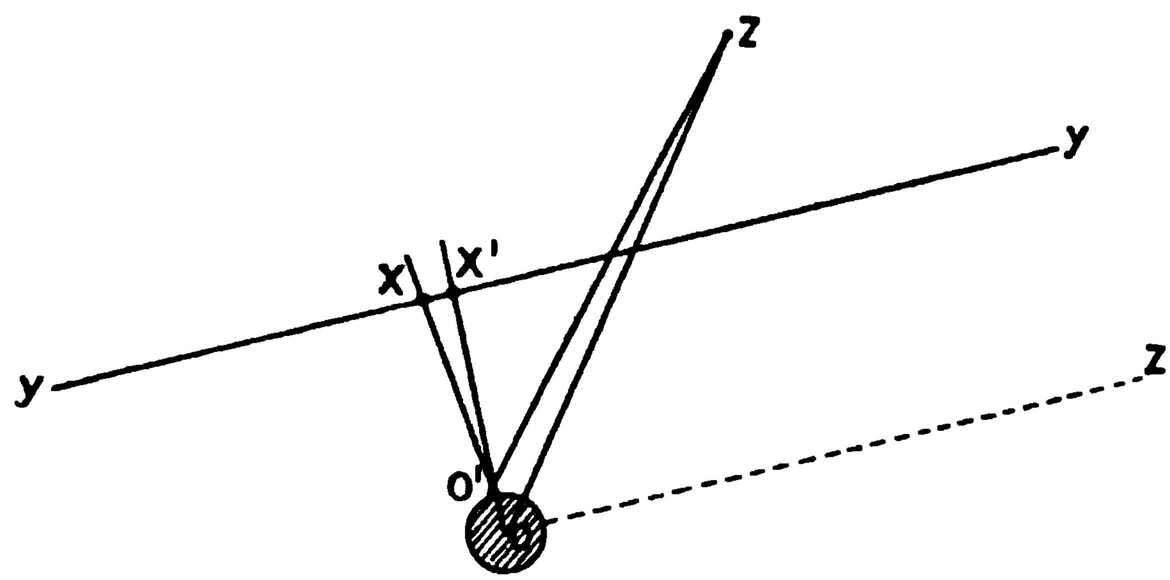


Fig. 254.

Let *yy* be a line of reference already projected through X.

It is required to determine what is the best zero to use, from which to project another line of reference to fix the position of X.

First Case.—Assume the particular case, O′ in the direction of X, to be the supposed position.

Let Z be a zero selected.

Z O X is the observed angle; but it is projected from the line O′Z: this gives the line O′X′; Z O′X′ being equal to Z O X.

X′ is the erroneous position of X along the definite line *yy*; and X X′ is the amount of the error.

$$\text{Now } ZO'X = ZOX + OZO',$$
$$OZO' = \text{angle at } Z \text{ subtended by } OO',$$
$$\text{also } ZO'X = ZO'X' + XOX';$$
$$\text{and since } ZO'X' = ZOX,$$
$$\therefore\ XOX' = OZO' = \text{angle at } Z \text{ subtended by } OO'.$$
$$\text{But } XO'X' = \text{angle at } O \text{ subtended by } XX';$$
$$\therefore\ \text{angle subtended by } XX' = \text{angle subtended at } Z \text{ by } OO'.$$
$$\frac{OO'}{O'Z} = \frac{\sin Z}{\sin XOZ}; \ \therefore \sin Z = \frac{OO'}{O'Z} \sin XOZ.$$

Z will be a minimum when X O Z = 0° or 180°.

Z will be a maximum when X O Z = 90°, shown as Z^2 in the figure.

Also $\frac{OO'}{O'Z}$ will have its least value when O′Z is infinity; OO′ being a constant.

Therefore, the further Z is away, the less is the value of $\frac{OO'}{O'Z}$; and the smaller is X O Z the less the value of sin Z.

Hence, *in this particular case*, the zero to select should be one where X O Z is the smallest possible, and where Z is very distant.

Second Case.—Assume the position O^1, fig. 255, at right angles to X O. With C^1 as centre, and radius C^1X, describe a circle. The circumference of this circle will go through X, O, and O^1.

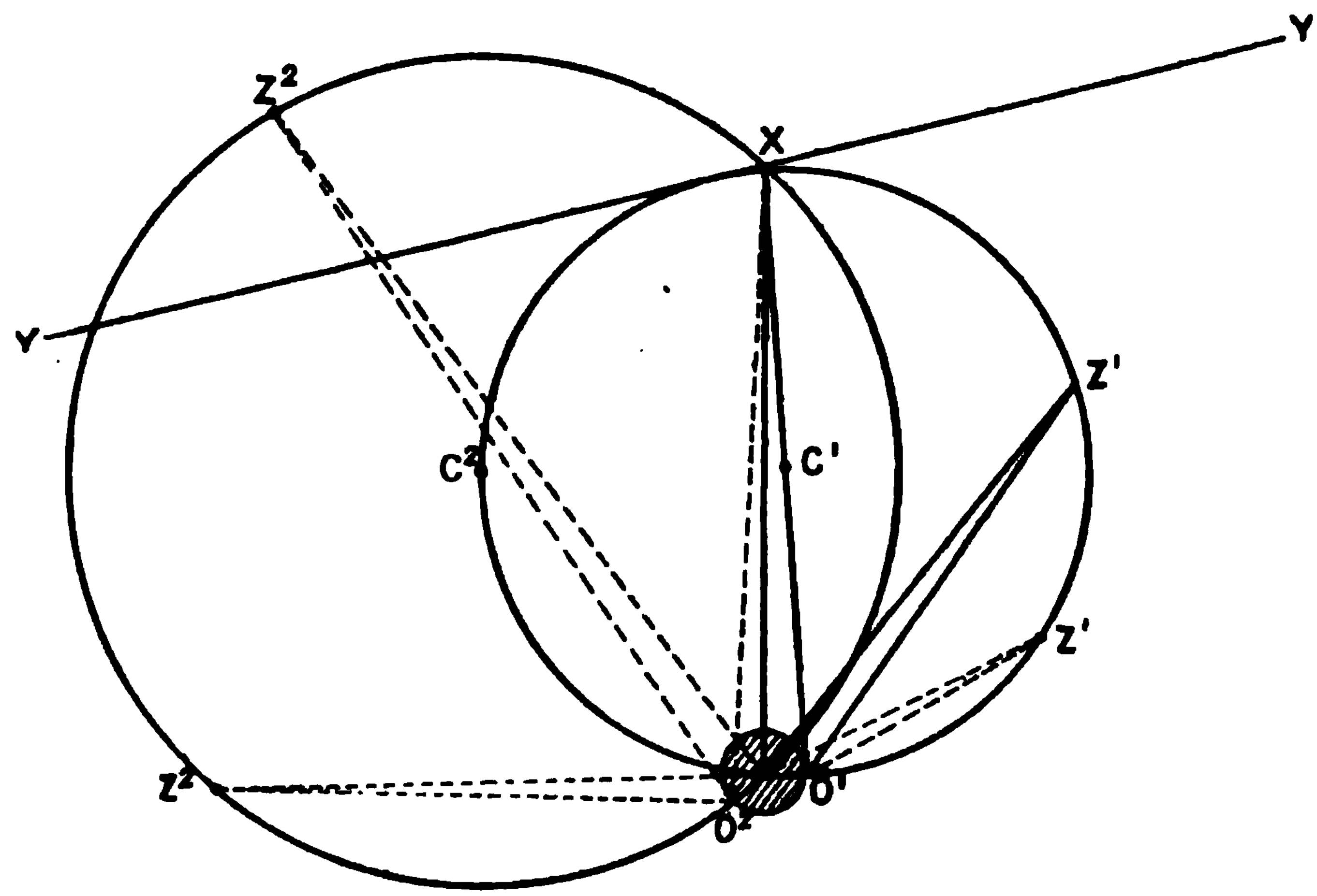


Fig. 255.

Then any point along the arc XZ^1O on either side of X O can be taken as a zero of reference; and there is no error produced in the position of X, by projecting the observed angle from either O or from O^1, for both angles stand on the same arc as Z^1X.

Third Case.—Assume the position O^2, 45° or 135° with O X. With C^2 as centre, and radius C^2X, describe the circle XZ^2O. The circumference of this circle will go through X, O, and O^2.

Any position of Z^2 along that circle will satisfy the condition that there will be no error if the line is projected from either O, or from O^2, through X; for both the angles at O^2 and O stand on the same arc Z^2X.

Thus an infinite number of circles can be drawn for the infinite number of positions in the area shown by the shaded circle; each erroneous position having its own circle of equivalent

zeros; the nearer it is to this circle the better will be the position of Z.

It is not possible to define any particular circle; the only position of the zero which satisfies all cases is, when it is at X—which is impossible: and the only point that can be defined as near *any* circle, is very close to X, the object to which the line of reference is intended; or to O, the position of the observer.

This deduction is specially applicable in the case of 'shooting up' the tangents of a small islet, the zero being its centre or summit; or in the case of finding the line of transit for boat soundings, the angle measured being to the object nearest the transit line laid off.

The only case where the zero could be near the observer is in a boat, close to a fixed object on the shore; or possibly in 'coast-lining.'

APPENDIX III.

Formula for Equation of Equal Altitudes.

Equation of equal altitudes $= \frac{dp}{2} \cdot (\tan l \operatorname{cosec} h - \cot p \cdot \cot h)$, where dp is the change in the polar distance or declination between the sights, and $h = \frac{1}{2}$ the elapsed time; l, the latitude; and p, the polar distance.

If T be the time between the sights = elapsed time = E, then $dp = \text{E} \times$ change of declination; and $\frac{dp}{2} = \frac{\text{E}}{2} \times$ change of P.D. or of declination; and $\frac{\text{E}}{2 \times 15}$ converts it into seconds of time $= \frac{\frac{1}{2}\text{E}}{15}$.

The first part of the equation is known as A; the second part, as B.

A − B is the equation of equal altitudes.

The signs of each term must be closely followed.

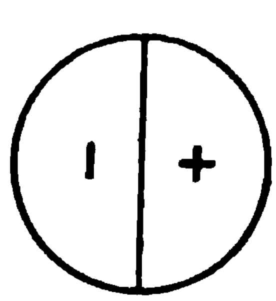


Fig. 256.

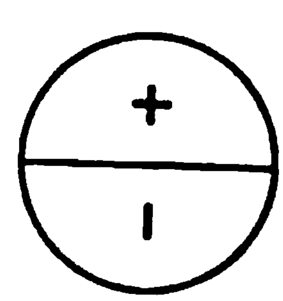


Fig. 257.

Fig. 256 is a sine curve: the right half is +, the left −.
Fig. 257 is a cosine curve: the upper half is +, the lower −.

Referring to the formula: if h is greater than $180°$ (12^h), then the cosec. h is $-$ (see fig. 256), and a cotangent which is $\frac{\cos . h}{\sin h}$ is $\frac{-\text{(fig. 257)}}{-\text{(fig. 256)}} = +$.

If h is $> 90°$ (6^h), then cosec h is $+$, and cot h is $-$.

The same analysis is made with p, the polar distance.

If P.D. is $> 90°$, then cot p is $-$; and it would be better in all cases, to avoid mistakes, if the polar distance is used, instead of the declination.

APPENDIX III. (*a*).

Methods of calculating Equal Altitudes, avoiding any Confusion over the Signs that arise in the Equation usually adopted. Observations for one Limb only are shown.

Lat. 54° 01′ N. Long. 56° 54′ W. 19th to 21st Sept. 1908.

	About IX. 30 A.M., 21st Sept.							About II. 30 P.M., 19th Sept.			
	h	m	s		°	′	″	h	m	s	s
	7	20	28·5		56	50	00	12	22	07·2	35·7
		21	16·5		57	00	00		21	23·5	40
		22	02·0			10	00		20	38·0	40
		22	45·8			20	00		19	53·8	39·6
		23	32·5			30	00		19	07	39·5
		24	15·5			40	00		18	21·2	36·7
		25	00·5			50	00		17	36·0	36·5
mean	7	22	45·9		57	20	00	12	19	52·4	
comparison	1	59	32·8	I.E. +	0	4	56	1	59	42·3	
chron. time	9	22	18·7	2	57	24	56	2	19	34·7	
or 19 days	45	22	18·7		28	42	28				
P.M. sights	2	19	34·7	S.D. ⊙	0	15	57				
sum	47	41	53·4		28	58	25				
diff.	43	02	44·0	R.P.		1	38				
½ sum	23	50	56·7		28	56	47				
½ diff.	21	31	22·0								
				Z.D.	61	03	13				

App. T. mid. date, 20th, $0^h\ 0^m\ 0^s$
long. 3 47 0 W.

G.A.T. 20th, 3 47 0

		Change.	
dec.	1° 09′ 13″ N.	58″·3	58″·31
decreasing	3 41	3·8	½ E 21·52
	1 05 32	60|221·54	15|12498·312
		3·41	$\frac{dp}{30} = 83{\cdot}322$
PX	88 54 28		

eq. time $6^m\ 31^s{\cdot}07$ ·879
\+ 3·32 3·78

− 6 34·4 3·32262

lat. 54° 01′

PZ 35° 59′

In the usual spherical triangle P Z X, find angle P X Z.

log cosec P X	88° 54′ 28″	10·000079
log cosec Z X	61 3 13	10·057957
	27 51 15	
P Z	35 59 0	
log ½ hav.	63 50 15	4·723222
log ½ hav.	8 07 45	3·850529
		8·631787 hav. = 23° 53′ 20″.

$$\text{Equation of equal alt.} = \frac{dp}{30} \cdot \sec \text{dec.} \cdot \cot PXZ.$$
$$- \qquad\qquad +$$

If polar distance west of the meridian is < P.D. east, then *dp* is −.

$$\log \frac{dp}{30}(83{\cdot}322) = 1{\cdot}920759$$
$$\log \sec\ 1^\circ\ 05'\ 32'' = 10{\cdot}000079$$
$$\log \cot\ 23\ \ 53\ \ 20 = 10{\cdot}353688$$

$$2{\cdot}274526 = 188{\cdot}16$$
$$= -3^m\ 8^s{\cdot}2,$$

which is the same as the result attained by the other means of calculating the equation of equal altitudes (see p. 385).

APPENDIX III. (*b*).

Example of finding the Error of Chronometer on M.T.P., or the Longitude, by the 'Mean of Absolutes' taken on the same Day.

21st Sept. 1908. Lat. 54° 01′ N. Long. 56° 54′ W. Error of chronometer at noon, 21st Sept., is 3^h 53^m $19^s.5$ slow on G.M.T. Rate, $5^s.4$ losing.

IX. 30 A.M. — II. 30 P.M.

Alt. ⊙	Int.	Time by Watch A.M.	Time by Watch P.M.	Int.	Alt. ⊙
56° 50′		7^h 20^m $28^s.4$	0^h 33^m $33^s.6$		53° 50′
57 00	48	21 16·4	32 55·0	38·6	54 00
10	45.6	22 02·0	32 14·8	40·2	10
20	43·8	22 45·8	31 35·2	39·6	20
30	46·6	23 32·4	30 55·6	39·6	30
40	43·2	24 15·6	30 16·4	39·2	40
50	45·2	25 00·8	29 35·6	40·8	50
57 20		7| 19 21·4	7| 11 06·2		54 20
	Mean	7 22 45·91	0 31 35·17		

A.M.

h m s	° ′ ″
Watch 7 22 45·91	Alt. 57 20
comp. 1 59 32·8	I.E. + 4 56
chron. 9 22 18·71	2|57 24 56
error 3 55 slow	28 42 28
	⊙ + 15 57
G.M.T. 1 17	
	28 58 25
	R P. 1 38
	28 56 47
	Z.D. 61 03 13
dec. 21st 0° 45′ 46″·3 N.	58·38
− 1 15·3	1·29
0 44 31	60|75·3102
	1′ 15″·3
eq. time 6^m $52^s.13$	·875
1·13	1·29
6 53·26	1·12875

P.M.

h m s	° ′ ″
Watch 0 31 35·17	Alt. 54 20
comp. 1 59 47·6	I.E. + 4 56
chron. 2 31 22·77	2|54 24 56
error 3 55	27 12 28
	⊙ − 15 57
G.M.T. 6 26 23	
	26 56 31
	R.P. 1 44
	26 54 47
	Z.D. 63 05 13
dec. 0° 45′ 46″·3 N.	58·38
6 15·8	6·44
0 39 30·5	60|375·8672
	6′ 15″·8
eq. time 6^m $52^s.13$	·875
5·63	6·44
6 57·76	5·63500

Z^1S and Z^2S are the uncorrected zenith distances.
Z S is the 'true' zenith distance.

(1) . $L = D^1 - ZS$

$$= D^1 - \left(Z^1S \mp \frac{i}{2} \mp \frac{e^1}{2} + r^1\right) - ie \pm \frac{i}{2} \pm \frac{e^1}{2} - r^1 \text{ to altitude}$$

$$= \left[D^1 - \left(Z^1S \mp \frac{i}{2} + r^1\right) \pm \frac{e^1}{2}\right]$$

$$\text{but } L^1 = D^1 - \left(Z^1S \mp \frac{i}{2} + r^1\right)$$

$$\therefore L = L^1 \pm \frac{e^1}{2}$$

(2) . $L^1 = ZS - D^2$

$$= \left(Z^2S \mp \frac{i}{2} \mp \frac{e^2}{2} + r^2\right) - D^2$$

$$= \left[\left(Z^2S \mp \frac{i}{2} + r^2\right) - D^2\right] \mp \frac{e^2}{2}$$

$$\text{but } L^2 = \left(Z^2S \mp \frac{i}{2} + r^2\right) - D^2$$

$$\therefore L = L^2 \mp \frac{e^2}{2}$$

Equation (1) . . $L = L^1 \pm \dfrac{e^1}{2}$

Equation (2) . . $L = L^2 \mp \dfrac{e^2}{2}$

Adding, . . $2L = L^1 + L^2 \pm \left(\dfrac{e^1}{2} \quad \dfrac{e^2}{2}\right)$

(3) . . . . and $L = \dfrac{L^1 + L^2}{2} \pm \dfrac{e^1 - e^2}{4}$

Subtracting, . . $0 = L^1 - L^2 \pm \left(\dfrac{e^1 + e^2}{2}\right)$

(4) . . and $L^2 - L^1 = \pm \dfrac{e^1 + e^1}{2}$

e^1 and e^2 are 'errors of observation,' and they are made up of instrumental errors and errors of observing.

Errors of observing must be taken as being the same in both cases; and the latitude deduced from each set of observations will be given a value depending upon the number and consistency of the observations. Instrumental errors will only be similar if the angles on the sextant are the same (see explanation, p. 14): the total instrumental errors, in most cases, vary directly as the size of the angle. If, then, the altitudes are very nearly alike, then in expression (3),

$$\text{L being} = \frac{L^1 + L^2}{2} \pm \left(\frac{e^1 - e^2}{4}\right), \frac{e^1 - e^2}{4} \text{ will} = 0,$$

and $$L = \frac{L^1 + L^2}{2}$$

In expression (4),

$$L^2 - L^1 = \pm\left(\frac{e^1}{2} + \frac{e^2}{2}\right)$$

If $$e^1 = e^2,$$

then $$L^2 - L^1 = \pm e;$$

e being $\pm$ to observed altitude.

If, which is not probable, the instrument is in '*perfect adjustment*,' then e may be looked upon as an 'error of construction' at the particular angle of the altitude; partly due to 'centring,' partly to an error in the faces of the mirrors, and partly to an error in the graduations.

If the altitudes are not the same, neither will e^1 be $= e^2$, and, except by an empirical valuation, in proportion, say, to the Kew error, the latitude cannot be so well defined.

Moreover, the observing errors will probably not be quite the same in amount owing to refraction.

APPENDIX IV. (*a*).

To find a suitable star to pair.

Example.—In latitude 12° 30′ S., from about 9 P.M.

M.T.P. = star's H.A. + star's R.A. − R.A.M.S.

S.H.A. = 0^h 0^m.

Then M.T.P. + R.A.M.S. = star's R.A.;

and given M.T.P., the time desired to start sight-taking as 9 P.M., then

9^h	0^m	M.T. passage
10	15	R.A.M.S. (from *Almanac*)
19	15	= R.A. star which will pass the meridian at about 9.00 P.M.

In the *Star Catalogue*, 'Vega' has a R.A. of 18^h 33^m and its declination is 38° 41′ N.

R.A.M.S.	10	15
rough M.T.	8	18

And if lat. = 12° 30′ S.

then Z.D. = 51 11 S.

alt. = 38 49, the altitude roughly of Vega; zenith being south, or star bears north.

Required a star, the zenith being north of it, *i.e.* which bears south, which will have an altitude of about 39°.

If alt. is 39°
the Z.D. = 51 N.

and lat. = 12 S.

then declination must be about 63° S.

In the *Catalogue*, γ Pavonis has a declination 65° 50′ S.

Its right ascension = 21^h 17^m
as before, R.A.M.S. = 10 15

hence rough M.T. passage = 9 0

And these two stars will make a pair, the single altitude of each being roughly 39° within a degree or two. Vega will pass the meridian at about 8.10 P.M., and γ Pavonis at about 9.00 P.M.

12° 35′ 18″·7 was the latitude found by Vega.
12° 35′ 38″·5 was that found by γ Pavonis.
} See *Example*, p. 449.

APPENDIX IV. (*b*).

EXAMPLE OF FINDING THE LATITUDE BY EX-MERIDIAN ALTITUDES OF TWO STARS; ONE BEARING NORTH, THE OTHER SOUTH, AND OF ABOUT EQUAL ALTITUDES.

1. *By Vega.*— Zenith north of star; star bears south at Rock Island. Approximate lat. 12° 35′ S., long. 143° 13′ 15″ E., on the 26th August, about VIII. 00 P.M. The comparison of the watch was $9^h\ 45^m\ 58^s$ slow on chronometer. The error of the chronometer was $1^h\ 17^m\ 31^s{\cdot}7$ slow on G.M.T.

First, find the approximate G.M.T. of the star's meridian passage to correct the R.A.M.S.

26th August, R.A.M.S.	10^h	19^m	(see *Nautical Almanac*).
19th August, R.A.S.	18	33	
rough M.T. of mer. passage	8	14	
long. in time	9	33 E.	
G.M.T. of mer. passage	10	41	
i.e., 25th August	22	41	

Secondly, correct the R.A.M.S. for Greenwich time, and find the exact time by watch of the meridian passage.

R.A.M.S., 25th August	10^h	15^m	$53^s{\cdot}7$	
correction for 22 hours	0	3	36·8	(see *N.A.*, p. 550).
„ „ 41 min.	0	0	6·7	
corrected R.A.M.S.	10	19	37·2	
corrected R.A. star	18	33	20·9	
mean time of passage	8	13	43·7	

The watch is slow	9^h	49^m	58^s	on chron. time.
chron. is slow	1	17	32	on G.M.T.
∴ watch is slow	11	7	30	on G.M.T.
G.M.T. is slow on local mean time	9	32	53	(longitude).
watch is slow	8	40	23	on local mean time.
M.T. passage is	8	13	44	
watch time of passage =	11	33	21	

The difference between watch time of each sight and this watch time of passage will give the H.A. at each observation.

Time by Watch.	Hour-angle.	Sid. T. of H.A.	$\frac{\text{Vers. H.A.}}{\text{Sin } 1''}$	Obsd. Alt.
11ʰ 31ᵐ 35ˢ	1ᵐ 46ˢ	1ᵐ 46ˢ	6·1	77° 28′ 30″
32 06	1 15	1 15	3·1	40
32 36	0 45	0 45	1·1	30
33 39	0 18	0 18	·2	50
34 15	0 54	0 54	1·6	50
34 46	1 25	1 25	3·9	30
35 16	1 51	1 51	6·7	20
			7|22·7	7|250
		mean	3·243	77 28 36

Appt. Mer. Alt.			
	77°	28′	50″
I.E. +			35
	77	29	25
	38	44	42
refr.		1	09
	38	43	33
	51	16	27
dec.	38	41	11
app. lat.	12	35	16

Correction to the meridian $= \dfrac{\text{vers. H.A.}}{\sin 1''}$. cos dec. . cos lat. . sec. alt.

Log $\dfrac{\text{vers. H.A.}}{\sin 1''}$ = ·510947
log cos dec. = 9·892406
log cos lat. = 9·989406
log sec alt. = 10·107807

·500566 = 3·166

Mean obs. alt.	77°	28′	36″
I.E. +			35
2|	77	29	11
	38	44	35·5
refr.		1	09·4
true alt.	38	43	26·1

true alt.	38°	43′	26″·1	N.
redn. to mer. +			3·2	
mer. alt.	38	43	29·3	
Z.D.	51	16	20·7	N.
star's dec.	38	41	11·4	S.
latitude =	12	35	19·3	N.

To correct refraction

for 38° 40′ =	1′ 13″
for 4′ =	·2
=	1 12·8
corr. for ther. 73° =	− 4
	1 08·8
corr. for bar. 29″·82 =	+ ·6
corrected refr. =	1 09·4

For the rigid correction to star's R.A. and declination, see pp. 233 and 583 of *Nautical Almanac.* They have been so corrected in the above case, but the computation is not here shown.

The star to pair with Vega was found to be γ Pavonis on the same date at about 11.00 P.M.

R.A.M.S.	10^h	19^m	50^s
corr. for G.M.T.			13
corrected R.A.M.S.	10	20	3
R.A.S.	21	17	42
mean time pass.	10	57	39
watch slow on M.T.	8	40	23
watch time pass.	2	16	56

Watch Time.	H.A.	Sid. Time.	Vers. H.A. / Sin 1″	Obsd. Alt.	Appt. Mer. Alt.
2^h 15^m 07^s	1^m 49^s	The sid.	6·5	73° 30′ 50″	72° 31′ 10″
15 49	1 07	interval	2·4	30 50	
16 17	0 39	is the	·8	31 20	The same
17 02	0 06	same.	0·0	31 10	lat. is used
17 30	0 34		·6	31 20	as with
17 58	1 02		2·1	31 10	Vega
18 25	1 29		4·3	31 10	73° 31′ 10″
					35
			7)16·7	7) 50	
			mean 2·4	73 31 07	73 31 45
					36 45 53
				refr.	1 15
				true alt.	36 44 38

$$\text{Log } \frac{\text{vers. H.A.}}{\sin 1''} \quad \cdot 380211$$

cos dec.	9·611844
cos lat.	9·989406
sec alt.	10·093391
	·074852 = 1″·19

Mean of obsd. alt.	73°	31′	07″	
I.E. +			35	
2	73	31	42	
	36	45	51	
refr.		1	14·6	
	36	44	36·4	
corr. +			1·2	
mer. alt.	36	44	37·6	
Z.D.	53	45	22·4	S.
dec.	65	50	57·2	N.
latitude =	12	35	34·8	N.

Latitude by Vega bearing south	12°	35′	19″·3	N.
latitude by γ Pavonis bearing north	12	35	34·8	
			54·1	
mean latitude	12°	35′	27″	N.

Here the mean is accepted because the number of observations was the same in both cases; and their values are about equal.

The 'error of observation' in the altitude observed $= L^1 - L^2$
$= 15''\cdot5$

that is – to Vega's observed altitude, and – to γ Pavonis: it is the total error, and includes observing error. The observing error may be $-60''$, and the instrumental will be $+45''$; or the first might be $+30''$, the instrumental will then be $-45''$.

By taking a larger number of observations, the mean error is probably reduced; they consequently have a slightly greater value.

APPENDIX V.

Squaring in the Triangulation of a River, or of a Series of Points, by the Astronomical Position obtained at Both Ends.

All the places shown in plate are main stations, possibly standing some distance back from the coast-line of the river or channel.

S and K are assumed to be taken from a previous survey, and their distance apart is 8·3301 miles or 50,384 feet; or this distance may have been deduced from a base measured elsewhere.

From this side, and all the necessary angles, the length of the sides approaching A from S are calculated; the computing is not shown here, but the distances are stated in the triangulation sheet.

It is required to find, through the triangulation, the latitude and longitude of A.

Given lat. S. 11° 55′ 49″ S., long. 143° 13′ 18″ E.

1. From S to H.

The true bearing is S. 38° 23′ 45″ E.; distance, 52009·1 feet; 6048·7 feet = 1 mile.

Find the Mercatorial bearing of H from S, and from this the d. lat. and departure.

For Mercatorial bearing, d. lat., and departure—

Convergency = dist. sin M.B. tan mid. lat.

log dist. =	·934418	lat S =	11° 55′ 49″ S.
sin bearing =	9·793293	½ d. lat.	3 22
tan mid. lat. =	9·327008		
		mid. lat.	11 59 11
log	·054719		

Convergency = 1′·134 = 1′ 08″

T.B. H from S, S.	38°	23′	45″	E.
½ convergency, +		0	34	E.
Merc. bearing, S.	38	24	19	E.
Merc. bearing S from H, N.	38	24	19	W.
½ convergency		0	34	
T.B. of S from H, N.	38	24	53	W.

d. lat. = dist. cos bng.

log. dist. =	·934418
cos bng. =	9·894115
log	·828533
	= 6·738
d. lat.	= 6′ 44″·3

dep. = dist. sin M.B.

log dist. =	·934418
sin bng. =	9·793245
log	·727663
	= 5·3415
dep.	= 5′ 20″·5

To find Mercatorial bearing, d. lat., and departure.

2. From H. to F.

For convergency—

log dist. = 4·994097 (feet).
log 6048·7 = 3·781662

log of miles = 1·212435
sin M.B. = 9·605032
tan mid. lat. = 9·333540

log ·151007
= 1·416
converg. = 1′ 24″·9
½ „ = 0 42·4

lat. S.	11°	55′	49″
d. lat.		6	44
lat. H	12	2	33
½ d. lat.		7	28
mid. lat.	12	10	1

D. Lat. between H. and F.

d. lat. = dist. cos M.B.
log dist. = 1·212435
cos bng. = $\bar{1}$·961611

log 1·174046
= 14·929 =
d. lat. = 14′ 55″·7

dep. = dist. sin bng.
log dist. = 1·212435
log M.B. = 9·604817

log ·817252
= 6·5652
dep. = 6′ 33″·9

T.B. S from H = N.	38°	24′	53″	W.
S H K =	42	36	25	
T.B. K from H = N.	4	11	32	E.
K H R =	46	8	10	
R H F =	105	56	0	
T.B. F from H = N.	156	15	42	E.
= S.	23	44	18	E.
½ convergency		0	42	
Mer. bearing *F from H* = S.	23	45	0	E.
½ convergency		0	42	
true bearing *H from F* = N.	23	45	42	W.

3. From F to A.

H F P =	53°	8′	15″	
P F K =	48	48	30	
X F A =	54	46	45	
T.B. F to A = N.	180	29	12	W.
F to A = S	0	29	12	E

For convergency—

log dist. in ft. $= 5{\cdot}055487$
log $6048{\cdot}7 = 3{\cdot}781662$

$1{\cdot}273825$
log sin M.B. $= 7{\cdot}928111$
tan mid. lat. $= 9{\cdot}339820$

$\bar{2}{\cdot}541756$
$= {\cdot}0348'$
convergency $= 2''{\cdot}09$

lat. H	12°	2′	33″
d. lat.		14	56
lat. F	12	17	29
½ d. lat.		9	23
mid. lat. =	12	26	52

T.B. F to A S. 0° 29′ 12″ E.
½ convergency 0 1

Merc. bng. S. 0 29 13 E.

d. lat. = dist. cos M.B.
log dist. $= 1{\cdot}273825$
cos bng. $= 9{\cdot}999984$

log $1{\cdot}273809$
$= 18{\cdot}785$
d. lat. $= 18'\ 47''{\cdot}1$

dep. = dist. sin bng.
log dist. $= 1{\cdot}273825$
sin bng. $= 9{\cdot}928111$

$\bar{1}{\cdot}201936$
$= {\cdot}159$
dep. $= 9''{\cdot}54$

From above—

	d. lat.	dep.
S to H	6′ 44″·3	5′ 20″·5
H to F	14 55·7	6 33·9
F to A	18 47·1	0 09·5
Total d. lat. . . .	40 27·1	12 03·9

lat. S	11°	55′	49″
d. lat.	0	40	27·1
lat. A	12	36	16·1
½ diff. lat.	0	20	13
mid. lat.	12	6	2

d. long. = dep. sec mid. lat.
log dep. $= 1{\cdot}081347$
180
sec mid. lat. $= 11{\cdot}010030$

log $1{\cdot}091557$
$= 12{\cdot}347$

$$\text{2nd correction} = \frac{\text{d. long. cos}^2\text{ mid. lat.}}{150}$$

log d. long. $= 1{\cdot}091557$
$\cos^2$ mid. lat. $= \left.\begin{matrix}{\cdot}989970 \\ {\cdot}989970\end{matrix}\right\}$

$1{\cdot}071497$
log $150 = 2{\cdot}176091$

$\bar{2}{\cdot}895406 = {\cdot}078$

$12{\cdot}347$
$-\ {\cdot}078$

d. long. $= 12{\cdot}269$
$= 12'\ 16''{\cdot}14$

longitude S	143°	13′	18″
d. long.	0	12	16·1
longitude A	143	25	34·1

Hence by triangulation—

Latitude A 12° 36′ 16″·1 ; longitude 143° 25′ 34″·1.

To find the Mercatorial bearing and distance from S to A by triangulation.

$$\frac{\text{dep.}}{\text{d. lat.}} = \text{tan bearing}$$

log dep. = 1·081527
log d. lat. = 1·606596

log tan 9·474931 = 16° 37′ 12″
bearing of S from A, N. 16 37 12 W.

dist. = d. lat. sec bearing 6048·7
log d. lat. = 1·606596
log sec bng. = 10·018530
log 6048·7 = 3·781662

log 5·406788
= 255145·8 feet.

From observations taken at both S and A—

Astronomically { lat. S 11° 55′ 49″ S. long. S 143° 13′ 18″ E.
lat. A 12 36 09·9 S. long. A 143 25 33·7 E.

d. lat. 40 20·9 = 40′·348 d. long. 12 15·7 = 12′·262

Now dep. = d. long. cos mid. lat., and dep. = d. lat. tan bearing.
∴ d. long. cos mid. lat. = d. lat. tan bng.,
or $\text{tan bng.} = \dfrac{\text{d. long. cos mid. lat.}}{\text{d. lat.}}$

log d. long. = 1·088596
log cos mid. lat. = 9·989970

11·078566
log d. lat. = 1·605822

log tan 9·472744 = 16° 32′ 28″

dist. = d. lat. sec . bng. × 6048·7 ft.
log d. lat = 1·605822
log sec bearing = 10·018355
log 6048·7 = 3·781662

log 5·405839
= 254,588·8 feet.

Bearing by triangulation N. 16° 37′ 12″ W.
„ astronomically N. 16 32 28 W.

error 0 4 44 E.

dist. by triangulation 255,145·8 ft.
„ astronomically 254,588·8 „

difference = 557·0 feet in 40·3 miles

To readjust the distances—

log dist. by triangulation 5·406788
„ astronomically 5·405839

log of cor. $\bar{1}$·999051

The error of bearing is applied to each triangulated bearing.

From S to H—

log S H = 4·716080
log cor. = $\bar{1}$·999051

log 4·714134 = 51,895·6 feet = S H

by triangulation bearing H to S 38° 24′ 19″
and back − 0 4 44

adjusted bearing 38 19 35

d. lat. = dist. cos bearing / 6048·7

log dist. = 4·715134
log cos bng. = 9·894596

4·609727
log 6048·7 = 3·781662

log ·828065 = d. lat. between S and H
= 6′·731 = 6′ 43″·8

d. long. = d. lat. tan bearing sec mid. lat.

log. d. lat. = ·828065
tan bearing = 9·897902
sec mid. lat. = 10·009574

·735541
= 5·439

2nd cor. for d. long. not calculated (see below)
− ·034

d. long. 5′·405
= 5′ 24″·0

To correct H to F—

log H F = 4·994097
log cor. = $\bar{1}$·999051

log 4·993148 = 98,434·7
corrected H F = 98,434·7 feet

by triangulation bearing H to F 23° 45′ 00″
− cor. 0 4 44

corrected T.B. 23 40 16

d. lat. = dist. cos bearing / 6048·7

log. dist. = 6·993148
log cos bearing = 9·961832

4·954980
log 6048·7 = 3·781662

log 1·173318 = 14·904
from H to F corrected d. lat. = 14′ 54″·2

d. long. = d. lat. tan bng. . sec mid. lat.

log d. lat. = 1·173318
tan bearing = 9·641833
sec mid. lat. = 10·009866

·825017 = 6·684

2nd cor. not calculated (see formula below).

2nd cor. d. long. − ·04

= 6·6444
= 6′ 38″·4

To correct F to A—

$\log FA = 5{\cdot}055487$
$-\log \text{cor.} = \bar{1}{\cdot}999051$
$\log 5{\cdot}054538 = 113{,}380{\cdot}5$

by triangulation bearing F to A 0° 29′ 14″
− cor. 0 4 44
corrected bearing 0 24 30

$$\text{d. lat.} = \frac{\text{dist. cos bearing}}{6048{\cdot}7}$$

$\log \text{dist.} = 5{\cdot}055487$
$\log \cos \text{bearing} = 9{\cdot}999989$
$5{\cdot}054527$
$\log 6048{\cdot}7 = 3{\cdot}781662$
$1{\cdot}272865 = 18'{\cdot}745$

from F to A corrected d. lat. = 18′ 44″·7

d. long. = dep. . sec mid. lat.
dep. = d. lat. tan bearing
d. long. = d. lat. tan bng. . sec mid. lat.

$\log \text{d. lat.} = 1{\cdot}272865$
$\log \tan \text{bearing} = \bar{7}{\cdot}852910$
$\log \sec \text{mid. lat.} = 10{\cdot}010229$
$\bar{1}{\cdot}136004 = {\cdot}1367'$

H to F d. long. = 8″·2

there is a 2nd difference of long. not calculated $= \dfrac{\text{d. long. cos mid. lat.}}{150}$
$= -{\cdot}001'$

Readjusting all the latitudes and longitudes—

lat A	12°	36′	09″·9	
d. lat. to F	0	18	44·7	
lat. F	12	17	25·5	+ ·7″
d. lat. to H	0	14	54·2	
lat. H	12	02	31·3	+ ·5″
d. lat. to S	0	6	43·8	
lat. S	11	55	47·5	+ ·3

There is an error left of 1″·5, which can be divided in the proportions of d. lat., *i.e.* 6·7, 14·9, and 18·7 = + ·3″, + ·5″, + ·7″.

long. A	143°	25′	33″·7	
d. long. to F	0	0	8·2	+ ·1″
long. F	143	25	25·5	...25·4
d. long. to H	0	6	38·4	+ 2·8
long. H	143	18	47·1	...44·2
	0	5	24·0	+ 2·2
long. S	143	13	23·1	...18·0

There is an error left of 5·1, which divide proportionally to d. long., *i.e.* ·1, 6·6, and 5·4 = + ·1″, + 2″·8, + 2″·2.

And all the other main stations are computed in the same manner.

APPENDIX VI.

Example of Telegraphic Meridian Distance and of Equal Altitudes taken on the same Day.

One observer is at K, the other at T.

Observer at K, in lat. 15° 27′ 35″ S., long. 145° 15′ E. (obsn. spot), takes the following observations, Wednesday, 16th November.

1st Set. $\overline{\overline{\odot}}$

Altitude.	Int.	Chron. Time A.M.	Chron. Time P.M.	Int.	Sum of secs.
....	....	....	....	....	....
....	....	....	....	....	....
....	....	....	....	....	....
....	....	$1^h\ 52^m\ 20^s{\cdot}0$	$8^h\ 30^m\ 35^s{\cdot}2$	....	....
....	....	....	....	....	....
....	....	....	....	....	....
....	....	....	....	....	....
			Mean of sum of seconds . .		54·273

Watch time A.M.	1^h	52^m	$20^s{\cdot}0$	M.T.A.N.	0^h	0^m	0^s
„ „ P.M.	8	30	35·2	longitude	9	41	1 E.
sum	10	22	54·273 [1]	G.M.T.A.N.	2	19	0
diff.	6	38	15·2		12	0	0
mid. T. by watch (½ sum)	5	11	27·136	G.M.T. 15 Nov.	14	19	0
[1] elapsed time (½ diff.)	3	19	07·6				

				Change at mdt
dec. midnight 15 Nov.	18°	48′	50″	− 37·305
−		1	26	2·317
dec. at $14^h\ 19^m$,	18	47	24	86·435685

		Change
Eq. T. midnight 15 Nov.	− $15^m\ 04^s{\cdot}805$	− ·474
	− ·897	2·317
Eq. T. at $14^h\ 19^m$	− $15^m\ 03^s{\cdot}908$	·897258

[1] Substitute the mean for the sum of seconds.

Equation of equal altitudes (A)

$= \tan \text{lat.} \operatorname{cosec} \frac{1}{2}e \cdot \log c$	$\frac{dp}{15} = c = \frac{\text{change declination} \times \frac{1}{2}e}{15}$
log tan lat. = 9·441801	change = 37·305
log cosec $\frac{1}{2}e$ = 10·117140	3·317
log c = ·916421	
log ·475362 = 2·989	15 | 123·740685
	c = 8·2494

Equation of equal altitudes (B)

$= \tan \text{dec.} \cot \frac{1}{2} \text{elapsed time} \cdot \log c$

log tan dec. = 9·531777
log cot $\frac{1}{2}e$ = 9·927170
log c = ·916421

log ·375368 = 2·373

Referring to the figure, Plate XVI.—

A is − 2^s·989	app. noon	0^h	0^m	0^s
B is + 2·373	eq. T. −		15	3·908
eqn. equal alts. − ·616	M.T.A.N.	11	44	56·092

mid. time by watch	5^h	11^m	27^s·136	
eqn. equal alts. −	0	0	·616	
watch T.A.N.	5	11	26·520	
[1] comparison mean of A.M. and P.M.	8	21	2·541	slow.
chron. T.A.N.	1	32	29·061	fast by one observation of $\odot$ A.M. and P.M.

[1] For comparisons—before A.M.'s 8^h 21^m 04^s·254 1^h 16^m by watch.
after ,, 8 21 03·327 3 18 ,, ,,
0 0 ·927 2 0

In 2 hours the comparison changed ·927, *i.e.* $\frac{\cdot 927}{120}$ per minute.

The A.M. time of 1st set was 1^h 52^m, that is 36 minutes after comparing.

∴ the change in the comparison was $\frac{\cdot 927}{120} \cdot 36 = \cdot 278$;

and hence the comparison at 1^h 52^m was 8^h 21^m 04^s·254
− 0 0 ·278
8 21 03·976

By the same form of deduction, the comparison for the P.M. sights = 8 21 01·107

mean 8 21 02·541 as above.

Results by 1st set of $\overline{\odot}$	1h	32m	29s·061		3rd set of $\overline{\odot}$	1h	32m	28s·933
„ „ 2nd „ $\underline{\odot}$	1	32	28·671		4th „ $\underline{\odot}$	1	32	28·543
mean 1st and 2nd	1	32	28·866		mean	1	32	28·738
„ 3rd and 4th	1	32	28·738					
accepted chron. T.A.N.	1	32	28·802					
from above, M.T.A.N.	11	44	56·092					
error of chron. slow on M.T.A.N.	10	12	27·290					

K was in communication with T by telegraph; and T gave a series of stops, at intervals of 10 seconds, and received the same from K—no allowance was made for travelling over the wire.

	Chronometer Time.		Local Time.		Meridian Distance.
	Sending.	Receiving.	Sending.	Receiving.	
	h m s	h m s	h m s	h m s	h m s
Mean .	1 54 20	1 42 32·28	[1] 11 50 36·37	[2] 11 54 59·57	0 4 23·20
From K to T.					
Mean .	1 47 20	1 59 07·25	11 59 47·49	11 55 23·62	0 4 23·87

[1] The error of T's chronometer being 9h 56m 16s·37 found at T.
[2] „ K's „ „ 10 12 27·29 „ K.

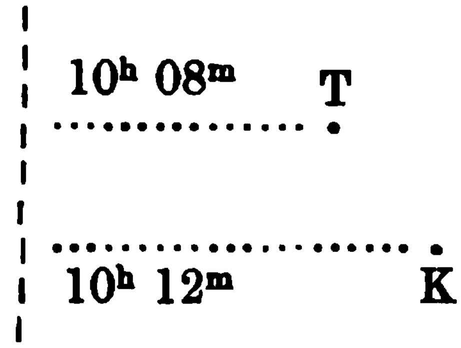
Mean meridian distance accepted, 0h 4m 23s·535.

Or another way, as shown in examples:—

	h m s		h m s
Chron. time of sending from T	1 54 20	sending from K	1 47 20
„ receiving at K	1 42 32·28	receiving at T	1 59 07·25
comparison of chron.	0 11 47·72		11 47·25

This small difference is due to personal errors of both observers.

Error of T chron. found at T	9h	56m	16s·27
mean comparison	0	11	47·485
∴ error of K chron. at T	10	08	03·755 slow
error of K chron. at K	10	12	27·29 slow
mer. dist.	0	4	23·535



Therefore K is east of T.

APPENDIX VII.

To find the length of a 'nautical' mile in a given latitude.

The formula is :—

Minutes of the meridian in feet $= r \,.\, \sin 1' \,.\, \cdot 5280$;

where $r =$ the radius of curvature at the particular latitude

$= e - 2d + 3d \,.\, \sin^2 l$,

$e =$ equatorial radius of the earth

$= 3962{\cdot}8$ statute miles,

$d =$ difference between equatorial and polar radii

$= 13{\cdot}24$ statute miles,

$l =$ latitude.

Example.—In lat. 51° 29′,

$$\log \sin^2 \text{lat.} = 9{\cdot}893444$$
$$2$$
$$\overline{9{\cdot}786888}$$
$$3d = (3 \,.\, 13{\cdot}24) = 39{\cdot}72;\ \log 1{\cdot}599009$$
$$\overline{\log 1{\cdot}385897} = 24{\cdot}326 = 3d \sin^2 \text{lat.}$$

$$e - 2d = 3962{\cdot}8 - 26{\cdot}48 = 3936{\cdot}32$$

$$\begin{array}{r} 3936{\cdot}32 \\ +\quad 24{\cdot}33 \\ \hline 3960{\cdot}65 \end{array}$$

minutes of arc in feet $= 3960{\cdot}65 \,.\, \sin 1' \,.\, 5280$

$$\begin{array}{rl} \log 3960{\cdot}65 & 3{\cdot}597766 \\ \log \sin 1' & 6{\cdot}463726 \\ \log 5280 & 3{\cdot}722634 \\ \hline & 3{\cdot}784126 = 6083{\cdot}1 \text{ feet.} \end{array}$$

APPENDIX VIII.

Table for Use with the 10-foot, 12-foot, 15-foot, or any Length Pole.

L s the distance along the pole in feet, between the centres of the 'discs,' where L is made $= \dfrac{60 \cdot 8}{\text{scale in inches.}}$

The following lengths are the corresponding measurements on the $\frac{1}{4}$-inch scale on the edge of the protractor:—

Angle Observed.	Measurement.	Angle.	Measurement.
0° 30′	4·60	2° 50′	·81
0 40	3·44	3 00	·76
0 50	2·75	3 10	·72
1 00	2·30	3 20	·69
1 10	1·96	3 30	·65
1 20	1·72	3 40	·62
1 30	1·53	3 50	·60
1 40	1·37	4 00	·57
1 50	1·25	4 10	·55
2 00	1·14	4 20	·53
2 10	1·06	4 30	·51
2 20	·98	4 40	·49
2 30	·92	4 50	·47
2 40	·86	5 00	·44

Example.—Suppose the scale is 4·8 inches = 1 mile.

Then $L = \dfrac{60 \cdot 8}{4 \cdot 8} = 12 \cdot 7$ feet = 12 feet 8 inches.

And the discs would be placed 12 feet 8 inches apart; and the above table will give the corresponding measurement for each angle.

APPENDIX IX.

Dip of Shore Horizon.

Correction in Minutes of Arc to be applied to an Angle of Elevation, observed with a Sextant, to a Shore Horizon. Also, Angle between the Horizontal Plane and the Line from the Eye to the Water-line of the Shore, for Rough Level on the Tide-pole.

Miles.	Distance of Shore.	Height of the Eye above Sea-level or of H. W. Line Mark on the Shore.						
		Increasing Uniformly.						
		5	10	15	20	25	30	
	yards.							
	200	28′ 20″	56′ 30″					
	300	21 10	42 30					
	400	14 10	28 20					
¼	500	11 20	22 40	34′	45′	56′	68′	See also table of parallax errors.
	600	9 30	19 00					
	700	8 30	16 50					
	800	7 20	14 20					
	900	6 40	13 00					
½	1000	5 51	11 40	17	22	28	34	
¾	...	4′	8′	12	15	19	23	
1	...	4	6	9	12	15	17	
1¼	...	3	5	7	9	12	14	
1½	...	3	4	6	8	10	12	
2	...	2	3	5	6	8	10	
2½	...	2	3	5	6	7	8	
3	...	2	3	4	5	6	7	
3½	...	2	3	4	5	6	6	
4	...	2	3	4	4	5	6	
5	...	2	3	4	4	5	5	
6	...	2	3	4	4	5	5	

TO

APPENDIX XII.

EXPLANATION OF THE CAUSE OF HIGH WATER SIMULTANEOUSLY AT OPPOSITE SIDES OF THE EARTH.

THE earth and moon are practically undisturbed, as regards their relative mean positions to each other.

In order that they shall remain so, the water on the earth, if raised on one side, does, by the laws of nature, alter its level in other directions, so that the balance shall be maintained.

The quickest and most effectual way of accommodating itself symmetrically is by instantly raising the water on the opposite side of the earth, to nearly the same level as that *directly* affected by external causes; just in the same way as a dancing dervish intuitively or naturally does, when using his arms as counterpoises to preserve his balance, or a man on a tight-rope to maintain his centre of gravity, the rope representing the earth's orbit. The instantaneous natural redistribution of the water is explained in the following manner:—

Now gravity is the force which attracts one thing to another, also one celestial body to another.

The magnitude of the force varies with the volume and density (by which we mean weight), and inversely as the square of the distance the bodies are apart.

In fig. 258, let M be the moon, C the centre of the earth. Join M C and produce it through the earth. Let the length M C

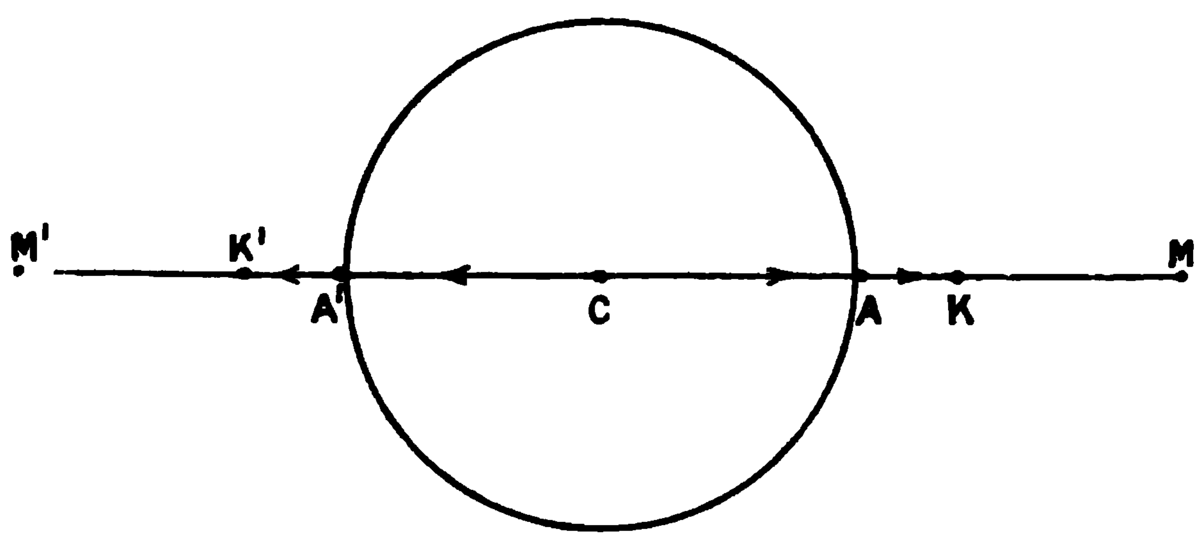


FIG. 258.

represent the force of attraction of the moon on the earth's centre. Then, since the earth's position in her orbit is unaffected by this force, there must be an equal and opposite force acting on the other side of C, represented by M′C: each force is acting in the direction of the arrows shown in fig. 258, and C M′ is the normal force always acting through the earth's centre. In the same figure, let A be a place on the earth's surface; and in this case let the length M A represent the magnitude of the force of attraction of the moon, the direction being from A towards M.

Since the force of gravity varies *inversely* as the square of the distance, the force acting at C will now be represented by line M K, but acting in the direction C to M′ to counterpoise gravity. Obviously the resultant of these forces must be A K acting from A towards K.

Take another place A′, on the opposite side of the earth, and let the length M A′ represent the attraction of the moon in the direction A′ towards M.

Reasoning as before, the force now acting on C will be represented by the length M K′, acting in the direction C to M′.

The resultant force is now A′K′, acting from A′ towards K′. So that in one case the force is from A to K, and at the same instant it is from A′ to K′; and both are vertical forces only.

In fig. 259, let B be a place on the earth's surface. As before, let the length M B represent the force of attraction of the moon

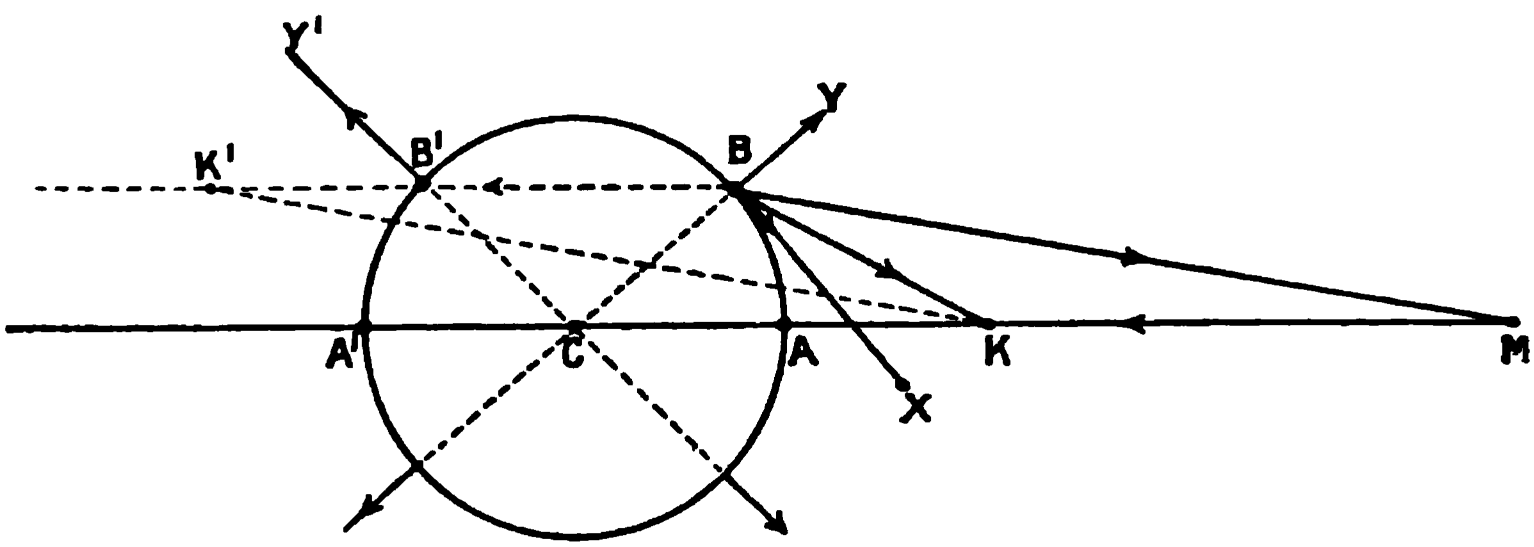


FIG. 259.

on B; and let the length M K represent the force acting on the centre; this force is transferred to B, and shown by B K′, acting in the direction shown by the arrow. Complete the parallelogram M K K′B, and B K represents the resultant of the two forces B M and B K′. This resultant B K is now resolved into two directions: B Y, a vertical force acting at B, from B to Y, and B X, a horizontal force from B to X. B X is the force by which the celestial bodies retain their relative positions; and B Y is the force which, Moxly holds, produces tides. By a similar process of reasoning, it will be seen that, at B′, the vertical component is B′Y′, from B′ to Y′; and simultaneously there will be a high water at B and at B′, though neither will be as high as at A or A′.

As B recedes farther from A, so B Y becomes less and B X greater, until at a position where B C M is 54° 44′ there is no vertical force; and this is the limit of the moon's 'tide cone.'[1]

In both figures the moon is shown as over the equator; but it may be placed in any position not beyond its extreme declination N. or S., showing the same results.

[1] See *Tides Simply Explained*, by Moxly.

APPENDIX XIII.

MINUTES TO BE ADDED TO OR SUBTRACTED FROM THE LUNITIDAL INTERVAL, TO FIND THE ESTABLISHMENT, ACCORDING TO THE MOON'S TRANSIT ON THE DAY IN QUESTION.

(1) *Tide 1½ days old*:—

Hour of the Moon's Transit after Sun.	h. 0	h. 1	h. 2	h. 3	h. 4	h. 5	h. 6	h. 7	h. 8	h. 9	h. 10	h. 11
Correction of the Lunitidal Interval to find the Vulgar Establishment.	m. 0	m. +16	m. +32	m. +47	m. +57	m. +60	m. +47	m. +16	m. −15	m. −28	m. −25	m. −15

(2) *Tide 2½ days old*:—

Hour of Moon's Transit.	h. 0	h. 1	h. 2	h. 3	h. 4	h. 5	h. 6	h. 7	h. 8	h. 9	h. 10	h. 11
Correction of the Lunitidal Interval.	m. 0	m. +15	m. +31	m. +47	m. +62	m. +72	m. +75	m. +62	m. +31	m. 0	m. −13	m. −10

APPENDIX XIV.

(Supplied by Lieutenant P. Griffiths, R.N.R.)

BUOYAGE.

Explanation.

He who surveys a harbour, a river, or an estuary, probably becomes more familiar with every detail in it, both above and below the water, under varying conditions of wind, weather, and of tides, than anyone who later will have but a bowing acquaintance with such a place. And he, more than any other, is the competent authority who can state on the chart the necessary 'leading lines' or 'clearing lines,' and give the 'sailing directions' as well as advise the buoyage. The converse is equally true, that a person not 'surveyingly' familiar with the shoals, tides, and other conditions, such as visibility, background, etc., and who moreover is neither versed in the buoyage system nor master of the necessary knowledge of correctly fixing the position of a buoy by sextant angles, is certainly not the most suitable for that duty.

The system of buoyage has been, more or less, brought to a state of uniformity in the British Isles, and in general principles a similar method is adopted by most foreign countries (see Plate XXII.).

With the exception of the colours of 'outside' buoys, and excepting the top-marks of St George's or St Andrew's cross on inside buoys, both of which are left to the discretion of local authorities, all rivers, harbours, and estuaries in Great Britain are buoyed in conformity with one general system.

A recognised system is not solely for a pilot's edification; probably he is the last who would require any system, but under many conditions it is essential to the navigator. The first conception of buoying was merely to warn the navigator, and to indicate the position of, or to point out, the hidden danger. In fact, in Norway the indication system was so elaborated that the direction of a 'broom' attached to the buoy, whether turned up or down or horizontal, indicated the particular direction of the shoal from the buoy; and in some other countries there are still traces of this idea left. Evidently, no person can navigate a channel so marked, without the aid of a pilot to pick his way between the shoals.

'Outside' buoys were intended to serve this purpose, warning being the first necessity, and in British waters do so still, but by their shape and top-mark merely indicate on which side it should be left when going with the main flood-stream, and

sometimes, by the top-mark only, that it must be kept close aboard on either hand.

To these 'outside' buoys there is no limitation as to the arrangement of the colours allotted to them, only that they shall differ as much as possible, and that they shall be effective.

Evidently all such matter in connection with these buoys must vary with the locality, and they are therefore entirely in the hands of the local authorities.

In the system adopted in the British Isles, after passing the outside warning buoy, those which follow are placed for the purpose of guidance, and not to mark a danger, though incidentally they more or less serve the same end; they therefore occupy positions to enable the pilot, or now the master, to steer the straightest and safest course, and to map out the greatest possible room.

Therefore they are not necessarily close up to the edge, or close to the ends of the banks, though it is not safe to go inside of them.

The committee on the uniform system of buoyage in the year 1883, bearing the system of guidance, and not of warning, in view, resolved that since there were two sides to a channel, and that there were two kinds of buoys, each shaped diametrically opposite to the other—that is, one flat topped (can), the other pointed top (conical)—therefore, one shall mark one side and one the other side.

Owing to the fact that most countries used the conical buoy on the starboard hand, the pointed top (conical) was elected to the starboard hand, entering a channel from seaward, or going in the direction of the main flood-tide stream when in the open.

The flat top (can) marks the port hand under the same conditions.

In order that the flat top shall float at about an equal height as the other, it is not an inverted cone (see Plate XXI.).

There is another shaped buoy—the spherical round-topped—which is practically a mixture between the two. This buoy, now provided with a bilge, floats high out of the water, and was therefore reserved for the ends of 'middle ground' buoys, situated on either hand at the entrance of a channel.

The next resolution was, that to assist in the identification, the starboard-hand buoys shall be all one colour.

This gave two colours for the starboard buoys, Red (R.) or Black (B.); White being inadmissible, and Green being used for another purpose.

So in English waters all Black buoys are allocated to the starboard side of *main* channels, while all Red are for contiguous or *side* channels.

The middle ground buoys shall always be Black with White horizontal bands (B.W.H.S.), or Red with White bands (R.W.H.S.), the stripes being better differentiated when White is narrower

than the Black stripe. Again, as in the case of channel buoys, the Black and White for main channels and the Red for side channels.

Port-hand buoys shall be parti-coloured; but since B.W.H.S. or R.W.H.S. was monopolised by the middle ground buoys, the arrangement of the colours had to be altered for them.

Hence they are either Black and White (B.W. cheq.) or Red and White chequered (R.W. cheq.), or Black and White (B.W.V.S.) or Red and White vertical stripes (R.W.V.S.); in both cases the White occupies less space than the Black.

Also this allowed for B.W.V.S. or B.W. cheq. for main channels, and R.W.V.S. or R.W. cheq. for side channels.

NOTE.—B.W. or R.W. was decided upon because such colours are more easily distinguished than Red and Black, though a Red and Black buoy as a mass is easier 'picked up.'

Therefore Black and Red (B.R.) was the parti-colour in any arrangement allotted to *outside* buoys.

The scheme of *outside* buoys follows the same as that of harbours; that is, all one colour starboard hand with the flood-stream, and parti-coloured on the other.

The next consideration was top-marks. Four top-marks were allocated to the uniform system, being the Globe, Cage, Triangle, and Diamond (see Plate XXI.), merely as an extra mark of distinction for a particular buoy.

The Globe or Cage is used for channel buoys, Globe on the starboard and Cage on the port hand; while the Diamond is on the outer point of a 'middle ground' buoy, and a Triangle is on the inner point; the middle ground being taken as a whole, independent of how often it may be traversed by smaller channels (see Plate XX.: N.W. Shingles, W. Girdler, and E. Shingles).

The crosses (St George and St Andrew) are at the disposal of the local authorities, and generally mark the entrance to, or each at one end of, a side and shallow cross channel (see Swin Spitway and Wallet Spitway), and may be left on either hand.

As regards names or numbers: in some places, the names are painted on the buoys; in others, there is a series of letters or of numbers, the latter sometimes arranged with odd on one side and even on the other; but this is entirely at the option of the local authorities, and is quite apart from the general system.

Mooring buoys are at the discretion of the local authorities.

On Plate XX. a rough outline of the three-fathom patches at entrance of the Thames is given, showing the system of buoyage employed by Trinity House; and at a glance it is seen that the *main* channels buoys are Black and the *subsidiary* ones Red.

Take first the Edinburgh Channel. On leaving the Tongue on the starboard hand are seen conical Black buoys, the one off Knock John being a *special* gas buoy; and between it and the Black Deep

Light-vessel is a wreck-marking vessel which shows by day three balls on a yard 20 feet above the sea, two placed vertically on the side on which vessels may pass in safety, and one on the other, and by night three white fixed lights similarly placed.

On the port hand we have E. Shingles spherical 'middle ground' buoy, with Black and White horizontal stripes, surmounted with staff and a diamond; and it marks the outer end of the Shingles middle ground.

Next we have the can buoys, B.W.V.S. or B.W. cheq., of which the N.E. Shingles is a special gas buoy marking the north-east extension of the Shingles. Then the N.W. Shingles a 'middle ground' buoy, which is a spherical buoy R.W.H.S. surmounted with a triangle; this buoy marks the inner end of the Shingles middle ground, and is Red and White because it also marks the end of the Alexandra Channel (a side channel). Continuing, we come to the W. Girdler, which is a spherical buoy B.W.H.S. surmounted with a triangle. This buoy marks the inner end of the combined Shingles and Girdler middle ground. Now we are back to the Black buoys marking the main channel.

The Shingles and the Girdler combined may be taken as one large middle ground, guarded by the E. Shingles for the outer and W. Girdler for the inner; but, as there is a channel between them, it is necessary that it should have a distinguishing colour, which is Red.

Again, suppose on leaving the Tongue on starboard hand, we take the Princess Channel; we have Black buoys on both hands marking the channel right up to the W. Girdler, where the two tracks meet, and continue with Black on either hand. But should we branch off, for instance, up the Queen's Channel or Alexandra Channel, we immediately find a change of colour, and the buoys in these subsidiary channels are painted Red.

In addition to the foregoing, there are numerous distinguishing marks used locally, especially when the same type of buoys are placed close together; such as the E. Shingles and E. Tongue, the former being surmounted by one diamond, the latter by two diamonds, but both indicate the outer ends of middle grounds.

United States and Canada.

In the United States and Canada the star-hand buoys are Red conical or spar; the port-hand, Black can and spar.

Mid-channel buoys are conical with Black and White vertical stripes, and must be passed close to.

Isolated danger or obstruction has a conical buoy with Black and Red horizontal stripes: it may be passed on either hand. Perches with balls and cages, etc., will, when placed on buoys, be at turning-points, the colour and number indicating on what side

they are to be passed. Quarantine buoys are painted yellow. If buoys are lettered or numbered, even numbers from seaward are painted in White on starboard, and odd numbers on port hand.

France.

The starboard-hand buoys are Red, and conical in shape, with top-mark a Red triangle.

On the port hand, Black conical buoys, with top-mark a Black cylinder.

Dangers and obstructions have conical buoys with horizontal stripes, Red and Black, top-mark a globe.

Outer middle grounds have conical buoys with Black and White horizontal stripes, with top-mark a Black diamond.

Inner middle grounds have conical buoys with horizontal stripes, Red and White, with top-mark two Red inverted cones.

Germany.

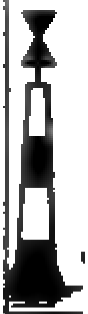
In Germany the colour of can and conical buoys is much the same as in the British Isles, but the reverse in shape, that is, can on the star hand and conical on the port hand.

Like the United States, they have a mid-channel buoy, spherical in shape, with Red and Black stripes.

The outer and inner ends of middle grounds are also spherical, with Black and Red horizontal stripes, each surmounted with a cross.

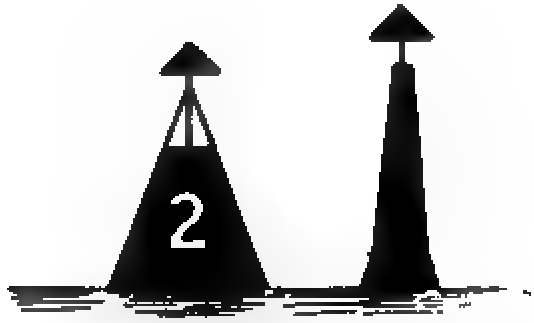
Holland and Belgium.

The Dutch and Belgian is considered an excellent system, and as regards marking the channels is much the same as the British, as seen on comparison; but, in addition to the above, distinguishing buoys are used for outside, of which several are shown in Plate XXII.; but they have no special colour or position assigned to them—they are merely a guide. The Dutch charts show their buoyage system very plainly.



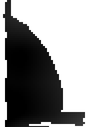
D OF
OUND.

Star.



CONICAL & SPAR.
RED.
TOP-MARK: CONICAL.

Letters and numbers, if any, are in WHITE; EVEN NUMBERS, commencing from seaward.



OF
UND.

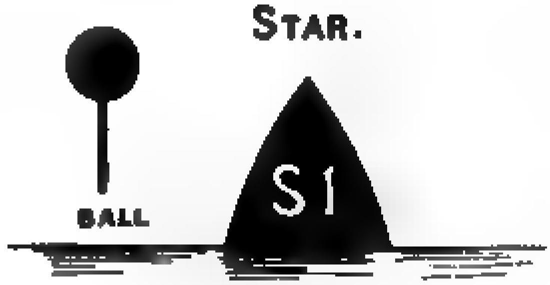
CAN & SPAR.
RED.

Letters or numbers are painted WHITE, commencing from seaward.

; EAST edge ; WEST edge ; on shoal .

ARK

Star.



CONICAL.
RED.

Buoys in sea channels are numbered consecutively (from seaward) and marked in WHITE by the first letter in name of channel.

sed on

CONSTRUCTION OF BUOYS.

Buoys are now constructed of steel plates riveted together, although some of the old wooden ones are still in use; but the latest pattern is that of forged steel, which has the advantage of being lighter, therefore they are more buoyant and there is less risk of leakage through the rivets in case of collision. They are also divided into compartments, and have ballast chambers. A well-constructed buoy, unless cut into, will stand a considerable amount of battering and still remain water-tight, especially if of the forged pattern; and their shape under water depends entirely on their intended use. They are moored with a flat sinker as in fig. 260, varying in weight according to the size of the buoy.

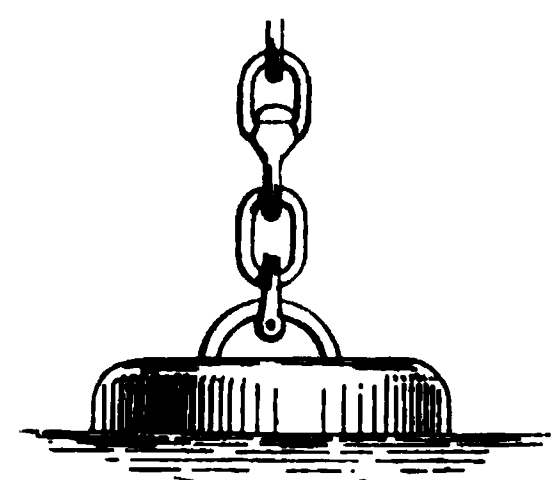


FIG. 260.

In describing the special buoys it would be well to take them singly. First take the automatic gas buoy. Any buoy may be fitted with it, and if required may also have a bell or whistle, and there are different systems in use. The buoy is fitted with a reservoir, and is charged with gasolene sufficient to burn from two to three months without recharging providing the coupling and valve remains tight; but they are rather troublesome. The lamp burns the oil without the intervention of a wick, coming from the reservoir through a special valve; and the alternating or occulting character is given by the rotation of a screen by the action of the heat of the flame.

The light is generally a flash or occulting; the former showing a single flash, with a duration of darkness greater than that of the light. The latter is a continuous light, with at regular intervals one sudden total eclipse, the duration of the light being greater than that of the darkness.

Acetylene gas buoys have been experimented with, but are not satisfactory or reliable.

Here it may be well to remind the navigator that the lights are liable to be obscured by the deposit of salt on the lantern, or the mechanism failing, and that buoys should be regarded only as secondary aids to navigation.

Vessels should be navigated by bearings and angles of fixed shore objects, as the buoys cannot be depended on, especially when placed in exposed positions, where they are liable to drift.

Also, with regard to automatic sounding signals, submarine and above water, that there are areas of silence in different directions and at different distances even when in close proximity; therefore the navigator should not assume that he is out of distance because he fails to hear the sound, or if he hears it faintly that he is a good distance from it, or *vice versa.*

The accompanying sketch (fig. 261) is that of a bell buoy used

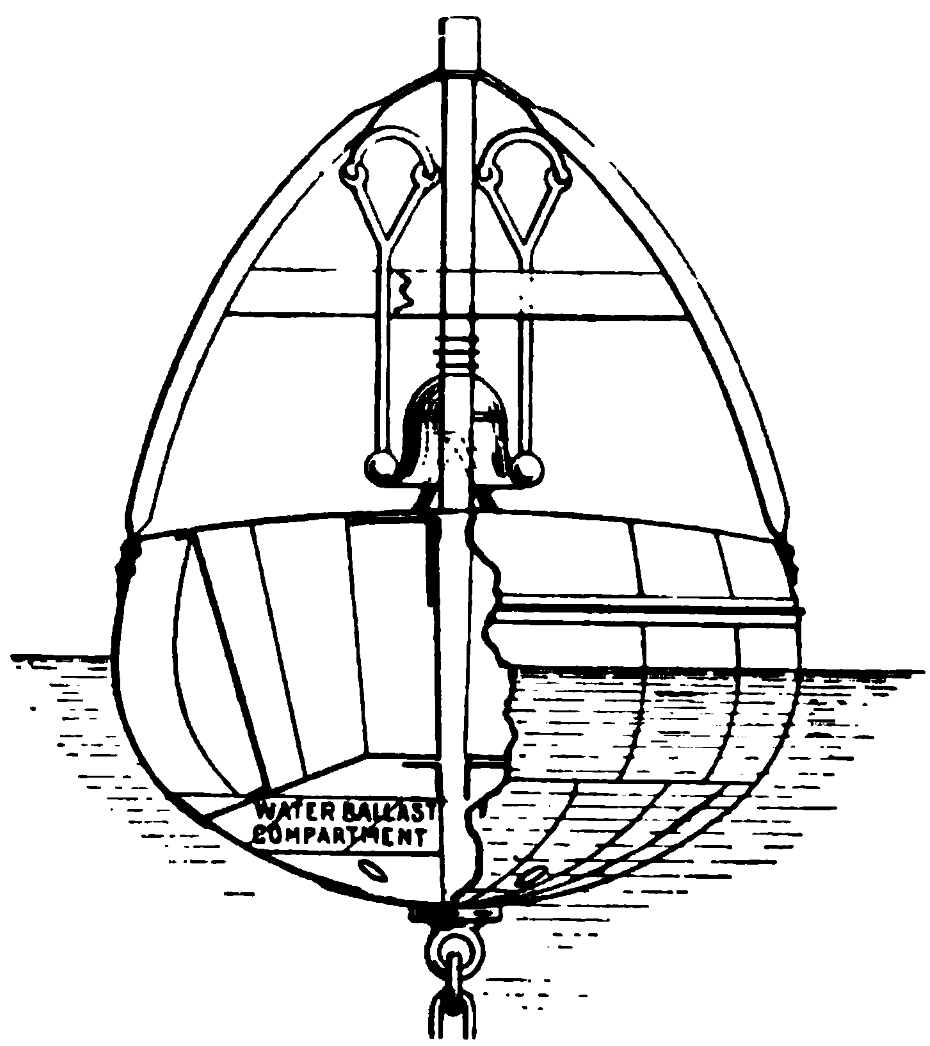


FIG. 261.

for marking outlying dangers or approaches; it works automatically. The bell is secured to the top of the buoy, which has a skeleton framework supporting four loosely hanging hammers, so that whatever angle the buoy takes up, one of the hammers would strike.

The whistle buoy, also automatic, is considered superior to the bell buoy. It consists of a buoy traversed by a long steel tube, the lower end of which is open, the upper end being closed, and to which is fitted the whistle and a non-return valve.

As the buoy bobs up and down, it causes the water in the tube to rise and fall accordingly; when it falls, the space in the tube

is filled with air entering through the non-return valve, which, on being compressed by the rise of the water, forces its way through the whistle, causing it to sound. This buoy is moored by a

Fig. 262.

chain to a band secured round the tube, and is fitted with a rudder opposite to the chain, to guide the tube clear of the mooring chain in a tideway.

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